A Tables for universal relations for quadrupole ϕ-mode.

We here present tables for the average error of all the universal relations we tested for the quadrupole ϕ-modes. The average error $\bar{\epsilon}$ is given by

$$\bar{\epsilon} = \frac{1}{N}\sum_{k=1}^{N}\left|1 - \frac{F_k}{F_{\mathrm{fit},k}}\right|,$$

where N is the total number of points for each theory.

Table 1: Average error $\bar{\epsilon}$ in % for universal relations for quadrupole ϕ-mode when plotting versus compactness C (left) and versus generalized compactness η (right).

	GR	**massless**
$M\omega_R/c$	0.1	0.5
$M/(c\tau)$	1.5	1.0
ω_R/ω_0	0.1	0.6
$\omega_R/\hat{\omega}_0$	1.3	2.9
$\tau\omega_0$	1.6	1.0
$\tau\hat{\omega}_0$	2.9	3.7
$R\omega_R/c$	0.1	0.5
$R\omega_R/(cC)$	0.1	0.6
$R\omega_R/(cC^2)$	0.6	1.0
$R\omega_R/(cC^3)$	2.4	3.9
$R/(c\tau)$	1.5	1.0
$R/(c\tau C)$	1.5	1.0
$R/(c\tau C^2)$	1.8	1.6
$R/(c\tau C^3)$	4.8	5.4
$M\tau\omega_R^2/c$	1.8	0.3
$R\tau\omega_R^2/c$	1.7	0.3
$\omega_R\tau$	1.6	0.5

	GR	**massless**
$M\omega_R/c$	1.2	1.7
$M/(c\tau)$	2.1	1.8
ω_R/ω_0	0.7	0.5
$\omega_R/\hat{\omega}_0$	1.1	1.7
$\tau\omega_0$	0.5	0.5
$\tau\hat{\omega}_0$	2.1	1.8
$R\omega_R/c$	0.1	0.4
$R\omega_R/(cC)$	1.4	1.1
$R\omega_R/(cC^2)$	2.7	2.7
$R\omega_R/(cC^3)$	4.7	5.2
$R/(c\tau)$	0.9	0.6
$R/(c\tau C)$	0.8	1.1
$R/(c\tau C^2)$	2.2	2.7
$R/(c\tau C^3)$	5.3	6.2
$M\tau\omega_R^2/c$	0.8	1.7
$R\tau\omega_R^2/c$	1.1	0.4
$\omega_R\tau$	1.0	0.3
$\hat{R}\omega_R/c$	1.2	1.7
$\hat{R}\omega_R/(c\eta)$	1.1	1.7
$\hat{R}\omega_R/(c\eta^2)$	1.1	1.7
$\hat{R}\omega_R/(c\eta^3)$	1.1	1.6
$\hat{R}/(c\tau)$	2.1	1.8
$\hat{R}/(c\tau\eta)$	2.0	1.8
$\hat{R}/(c\tau\eta^2)$	2.0	1.8
$\hat{R}/(c\tau\eta^3)$	1.8	1.7

B Tables for universal relations for dipole ϕ-mode.

We here present tables for the average error of all the universal relations we tested for the dipole ϕ-modes.

Table 2: Average error $\bar{\epsilon}$ in % for universal relations for dipole ϕ-mode when plotting versus compactness C (left) and versus generalized compactness η (right).

	GR	**massless**
$M\omega_R/c$	0.4	0.3
$M/(c\tau)$	1.0	1.1
ω_R/ω_0	0.4	0.3
$\omega_R/\hat{\omega}_0$	1.9	1.7
$\tau\omega_0$	1.0	1.1
$\tau\hat{\omega}_0$	1.8	1.5
$R\omega_R/c$	0.4	0.3
$R\omega_R/(cC)$	0.5	0.3
$R\omega_R/(cC^2)$	0.9	0.9
$R\omega_R/(cC^3)$	4.0	4.5
$R/(c\tau)$	1.0	1.1
$R/(c\tau C)$	1.0	1.1
$R/(c\tau C^2)$	1.6	1.7
$R/(c\tau C^3)$	5.1	5.5
$M\tau\omega_R^2/c$	1.5	1.5
$R\tau\omega_R^2/c$	1.5	1.5
$\omega_R\tau$	1.2	1.3

