A SOLVING THE OBSERVATION ERROR EQUATION FOR A SINGLE STAR

In this section, the procedure of constructing and solving the observation error equation containing the matrix of astrometric parameters is presented. In Sect. 3.2, the single observation equation (Eqs. (4,5)) for a single star is introduced, which has the matrix form:

$$L(t_i) + v_i = B(t_i)X\,, \tag{A.1}$$

where $L(t_i)$ is the observation matrix (Van Altena, 2013, Sect. 19.2):

$$L(t_i) = \begin{bmatrix} \xi(t_i) \\ \eta(t_i) \end{bmatrix} = \begin{bmatrix} \frac{\cos\delta(t_i)\sin(\alpha(t_i)-\alpha_{ep})}{\sin\delta_{ep}\sin\delta(t_i)+\cos\delta_{ep}\cos\delta(t_i)\cos(\alpha(t_i)-\alpha_{ep})} \\ \frac{\cos\delta_{ep}\sin\delta(t_i)-\sin\delta_{ep}\cos\delta(t_i)\cos(\alpha(t_i)-\alpha_{ep})}{\sin\delta_{ep}\sin\delta(t_i)+\cos\delta_{ep}\cos\delta(t_i)\cos(\alpha(t_i)-\alpha_{ep})} \end{bmatrix}; \tag{A.2}$$

v_i is the correction value of $L(t_i)$, indicating that there are slight observation errors in $\xi(t_i)$ and $\eta(t_i)$; X is the unknown parameter matrix, which is defined as:

$$X = \begin{bmatrix} \Delta\alpha^* & \Delta\delta & \mu_{\alpha^*} & \mu_\delta & \pi \end{bmatrix}^{\mathrm{T}}; \tag{A.3}$$

$B(t_i)$ is the coefficient matrix, which is defined as:

$$B(t_i) = \begin{bmatrix} 1 & 0 & \frac{t_i - t_{ep}}{1-\mathbf{r}'_{ep}\mathbf{b}_{\mathrm{O}}(t_i)\pi/Au} & 0 & \frac{-\mathbf{p}'_{ep}\mathbf{b}_{\mathrm{O}}(t_i)/Au}{1-\mathbf{r}'_{ep}\mathbf{b}_{\mathrm{O}}(t_i)\pi/Au} \\ 0 & 1 & 0 & \frac{t_i - t_{ep}}{1-\mathbf{r}'_{ep}\mathbf{b}_{\mathrm{O}}(t_i)\pi/Au} & \frac{-\mathbf{q}'_{ep}\mathbf{b}_{\mathrm{O}}(t_i)/Au}{1-\mathbf{r}'_{ep}\mathbf{b}_{\mathrm{O}}(t_i)\pi/Au} \end{bmatrix}. \tag{A.4}$$

The error equation is established by associating the single-star observation equation for n different moments:

$$\begin{bmatrix} v_1 \\ v_2 \\ \cdots \\ v_i \\ \cdots \\ v_n \end{bmatrix} = \begin{bmatrix} B(t_1) \\ B(t_2) \\ \cdots \\ B(t_i) \\ \cdots \\ B(t_n) \end{bmatrix} X - \begin{bmatrix} L(t_1) \\ L(t_2) \\ \cdots \\ L(t_i) \\ \cdots \\ L(t_n) \end{bmatrix} \tag{A.5}$$

or

$$V = BX - L\,. \tag{A.6}$$

When the matrix form of the error equation of the single star is established, the expression of the unknown parameter matrix can be obtained from the least-squares method(the weight matrix of the observation value is the identity matrix):

$$X = (B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}L\,. \tag{A.7}$$

The iterative equations for solving the unknown parameter matrix can be constructed by the definition Eq. (A.3) and the expression Eq. (A.7). When the iterative equations converge, we can calculate the specific value of the unknown parameter matrix X, to obtain the position, parallax, and proper motion of the single star.

