Supplementary material to Lambert and Sottili (2023): "Possible role of the tidal and rotational forcing on bradyseismic crises and volcanic unrest in the Campi Flegrei and Somma-Vesuvius areas"

1 FORMULAE FOR GRAVITY AND STRAIN CHANGES UNDER TIDAL AND CENTRIFUGATION POTENTIALS

The tidal potential can be retrieved using the Cartwright and Tyler tide generating potential development (Cartwright and Tayler, 1971). For a station of colatitude θ and longitude λ, one has

$$V_{20} = \sqrt{\frac{5}{4\pi}} K_{20} H_{20} \frac{3\cos^2\theta - 1}{2}, \quad \text{(S1)}$$

$$V_{21} = -3\sqrt{\frac{5}{24\pi}} K_{21} H_{21} \cos\theta \sin\theta, \quad \text{(S2)}$$

$$V_{22} = 3\sqrt{\frac{5}{96\pi}} K_{22} H_{22} \sin^2\theta, \quad \text{(S3)}$$

where ϕ is the phase of the tidal constituent obtained thanks to, e.g., integer combination of Doodson's variables and referred to J2000.0, H_c and H_s being the tidal heights (in meters) readable in Cartwright and Tyler tables, $K_{20} = 0.36178$, $K_{21} = -K_{22} = -0.511646$, and $H_{20} = H_c \cos\phi + H_s \sin\phi$, $H_{21} = H_c \cos(\phi + \lambda) + H_s \sin(\phi + \lambda)$, and $H_{22} = H_c \cos(\phi + 2\lambda) + H_s \sin(\phi + 2\lambda)$.

Regarding the rotational potential, let an Earth rotating at speed $\vec{\omega} = \Omega(m_1, m_2, 1 + m_3)^T$ where the dimensionless m_i are of the order of 10^{-6} for $i = 1, 2$ (polar motion) and 10^{-8} for $i = 3$ (LOD). The rotational potential felt by a body at distance $\vec{r}$ (or conversely longitude λ, colatitude θ, and radius r) is

$$V = -\frac{\Omega^2 r^2}{2}(m \sin 2\theta + 2m_3 \sin^2\theta), \quad \text{(S4)}$$

where $m = m_1 \cos\lambda + m_2 \sin\lambda$.

The surface deformations induced by the potential(s) can be computed using the vertical ($h = 0.6078$) and lateral ($l = 0.0847$) Love numbers (Wahr, 1985) as

$$U_r = \frac{h}{g} V, \quad \text{(S5)}$$

$$U_\theta = \frac{l}{g}\frac{\partial V}{\partial \theta}, \quad \text{(S6)}$$

$$U_\lambda = \frac{l}{g \sin\theta}\frac{\partial V}{\partial \lambda}, \quad \text{(S7)}$$

where g is the mean acceleration of the gravity at the surface.

Corresponding variations of g can be deduced from the vertical gradient of the potential:

$$\Delta g = \frac{V}{r} \tag{S8}$$

$$= \frac{g}{hr} U_r \tag{S9}$$

$$= \frac{g A_{20} H_{20}}{r} \frac{3\cos^2\theta - 1}{2} - \Omega^2 r (m \sin 2\theta + 2 m_3 \sin^2\theta), \tag{S10}$$

where $A_{20} = \sqrt{\frac{5}{4\pi}} \frac{K_{20}}{ag} \sim 10^{-8}$ (dimensionless).

The components of the strain tensor at a point of longitude λ, colatitude θ, and radius r are deduced from the deformations following (see, e.g., Agnew, 2007)

$$e_{rr} = \frac{U_r}{r}, \tag{S11}$$

$$e_{\theta\theta} = \frac{1}{r}\frac{\partial U_\theta}{\partial \theta} + \frac{U_r}{r}, \tag{S12}$$

$$e_{\lambda\lambda} = \frac{1}{r \sin\theta}\frac{\partial U_\lambda}{\partial \lambda} + \frac{U_\theta}{r}\cot\theta + \frac{U_r}{r}, \tag{S13}$$

$$e_{\theta\lambda} = \frac{1}{2r\sin\theta}\frac{\partial U_\theta}{\partial \lambda} + \frac{1}{2r}\frac{\partial U_\lambda}{\partial \theta} + \frac{U_\lambda}{2r}\cot\theta. \tag{S14}$$

For long period tides, which are of interest for our study, one has

$$e_{rr} = A_{20} H_{20} h \frac{3\cos^2\theta - 1}{2}, \tag{S15}$$

$$e_{\theta\theta} = 3 A_{20} H_{20} l (2\cos^2\theta - 1) + e_{rr}, \tag{S16}$$

$$e_{\lambda\lambda} = 3 A_{20} H_{20} l \cos^2\theta + e_{rr}, \tag{S17}$$

$$e_{\theta\lambda} = 0. \tag{S18}$$

The contributions from the rotational potential are (e.g., Shen, 2005)

$$e_{rr} = -\frac{\Omega^2 r h}{2g}(m \sin 2\theta - 2 m_3 \sin^2\theta), \tag{S19}$$

$$e_{\theta\theta} = \frac{2\Omega^2 r l}{g}(m \sin 2\theta - m_3 \cos 2\theta) + e_{rr}, \tag{S20}$$

$$e_{\lambda\lambda} = \frac{\Omega^2 r l \cot\theta}{g}(m(1 - \cos 2\theta) - m_3 \sin 2\theta) + e_{rr}, \tag{S21}$$

$$e_{\theta\lambda} = \frac{\Omega^2 r l}{g}\tilde{m}\sin\theta, \tag{S22}$$

where $\tilde{m} = -m_1 \sin\lambda + m_2 \cos\lambda$.

2 EARTH ROTATION DATA

The Earth rotation parameters distributed by the IERS are the so-called pole coordinates x_p and $-y_p$. For periods larger than, say, 10 days, these coordinates are simply related to the above instantaneous pole coordinates m_1 and m_2 by $m_1 = x_p$ and $m_2 = y_p$. The length-of-day (LOD) is the rotation period of the Earth about its axis, let, for a perturbed mean rotation speed $\Omega = 7.229115 \times 10^{-5}$ rad/s by a factor $1 + m_3$

$$\mathrm{LOD} = \frac{2\pi k}{\Omega(1/m_3)}, \tag{S23}$$

where $k = 1.0027$ is the ratio of the Solar day to sidereal day (86400 s/86164 s). Considering $m_3 << 1$, the excess of LOD with respect to the nominal value of 86400 s - equals to $2\pi k/\Omega$ - is

$$\Delta\mathrm{LOD} = -\frac{2\pi k}{\Omega} m_3. \tag{S24}$$

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