Appendix A: Mobile platform modeling process

As shown in Fig. 5b, the motion of a moving platform can be viewed as a circular motion around a hypothetical point P at any moment t in X-Y plane, we have

$$\dot{\theta} = \omega \tag{A1}$$

$$R\omega = r\dot{\beta}_1 \tag{A2}$$

$$(R + b)\omega = \dot{x}_0 \cos(\theta)\dot{y}_0 \sin(\theta) \tag{A3}$$

$$(R + 2b)\omega = r\dot{\beta}_r \tag{A4}$$

Assume the moving platform is rigid and it moves only in one plane, the motion of the moving platform can be seen essentially as the motion of a rigid body in one plane. According to the velocity projection theorem, the velocity projections of P_s and p_0 on the symmetry axis should be equivalent.

$$\dot{x}_0 \cos(\theta) + \dot{y}_0 \sin(\theta) = \dot{x}_s \cos(\theta) + \dot{y}_s \sin(\theta) \tag{A5}$$

Consider Eq. (A1), (A2), (A3) and (A5), we can obtain

$$\dot{x}_s \cos(\theta)\dot{y}_s \sin(\theta) - b\dot{\theta} = r\dot{\beta}_1 \tag{A6}$$

Furthermore, considering Eq. (A4) and (A2), we can also obtain

$$r\dot{\beta}_1 = r\dot{\beta}_r - 2b\omega = r\dot{\beta}_r - 2b\dot{\theta} \tag{A7}$$

Given the square Eq. (A6) and (A7), we can obtain:

$$\dot{x}_s \cos(\theta) + \dot{y}_s \sin(\theta) + b\dot{\theta} = r\dot{\beta}_r \tag{A8}$$

From equation (A7)

$$\dot{\theta} = r/2b(\dot{\beta}_\mathrm{r} - \dot{\beta}_1) \tag{A9}$$

It is worth noting that Eq. (A6) and (A8) are usually referred to as rolling constraints, where the drive wheels do not slide. In addition, the moving platform must move in the direction of the axis of symmetry:

$$\dot{x}_s \sin(\theta) - \dot{x}_s \cos(\theta) + d\dot{\theta} = 0 \tag{A10}$$

Adding Eqs. (A6) and (A8), we have:

$$2(\dot{x}_s \cos(\theta) + \dot{y}_s \sin(\theta)) = r(\dot{\beta}_r + \dot{\beta}_1) \tag{A11}$$

On basis of the equations (A9) - (A11), the kinematic equations of the moving platform can be obtained in Eq. (28) and Eq. (29).

Appendix B: calculation procedure of δ_s

The derivations from (30) to (31) are shown in this appendix. Specifically, we have the flush transformation matrix.

$${}^{s}_{1}\mathbf{T} = \begin{bmatrix} c\gamma_1 & -s\gamma_1 & 0 & 0 \\ s\gamma_1 & c\gamma_1 & 0 & 0 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^{1}_{2}\mathbf{T} = \begin{bmatrix} c\gamma_2 & s\gamma_2 & 0 & 0 \\ 0 & 0 & -1 & -l_{12} \\ -s\gamma_2 & c\gamma_2 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^{2}_{3}\mathbf{T} = \begin{bmatrix} c\gamma_3 & s\gamma_3 & 0 & 0 \\ -s\gamma_3 & c\gamma_3 & 0 & l_2 \\ 0 & 0 & 1 & -l_{22} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{3}_{4}\mathbf{T} = \begin{bmatrix} c\gamma_4 & -s\gamma_4 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ -s\gamma_4 & -c\gamma_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^{4}_{5}\mathbf{T} = \begin{bmatrix} c\gamma_5 & -s\gamma_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\gamma_5 & c\gamma_5 & 0 & l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^{5}_{6}\mathbf{T} = \begin{bmatrix} c\gamma_6 & -s\gamma_6 & 0 & 0 \\ 0 & 0 & 1 & l_5 \\ -s\gamma_6 & -c\gamma_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

${}^{6}Pend = [0, 0, l_6, 1]$.

