**Constitutive models**

**Mohr-Coulomb model** defines a flow of bulk material by a straight line in the normal stress () and shear stress () plane with a slope of , which is an angle of internal friction, and the shear stress intercept of , which is cohesion coefficient.

(1)

Conventionally, a Mohr-Coulomb failure envelope is determined by estimating a common tangent line of multiple Mohr circles determined from shear test results. From the results of compressive shear tests with different consolidation stress levels, multiple Mohr circles can be obtained. A common tangential line representing a Mohr-Coulomb failure envelope can be determined using a pair of Mohr circles of two tests with different consolidation pressures.

Jenike’s approach defines the ratio between unconfined yield strength of and the consolidation stress , i.e., . A particulate material with larger than 4 indicates less problematic flow characteristic. This is because smaller unconfined yield strength implies cohesionless flow. To employ Jenike’s approach, a yield stress of a certain consolidation stress is determined typically using a shear cell. Because it is not feasible to directly determine an unconfined yield stress, it is estimated from the yield locus, which is the Mohr-Coulomb failure envelope.

**Drucker-Prager model** defines failure envelope as a linearly increasing shear stress corresponding to increasing hydrostatic stress. A linear Drucker-Prager envelope is expressed as an extended von Mises yield criterion.

(2)

where is the second invariant of a deviatoric tensor and is the first invariant of a Cauchy stress tensor, is the slope of a failure envelope, and is the intercept of a failure envelope. Considering that the represents the failure of biomass, i.e., the onset of flow, and is analogous to the angle of internal friction () and the coefficient of cohesion () of the Mohr-Coulomb failure envelope.

**The modified Cam-Clay model** defines a straight critical state line on the plane of the hydrostatic stress () axis and shear stress (deviatoric stress ) axis. In the modified Cam-Clay model, the yield locus is defined by the critical state line:

(3)

where the slope of the critical state line (*M*) is determined from failure stress in deviatoric stress (*q*) corresponding to hydrostatic stress (*p*), and hydrostatic stress corresponding to the intersection of the yield loci with the *p*-axis ().