**Supplementary Material**

In a set of CTD data, the conductivity varies with depth, let's say (c1, d1), (c2, d2), …, (cm, dm). Select some depth points (x1, x2, …, xn) on the density curve as independent points, n<<m, and the corresponding conductivity is set to y1, y2,…,yn. Connect y1, y2,…,yn with cubic spline interpolation, and the expression is:

$y\left(x\right)=\sum\_{i=1}^{n}f\_{x,i}y\_{i} x\_{i}\leq x\leq x\_{n}$ （2.1）

y(x) is the fitting formula of conductivity, which is a linear expression about y1, y2,…,yn, and $f\_{x,i}$ are interpolation coefficients. Using the least squares method, yi can be solved. And then get the fitting formula.

The interpolation coefficients $f\_{x,i}$ are obtained as follows:

Assuming that the cubic spline interpolation function *y(x)* exists on the interval $[x\_{1}, x\_{n}]$. $m\_{i }(i=1,2,…,n)$represent *y(x)*’s first derivatives at *xi*, and the curve satisfies the following condition on each interval $[x\_{i, } x\_{i+1}]$:

$y\left(x\_{i }\right)=y\_{i }$ , $y\left(x\_{i+1 }\right)=y\_{i+1}$

$y'\left(x\_{i }\right)=m\_{i }$ , $y'\left(x\_{i+1 }\right)=m\_{i+1}$

Therefore, we can use Hermite interpolation polynomial to write the expression of *y(x)* on interval $[x\_{i, } x\_{i+1}]$:

$$y\left(x\right)=\left(\frac{x-x\_{i+1}}{h\_{i}}\right)^{2}\left(1+2\frac{x-x\_{i}}{h\_{i}}\right)y\_{i}+\left(\frac{x-x\_{i}}{h\_{i}}\right)^{2}\left(1-2\frac{x-x\_{i+1}}{h\_{i}}\right)y\_{i+1}$$

$$+\left(\frac{x-x\_{i+1}}{h\_{i}}\right)^{2}\left(x-x\_{i}\right)m\_{i}+\left(\frac{x-x\_{i}}{h\_{i}}\right)^{2}\left(x-x\_{i+1}\right)m\_{i+1}$$

 （2.2）

where, $h\_{i}=x\_{i+1}-x\_{i}$ , $ x\_{i}\leq x\leq x\_{i+1}$

The second derivative of Eq. (2.2) is

$$y^{''}\left(x\right)=\left(\frac{6}{h\_{i}^{2}}-\frac{12}{h\_{i}^{3}}\left(x\_{i+1}-x\right)\right)y\_{i}+\left(\frac{6}{h\_{i}^{2}}-\frac{12}{h\_{i}^{3}}\left(x-x\_{i}\right)\right)y\_{i+1}$$

$$+\left(\frac{2}{h\_{i}}-\frac{6}{h\_{i}^{2}}\left(x\_{i+1}-x\right)\right)m\_{i}-\left(\frac{2}{h\_{i}}-\frac{6}{h\_{i}^{2}}\left(x-x\_{i}\right)\right)m\_{i+1}$$

The second derivative of y(x) at xi point is

$y^{''}\left(x\_{i}^{+}\right)=-\frac{6(y\_{i}-y\_{i+1})}{h\_{i}^{2}}-\frac{4m\_{i}+2m\_{i+1}}{h\_{i}}$ (2.3)

$y^{''}\left(x\_{i}^{-}\right)=-\frac{6(y\_{i}-y\_{i-1})}{h\_{i-1}^{2}}+\frac{4m\_{i}+2m\_{i-1}}{h\_{i-1}}$ (2.4)

Continuity of the second derivative of *y(x)* at point *xi* requires that

$y^{''}\left(x\_{i}^{+}\right)=y^{''}\left(x\_{i}^{-}\right) , (i=2,3,…,K-1)$ (2.5)

Eq. (2.3) can be simplified as:

$\left(1-α\_{i}\right)m\_{i-1}+2m\_{i}+α\_{i}m\_{i+1}=β\_{i}$ (2.6)

where, $α\_{i}=\frac{h\_{i-1}}{h\_{i-1}+h\_{i}} $,$ β\_{i}=3\left[\frac{1-α\_{i}}{h\_{i-1}}\left(y\_{i}-y\_{i-1}\right)+\frac{α\_{i}}{h\_{i}}(y\_{i+1}-y\_{i})\right]$

Additional boundary conditions are required to determine cubic spline functions. We adopt common natural boundary conditions: $y^{''}\left(x\_{1}\right)=0 , y^{''}\left(x\_{n}\right)=0$. Substitute $x\_{1}$ and $x\_{n}$ into Eq. (2.3) and Eq. (2.4) respectively, we can get that

$2m\_{1}+m\_{2}=\frac{3}{h\_{1}}\left(y\_{2}-y\_{1}\right)$ (2.7)

$m\_{n-1}+2m\_{n}=\frac{3}{h\_{n-1}}\left(y\_{n}-y\_{n-1}\right)$ (2.8)

We can get from Eq. (2.4), Eq. (2.5) and Eq. (2.6) that

$DM=Y$ (2.9)

where, $M=(m\_{1},m\_{2},m\_{3},…,m\_{n-2},m\_{n-1},m\_{n})^{T}, Y=(β\_{0},β\_{1},…,β\_{n-1},β\_{n})^{T}$

$$D=\left[\begin{matrix}2&1&0&0&…&0&0&0&0\\1-α\_{2}&2&α\_{2}&0&…&0&0&0&0\\0&1-α\_{3}&2&α\_{3}&…&0&0&0&0\\0&0&1-α\_{4}&2&…&0&0&0&0\\…&…&…&…&…&…&…&…&…\\0&0&0&0&…&2&α\_{n-3}&0&0\\0&0&0&0&…&1-α\_{n-2}&2&α\_{n-2}&0\\0&0&0&0&…&0&1-α\_{n-1}&2&α\_{n-1}\\0&0&0&0&…&0&0&1&2\end{matrix}\right]$$

so, $M=D^{-1}Y$

Suppose $Y=B(y\_{1},y\_{2},…,y\_{n-1},y\_{n})^{T}$，then $M=D^{-1}B(y\_{1},y\_{2},…,y\_{n-1},y\_{n})^{T}$，

Let $E=D^{-1}B$, $E\_{i,j}$ represent the i-th row and the j-th column

$m\_{i}=\sum\_{j=1}^{n}E\_{i,j}y\_{i}$ (2.10)

Similarly,

$m\_{i+1}=\sum\_{j=1}^{n}E\_{i+1,j}y\_{i}$ (2.11)

For convenience, Eq (2.1) is rewritten into the following form:

$y\left(x\right)=g\_{1}\left(x\right)y\_{i}+g\_{2}\left(x\right)y\_{i+1}+g\_{3}\left(x\right)m\_{i}+g\_{4}\left(x\right)m\_{i+1}$ , $x\_{i}\leq x\leq x\_{i+1}$

The interpolation coefficient for the j-th IP of x on the interval$[x\_{i, } x\_{i+1}]$ is

$f\_{i,j}=\left\{\begin{array}{c}E\_{i,j}g\_{3}\left(x\right)+E\_{i+1,j}g\_{4}\left(x\right) , j\ne i,i+1\\E\_{i,j}g\_{3}\left(x\right)+E\_{i+1,j}g\_{4}\left(x\right)+g\_{1}\left(x\right) , j=i \\E\_{i,j}g\_{3}\left(x\right)+E\_{i+1,j}g\_{4}\left(x\right)+g\_{2}\left(x\right) , j=1+1 \end{array} \right.$ (2.12)