where c stands for function $cos(\bullet)$, s stands for the function $sin(\bullet)$; h is the height of the platform and li denotes the length of the ith link, $(i \in 1, 2..., 6)$. In addition, l_{12} denotes the displacement distance between joints 1 and 2 of the manipulator and $l_2 2$ denotes the displacement distance between joints 2 and 3. From (5), it can be derived

$${}^{s}\mathbf{W}_{end} = \begin{bmatrix} (l_6 + l_5)(\cos\gamma_5 \sin(\gamma_2 + \gamma_3)\cos\gamma_1 - \sin\gamma_5 \cos\gamma_4 \cos(\gamma_2 + \gamma_3)\cos\gamma_1 \\ + \sin\gamma_5 \sin\gamma_4 \sin\gamma_1) + (l_3 + l_4)\sin(\gamma_2 + \gamma_3)\cos\gamma_1 + l_2 \sin\gamma_2 \cos\gamma_1 \\ (l_6 + l_5)(\cos\gamma_5 \sin(\gamma_2 + \gamma_3)\sin\gamma_1 - \sin\gamma_5 \cos\gamma_4 \cos(\gamma_2 + \gamma_3)\sin\gamma_1 \\ + \sin\gamma_5 \sin\gamma_4 \cos\gamma_1) + (l_3 + l_4)\sin(\gamma_2 + \gamma_3)\sin\gamma_1 + l_2 \sin\gamma_2 \sin\gamma_1 \\ (l_6 + l_5)(\sin\gamma_5 \sin(\gamma_2 + \gamma_3)\cos\gamma_4 + \cos\gamma_5 \cos(\gamma_2 + \gamma_3)) \\ +(l_3 + l_4)\cos(\gamma_2 + \gamma_3) + l_2 \cos\gamma_2 + l_1 + h \\ 1 \end{bmatrix} \text{ where } h = 0.698m,$$

$l_1 = 0.065m$, $l_2 = 0.555m$, $l_3 = 0.19m$, $l_4 = 0.13m$, $l_5 = 0.082m$, $l_6 = 0.018m$, $l_{12} = 0.11m$ and $l_{22} = 0.11m$.

$$\begin{aligned} \begin{bmatrix} \delta_z; 1 \end{bmatrix} &= {}^{z}_{s}\mathbf{T}{}^{s}\mathbf{W}_{end} \\ &= \begin{bmatrix} (l_6 + l_5)(\cos\gamma_5 \sin(\gamma_2 + \gamma_3)\cos(\gamma_1 + \theta)) - \sin\gamma_5 \cos\gamma_4 \cos(\gamma_2 + \gamma_3)\cos(\gamma_1 + \theta) \\ + \sin\gamma_5 \sin\gamma_4 \sin(\gamma_1 + \theta) + (l_3 + l_4)\sin(\gamma_2 + \gamma_3)\cos(\gamma_1 + \theta) + \mathrm{l}_2 \sin\gamma_2 \cos(\gamma_1 + \theta) \\ (l_6 + l_5)(\cos\gamma_5 \sin(\gamma_2 + \gamma_3)\sin(\gamma_1 + \theta)) - \sin\gamma_5 \cos\gamma_4 \cos(\gamma_2 + \gamma_3)\sin(\gamma_1 + \theta) \\ + \sin\gamma_5 \sin\gamma_4 \cos(\gamma_1 + \theta) + (l_3 + l_4)\sin(\gamma_2 + \gamma_3)\sin(\gamma_1 + \theta) + l_2 \sin\gamma_2 \sin(\gamma_1 + \theta) \\ (\mathrm{l}_6 + \mathrm{l}_5)(\sin\gamma_5 \sin(\gamma_2 + \gamma_3)\cos\gamma_4 + \cos\gamma_5 \cos(\gamma_2 + \gamma_3)) + (\mathrm{l}_3 + \mathrm{l}_4)\cos(\gamma_2 + \gamma_3) + \\ l_2 \cos\gamma_2 + l_1 + h \\ 1 \end{bmatrix} + \begin{bmatrix} \mathrm{x_s} \\ \mathrm{y_s} \\ 0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} \delta_\mathrm{s}; 1 \end{bmatrix} + \begin{bmatrix} \mathrm{x_s}; \mathrm{y_s}; 0; 0 \end{bmatrix} \end{aligned} \tag{B1}$$

Thus, Eq. (31) can be obtained.

Appendix C: Parameters of Jacobian matrix

Here, $\dot{x}_s$ and $\dot{y}_s$ as a function of θ , $\dot{\beta}_1$ and $\dot{\beta}_r$, according to the previous analysis. $\dot{\delta}_s$ is a function of the joint angle γ_i and joint velocity $\dot{\gamma}_i$ of the manipulator, i = 1,2,... , 6, so that δ_z is a function of $\theta, \dot{\beta}_1, \dot{\beta}_r, \gamma_i$ and dot$\gamma_\mathrm{i}(i = 1, 2, ..., 6)$ as a function . Thus, Eq. (33) can be rewritten as:

$$\dot{\delta}_z = \begin{bmatrix} \dot{x}_z & \dot{y}_z & \dot{z}_z \end{bmatrix}^T = \mathbf{J}_H\dot{\theta} + \mathbf{J}_s\dot{\theta} \tag{C1}$$

where $\mathbf{J_H} = \begin{bmatrix} \frac{r}{2}\cos\theta + \frac{dr}{2b}\sin\theta & \frac{r}{2}\cos\theta - \frac{\mathrm{dr}}{2\mathrm{b}}\sin\theta & 0 & \cdots & 0 \\ \frac{r}{2}\sin\theta + \frac{dr}{2b}\cos\theta & \frac{r}{2}\sin\theta - \frac{\mathrm{dr}}{2\mathrm{b}}\cos\theta & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$,

