**Constitutive models**

**Mohr-Coulomb model** defines a flow of bulk material by a straight line in the normal stress ($σ$) and shear stress ($τ$) plane with a slope of $ϕ$, which is an angle of internal friction, and the shear stress intercept of $c$, which is cohesion coefficient.

$τ=c+σtanϕ$ (1)

Conventionally, a Mohr-Coulomb failure envelope is determined by estimating a common tangent line of multiple Mohr circles determined from shear test results. From the results of compressive shear tests with different consolidation stress levels, multiple Mohr circles can be obtained. A common tangential line representing a Mohr-Coulomb failure envelope can be determined using a pair of Mohr circles of two tests with different consolidation pressures.

 Jenike’s approach defines the ratio between unconfined yield strength of $σ\_{c}$ and the consolidation stress $σ\_{1}$, i.e., $ff\_{c}= \frac{σ\_{1}}{σ\_{c}}$. A particulate material with $ff\_{c}$ larger than 4 indicates less problematic flow characteristic. This is because smaller unconfined yield strength implies cohesionless flow. To employ Jenike’s approach, a yield stress of a certain consolidation stress is determined typically using a shear cell. Because it is not feasible to directly determine an unconfined yield stress, it is estimated from the yield locus, which is the Mohr-Coulomb failure envelope.

**Drucker-Prager model** defines failure envelope as a linearly increasing shear stress corresponding to increasing hydrostatic stress. A linear Drucker-Prager envelope is expressed as an extended von Mises yield criterion.

$F\_{1}=\sqrt{J\_{2}}-βI\_{1}-d$ (2)

where $J\_{2}$ is the second invariant of a deviatoric tensor and $I\_{1}$ is the first invariant of a Cauchy stress tensor, $β$ is the slope of a failure envelope, and $d$ is the intercept of a failure envelope. Considering that the $F\_{1}=0$ represents the failure of biomass, i.e., the onset of flow, $β,$ and $d$ is analogous to the angle of internal friction ($ϕ$) and the coefficient of cohesion ($c$) of the Mohr-Coulomb failure envelope.

**The modified Cam-Clay model** defines a straight critical state line on the plane of the hydrostatic stress ($x$) axis and shear stress (deviatoric stress $y$) axis. In the modified Cam-Clay model, the yield locus is defined by the critical state line:

$f=M^{2}p^{2}-M^{2}p\_{0}p+q^{2}$ (3)

where the slope of the critical state line (*M*) is determined from failure stress in deviatoric stress (*q*) corresponding to hydrostatic stress (*p*), and hydrostatic stress corresponding to the intersection of the yield loci with the *p*-axis ($p\_{0}$).