	GR	**massless**
$M\omega_R/c$	1.8	1.6
$M/(c\tau)$	1.8	1.6
ω_R/ω_0	0.4	0.5
$\omega_R/\hat{\omega}_0$	1.8	1.6
$\tau\omega_0$	1.2	1.3
$\tau\hat{\omega}_0$	1.8	1.6
$R\omega_R/c$	0.5	0.4
$R\omega_R/(cC)$	1.1	1.1
$R\omega_R/(cC^2)$	2.5	2.6
$R\omega_R/(cC^3)$	4.3	4.8
$R/(c\tau)$	0.9	0.9
$R/(c\tau C)$	1.8	2.0
$R/(c\tau C^2)$	3.3	3.5
$R/(c\tau C^3)$	5.3	5.8
$M\tau\omega_R^2/c$	2.6	2.6
$R\tau\omega_R^2/c$	1.4	1.4
$\omega_R\tau$	1.1	1.1
$\hat{R}\omega_R/c$	1.8	1.6
$\hat{R}\omega_R/(c\eta)$	1.8	1.6
$\hat{R}\omega_R/(c\eta^2)$	1.8	1.6
$\hat{R}\omega_R/(c\eta^3)$	1.8	1.6
$\hat{R}/(c\tau)$	1.8	1.6
$\hat{R}/(c\tau\eta)$	1.8	1.6
$\hat{R}/(c\tau\eta^2)$	1.8	1.6
$\hat{R}/(c\tau\eta^3)$	1.9	1.7

C Tables for universal relations for radial ϕ-mode.

We here present tables for the average error of all the universal relations we tested for the radial ϕ-modes.

Table 3: Average error $\bar{\epsilon}$ in % for universal relations for radial ϕ-mode when plotting versus compactness C (left) and versus generalized compactness η (right). The values in brackets indicate the mean error for a fit up to $M/R = 0.24$.

	GR	**massless**
$M\omega_R/c$	0.04	0.9 [0.03]
$M/(c\tau)$	0.02	0.4 [0.01]
ω_R/ω_0	0.05	0.7 [0.03]
$\omega_R/\hat{\omega}_0$	1.5	1.2 [0.7]
$\tau\omega_0$	0.02	0.4 [0.01]
$\tau\hat{\omega}_0$	1.5	1.7 [0.7]
$R\omega_R/c$	0.04	0.7 [0.03]
$R\omega_R/(cC)$	0.1	0.7 [0.03]
$R\omega_R/(cC^2)$	0.6	1.4 [0.1]
$R\omega_R/(cC^3)$	2.8	4.9 [0.4]
$R/(c\tau)$	0.02	0.4 [0.01]
$R/(c\tau C)$	0.1	0.3 [0.02]
$R/(c\tau C^2)$	0.8	1.2 [0.1]
$R/(c\tau C^3)$	3.2	5.6 [0.5]
$M\tau\omega_R^2/c$	0.1	2.5 [0.07]
$R\tau\omega_R^2/c$	0.1	1.9 [0.07]
$\omega_R\tau$	0.06	1.1 [0.04]

	GR	**massless**
$M\omega_R/c$	1.4	1.1
$M/(c\tau)$	1.2	1.5
ω_R/ω_0	0.7	1.2
$\omega_R/\hat{\omega}_0$	1.4	1.1
$\tau\omega_0$	0.8	0.7
$\tau\hat{\omega}_0$	1.2	1.4
$R\omega_R/c$	0.06	0.7
$R\omega_R/(cC)$	1.3	1.8
$R\omega_R/(cC^2)$	2.8	3.4
$R\omega_R/(cC^3)$	4.8	6.3
$R/(c\tau)$	0.2	0.3
$R/(c\tau C)$	1.5	1.3
$R/(c\tau C^2)$	3.1	3.0
$R/(c\tau C^3)$	5.2	6.3
$M\tau\omega_R^2/c$	1.6	1.9
$R\tau\omega_R^2/c$	0.3	1.8
$\omega_R\tau$	0.2	1.0
$\hat{R}\omega_R/c$	1.4	1.1
$\hat{R}\omega_R/(c\eta)$	1.4	1.1
$\hat{R}\omega_R/(c\eta^2)$	1.4	1.1
$\hat{R}\omega_R/(c\eta^3)$	1.3	1.2
$\hat{R}/(c\tau)$	1.2	1.4
$\hat{R}/(c\tau\eta)$	1.2	1.3
$\hat{R}/(c\tau\eta^2)$	1.2	1.3
$\hat{R}/(c\tau\eta^3)$	1.2	1.1