$\dot{\Theta} = \begin{bmatrix} \dot{\beta}_1 & \dot{\beta}_r & \dot{\theta} & \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 & \gamma_5 & \gamma_6 \end{bmatrix}$,and

$$\mathbf{J_s} = \begin{bmatrix} 0 & 0 & j_{s11} & j_{s12} & j_{s13} & j_{s14} & j_{s15} & j_{s16} & j_{s17} \\ 0 & 0 & j_{s21} & j_{s22} & j_{s23} & j_{s24} & j_{s25} & j_{s26} & j_{s27} \\ 0 & 0 & j_{s31} & j_{s32} & j_{s33} & j_{s34} & j_{s35} & j_{s36} & j_{s37} \end{bmatrix}$$ Among the parameters:

$j_{s11} = j_{s12} = (l_5 + l_6)(-c_5 s_{32} s_{1\theta} + s_5 c_4 c_{32} s_{1\theta} + s_5 s_4 c_{1\theta}) - (l_3 + l_4) s_{32} s_{1\theta} - l_2 s_2 s_{1\theta}$,
$j_{s13} = (l_5 + l_6)(c_5 c_{32} c_{1\theta} + s_5 c_4 s_{32} c_{1\theta}) + (l_3 + l_4) c_{32} c_{1\theta} + l_2 c_2 c_{1\theta}$,
$j_{s14} = (l_5 + l_6)(c_5 c_{32} c_{1\theta} + s_5 c_4 c_{32} c_{1\theta}) + (l_3 + l_4) c_{32} c_{1\theta}$,
$j_{s15} = (l_5 + l_6)(s_5 s_4 c_{32} c_{1\theta} + s_5 c_4 s_{1\theta})$,
$\mathrm{j}_{s16} = (\mathrm{l}_5 + \mathrm{l}_6)(-\mathrm{s}_5 \mathrm{s}_{32} \mathrm{c}_{1\theta} - \mathrm{c}_5 \mathrm{c}_4 \mathrm{c}_{32} \mathrm{c}_{1\theta} + \mathrm{c}_5 \mathrm{s}_4 \mathrm{s}_{1\theta}), \mathrm{j}_{s17} = 0$,
$\mathrm{j}_{s21} = \mathrm{j}_{s22} = (\mathrm{l}_5 + \mathrm{l}_6)(\mathrm{c}_5 \mathrm{s}_{32} \mathrm{c}_{1\theta} - \mathrm{s}_5 \mathrm{c}_4 \mathrm{c}_{32} \mathrm{c}_{1\theta} + \mathrm{s}_5 \mathrm{s}_4 \mathrm{s}_{1\theta}) + (\mathrm{l}_3 + \mathrm{l}_4) \mathrm{s}_{32} \mathrm{c}_{1\theta} + \mathrm{l}_2 \mathrm{s}_2 \mathrm{c}_{1\theta}$,
$j_{s23} = (l_5 + l_6)(c_5 c_{32} s_{1\theta} + s_5 c_4 s_{32} s_{1\theta}) + (l_3 + l_4) c_{32} s_{1\theta} + l_2 c_2 s_{1\theta}$,
$j_{s24} = (l_5 + l_6)(c_5 c_{32} s_{1\theta} + s_5 c_4 s_{32} s_{1\theta}) + (l_3 + l_4) c_{32} s_{1\theta}, j_{s25} = (l_5 + l_6)(s_5 s_4 c_{32} s_{1\theta} - s_5 c_4 c_{1\theta})$,
$j_{s26} = (l_5 + l_6)(-s_5 s_{32} s_{1\theta} - c_5 c_4 c_{32} s_{1\theta} - c_5 s_4 c_{1\theta}), j_{s27} = 0$,
$j_{s31} = j_{s32} = 0, j_{s33} = (l_5 + l_6)(s_5 c_4 s_{32} - c_5 s_{32}) - (l_3 + l_4) s_{32} - l_2 s_2$,
$j_{s34} = (l_5 + l_6)(s_5 c_4 s_{32} - c_5 s_{32}) - (l_3 + l_4) s_{32}, j_{s35} = (l_5 + l_6)(-s_5 s_4 s_{32})$,
$j_{s36} = (l_5 + l_6)(c_5 c_4 s_{32} - s_5 s_{32}), j_{s37} = 0$,