Supplementary Material

**10Be cosmogenic nuclide dating pretreatment and calculation**

All samples were crushed into 600-850 μm and carbonate eliminated by HCl (10%) and repeatedly purified by HF: HNO3 solution until the Al concentration was lower than 100ppm. The purified quartz was spiked with a Be carrier and then dissolved with concentrated HF acid. Following anion and cation exchange in a series of column chromatography procedures, the sample solution was neutralized by NH4OH to precipitate Be(OH)2. The precipitations were oxidized to BeO in a furnace, then pressed into Cu target holders after Nb powder was added. The 10Be/9Be measurements were made in the 5MV AMS Lab in SUERC, the value of the 10Be/9Be blank yielded 1.60-2.64×10-14 and is 1-2 magnitude lower than those values of samples. Ratios were normalized to the standard NIST\_27900.

Under conditions of constant production rate and constant erosion rate, cosmogenic nuclide concentration of surface that is exposed at time *t* can be expressed as (Lal, 1991; Balco et al., 2008; Braucher et al., 2009)

$N\_{z\left(t\right)}=P\_{n,0}ⅇ^{-\frac{ρ\_{z}}{Λ\_{n}}}\left(\frac{1-ⅇ^{\left(\frac{ρ\_{r}}{Λ\_{n}}+λ\right)t}}{\frac{ρr}{Λ\_{n}}+λ}\right)+P\_{m1,0}ⅇ^{-\frac{ρ\_{z}}{Λm1}}\left(\frac{1-ⅇ^{\left(\frac{ρ\_{r}}{Λ\_{m1}}+λ\right)t}}{\frac{ρr}{Λ\_{m1}}+λ}\right)+P\_{m2,0}ⅇ^{-\frac{ρ\_{z}}{Λ\_{m2}}}\left(\frac{1-ⅇ^{\left(\frac{ρ\_{r}}{Λ\_{m2}}+λ\right)t}}{\frac{ρr}{Λ\_{m2}}+λ}\right)$  (1)

where 𝑃𝑛,0, 𝑃𝑚1,0, and 𝑃𝑚2,0 are the surface production rate induced by nucleons, negative muons, and fast muons; 𝛬𝑛, 𝛬𝑚1, 𝑎𝑛𝑑 𝛬𝑚2 are the attenuation lengths of the nucleons and muons (negative and fast), respectively; *z* is the surface depth; 𝜆 is the decay constant, and *r* is a constant erosion rate. Based on equation 1, the production of cosmogenic nuclides may be simplified into two major components: the production rate at specific depth (*Pz*), and the effective exposure age of the site (*Te*), which is the time that is required to accumulate concentration *Nz* at production rate *Pz*without erosion and radioactive decay (Wang and Oskin, 2021):

$N\_{z\left(t\right)}=\sum\_{}^{}P\_{zi}T\_{ei}$ (2a)

Where $P\_{zi}= P\_{i,0}ⅇ^{-\frac{ρ\_{z}}{Λ\_{i}}}$, $T\_{ei} =\left(\frac{1-ⅇ^{\left(\frac{ρ\_{r}}{Λ\_{i}}+λ\right)t}}{\frac{ρr}{Λ\_{i}}+λ}\right)$, *i* = *n*, *m1*, *m2* (2a)

The 10Be concentration (*C*) measured from a suite of samples includes the in-situ produced concentration (*Nz*), and the inherited concentration (𝐶𝑖𝑛ℎ)

$C=\sum\_{}^{}P\_{zi}T\_{ei}+ C\_{inh}$ (3)

In realistic cases, estimate total eroded thickness (*D*) from field evidence could be more straightforward than to obtain an erosion rate. With eroded thickness, the effective age of each pathway may be rewritten as:

$T\_{ei} =\left(\frac{1-ⅇ^{\left(\frac{ρ\_{D}}{Λ\_{i}}+λt\right)}}{\frac{ρD}{Λ\_{i}t}+λ}\right)$, *i* = *n*, *m1*, *m2* (4)

We rewrite the effective age related to muons (*Tem*) into a fraction (g) of the effective age related to nucleons (*Ten*), detailed derivation is referred to Wang and Oskin (2021) The fraction *g* can be approximated solely from knowledge of the eroded thickness (*D*):

$g\_{i}= \frac{T\_{emi}}{T\_{en}} ≈ ⅇ^{-\frac{1}{2}\left(\frac{ρ\_{D}}{Λm\_{i}}-\frac{ρ\_{D}}{Λ\_{n}}\right)+\frac{1}{24}\left[\left(\left(\frac{ρ\_{D}}{Λm\_{i}}\right)^{2}-\left(\frac{ρ\_{D}}{Λ\_{n}}\right)^{2}\right)\right]}$, *i* =1, 2 (5)

Bring *gi* into equation 3:

$C\left(z\right)=P\_{zn}T\_{en}+P\_{zm1}g\_{1}T\_{em1}+P\_{zm2}g\_{2}T\_{em2}+C\_{inh}= P\_{ze}T\_{en}+C\_{inh}$ (6)

$P\_{ze}= P\_{zn}+P\_{zm1}g\_{1}+P\_{zm2}g\_{2}$

Using equation 6, *Ten* and *Cinh* can be found by applying least squares linear regression with known production rates, eroded thickness, and sample concentrations. To estimate the exposure age, we need to find the solution for

$f\left(t\right)= \left(\frac{1-ⅇ^{\left(\frac{ρ\_{D}}{Λ\_{i}}+λt\right)}}{\frac{ρD}{Λ\_{i}t}+λ}\right)-T\_{en}=0$ (7)

t may be found in the derivative of equation 7 iteratively by applying the Newton’s method:

$f'\left(t\right)=-λⅇ^{\left(\frac{ρ\_{D}}{Λ\_{i}}\right)D-λt}-\frac{ρDT\_{en}}{Λ\_{i}t^{2}}$ (8)

The exposure age can be iterated from

$t\_{n+1}= t\_{n}-\frac{f\left(t\right)}{f'\left(t\right)}$ (9)

With initial *t0* = *Ten*

**Reference**

1. Balco, G., Stone, J. O., Lifton, N. A., and Dunai, T. J. (2008). A complete and easily accessible means of calculating surface exposure ages or erosion rates from 10Be and 26Al measurements. *Quaternary Geochronology* 3, 174–195. doi: 10.1016/j.quageo.2007.12.001.
2. Braucher, R., Del Castillo, P., Siame, L., Hidy, A. J., and Bourlés, D. L. (2009). Determination of both exposure time and denudation rate from an in situ-produced 10Be depth profile: A mathematical proof of uniqueness. Model sensitivity and applications to natural cases. *Quaternary Geochronology* 4, 56–67. doi: 10.1016/j.quageo.2008.06.001.
3. Lal, D. (1991). Cosmic ray labeling of erosion surfaces: in situ nuclide production rates and erosion models. *Earth and Planetary Science Letters* 104, 424–439. doi: 10.1016/0012-821X(91)90220-C.
4. Wang, Y., and Oskin, M. E. (2021). Combined linear regression and Monte Carlo approach to modelling exposure age depth profiles. *Geochronology Discussions*, 1–25.