The mean square error (σ_o) of the measured value L is:

$$\sigma_o = \sqrt{\frac{(BX-L)^T(BX-L)}{2n-5}}\,. \tag{A.8}$$

The covariance matrix (Q_{XX}) of X is:

$$Q_{XX} = (B^TB)^{-1}\,. \tag{A.9}$$

The variance matrix (D_{XX}) of X is:

$$D_{XX} = \sigma_o^2 Q_{XX} = \begin{bmatrix} \sigma^2_{\alpha^*} & \sigma_{\alpha^*\delta} & \sigma_{\alpha^*\mu_{\alpha^*}} & \sigma_{\alpha^*\mu_\delta} & \sigma_{\alpha^*\pi} \\ \sigma_{\delta\alpha^*} & \sigma^2_{\delta} & \sigma_{\delta\mu_{\alpha^*}} & \sigma_{\delta\mu_\delta} & \sigma_{\delta\pi} \\ \sigma_{\mu_{\alpha^*}\alpha^*} & \sigma_{\mu_{\alpha^*}\delta} & \sigma^2_{\mu_{\alpha^*}} & \sigma_{\mu_{\alpha^*}\mu_\delta} & \sigma_{\mu_{\alpha^*}\pi} \\ \sigma_{\mu_\delta\alpha^*} & \sigma_{\mu_\delta\delta} & \sigma_{\mu_\delta\mu_{\alpha^*}} & \sigma^2_{\mu_\delta} & \sigma_{\mu_\delta\pi} \\ \sigma_{\pi\alpha^*} & \sigma_{\pi\delta} & \sigma_{\pi\mu_{\alpha^*}} & \sigma_{\pi\mu_\delta} & \sigma^2_{\pi} \end{bmatrix}. \tag{A.10}$$

Thus, the standard uncertainties of the astrometric parameters and the correlation coefficients between each parameter can be obtained:

$$\sigma_i = \sqrt{\sigma_i^2} \quad (i = \alpha^*,\ \delta,\ \mu_{\alpha^*},\ \mu_\delta,\ \pi) \tag{A.11}$$

$$\rho_{ij} = \frac{\sigma_{ij}}{\sqrt{\sigma_i^2\sigma_j^2}} \quad (i, j = \alpha^*,\ \delta,\ \mu_{\alpha^*},\ \mu_\delta,\ \pi \text{ and } i \neq j)\,. \tag{A.12}$$

B LOWER BOUND OF THE POSITIONAL PRECISION OF ASTROMETRIC OBSERVATIONS

Diffraction of light and photon statistics limit the positional precision of astrometric observations, and the positional precision for a diffraction-limited image is (Lindegren, 1978; Van Altena, 2013; Lindegren, 2013):

$$\sigma_{lb} = \frac{1}{\pi}\frac{\lambda}{D}\frac{1}{SNR}\ rad\,, \tag{B.1}$$

where λ is the wavelength of the observed photon; D is the aperture of the telescope; SNR is the signal-to-noise ratio (SNR) in the image, which is given by (Howell, 2006):

$$SNR = \frac{S_{star}}{\sqrt{S_{star} + n_{pix} \times (S_{sky} + S_{dark} + \sigma^2_{readout} + (G \times \sigma_f)^2)}}\,, \tag{B.2}$$

where S_{star} is the total number of electrons collected from the target star; n_{pix} is the number of pixels under consideration for the SNR calculation; S_{sky} is the total number of electrons per pixel from the background or sky; S_{dark} is the total number of dark current electrons per pixel; $\sigma^2_{readout}$ is the total number of electrons per pixel resulting from the read noise; G is the gain of the CCD (in electrons/ADU); σ_f is the medium error of the digital-to-analog conversion noise (ADU).

Eq. (B.1) is a reference quantity that serves as a lower bound on the positional precision and can be computed without specifying the centroiding algorithm (Lindegren, 2013). However, in practice, the lower bound of the positional precision is difficult to reach. Firstly, the sampling process of the detector needs to satisfy the sampling theorem, and when the angular resolution of the telescope pixels is larger than $\frac{\lambda}{2D}$, the positional precision will degrade during the sampling process (Lindegren, 2013). For CSST, the lower bound of the positional precision after the theoretical degradation is shown in Fig. 4. Secondly, the centroiding algorithm will also affect the positional precision, and from estimation theory, it can be deduced that only a good centroiding algorithm may come close to this reference lower bound (Lindegren, 2013).