Supplementary materials

Algal proliferation risk assessment using vine copula-based coupling methods in the South-to-North Water Diversion Project of China

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Table S1

The Copula family of functions and their corresponding mathematical descriptions.

Name (Abbreviation)	Mathematical Description	Parameter range	Reference
Gaussian (N)	$\int_{-\infty}^{\phi^{-1}(u)}\int_{-\infty}^{\phi^{-1}(v)}\frac{1}{2\pi\sqrt{1-\theta^2}}exp\,(\frac{2\theta xy-x^2-y^2}{2(1-\theta^2)})dxdy$	$\theta\in[-1,\ 1]$	
Student-t (t)	$\int_{-\infty}^{\phi^{-1}(u)}\int_{-\infty}^{\phi^{-1}(v)}\frac{1}{2\pi\sqrt{1-\theta^2}}exp\,(\frac{2\theta xy-x^2-y^2}{2(1-\theta^2)})dxdy$	$\theta_1\in[-1,\ 1]\&\theta_2\in(0,\ \infty)$	
Gumbel (G)	$exp\left\{-\left[(-\ln u)^\theta+(-\ln v)^\theta\right]^{1/\theta}\right\}$	$\theta\in[1,\ \infty)$	(Lee et al., 2013)
Frank (F)	$-\frac{1}{\theta}ln\left[1+\frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1}\right]$	$\theta\in$ R\{0}	
Joe (J)	$1-\left[(1-u)^\theta+(1-v)^\theta-(1-u)^\theta(1-v)^\theta\right]^{1/\theta}$	$\theta\in[1,\ \infty)$	
Clayton (C)	$\left(u^{-\theta}+v^{-\theta}-1\right)^{-1/\theta}$	$\theta\in(0,\ \infty)$	(Manstavicius and Leipus, 2017)

Table S2

Decomposition structure and parameter test of three Vine Copula functions in different dimensions of upstream.

Dimension	Tree	R-Vine			C-Vine			D-Vine		
		Edge	**Best-fitting**	**Par (Par2)**	**Edge**	**Best-fitting**	**Par (Par2)**	**Edge**	**Best-fitting**	**Par (Par2)**
3- dimensional	Tree 1	**2,1**	**J**	**2.21**	2,1	J	2.21	2,1	J	2.21
		3,2	**SJ**	**1.92**	3,2	SJ	1.92	3,2	SJ	1.92
	Tree 2	**3,1;2**	**G270**	**-1.27**	3,1;2	G270	-1.27	3,1;2	G270	-1.27
4- dimensional	Tree 1	**2,1**	**J**	**2.21**	2,1	J	2.21	1,3	C90	-0.59
		2,3	**F**	**-3.56**	2,3	F	-3.56	2,1	J	2.21
	Tree 2	**4,2**	**SJ**	**1.92**	4,2	SJ	1.92	4,2	SJ	1.92
		4,1;2	**G270**	**-1.27**	4,1;2	G270	-1.27	2,3;1	G90	-1.37
		4,3;2	**G**	**1.23**	4,3;2	G	1.23	4,1;2	G270	-1.27
	Tree 3	**3,1;4,2**	**SJ**	**1.16**	3,1;4,2	SJ	1.16	4,3;2,1	N	0.32
5-dimensional	Tree 1	**3,4**	**F**	**-3.56**	3,4	F	-3.56	1,4	C90	-0.59
		3,1	**J**	**2.21**	3,1	J	2.21	3,1	J	2.21
		2,3	**C**	**1.19**	3,2	C	1.19	2,3	C	1.19
	Tree 2	**5,2**	**N**	**0.88**	5,3	SJ	1.92	5,2	N	0.88
		2,4;3	**N**	**0.41**	2,4;3	N	0.41	3,4;1	G90	-1.37
		2,1;3	**G270**	**-1.43**	2,1;3	G270	-1.43	2,1;3	G270	-1.43
		5,3;2	**C**	**0.12**	5,2;3	N	0.80	5,3;2	C	0.12
	Tree 3	**1,4;2,3**	**SJ**	**1.30**	1,4;2,3	SJ	1.30	2,4;3,1	N	0.39
	Tree 3	**5,1;2,3**	**G**	**1.02**	5,1;2,3	G	1.01	5,1;2,3	G	1.02
	Tree 4	**5,4;1,2,3**	**F**	**0.18**	5,4;1,2,3	SC	0.02	5,4;2,3,1	F	0.40

Notes: 1—ACD, 2—WT, 3—V, 4—DO, 5—Q.

Table S3

Decomposition structure and parameter test of three Vine Copula functions in different dimensions of midstream.

Dimension	Tree	R-Vine			C-Vine			D-Vine		
		Edge	Best-fitting Copula	Par (Par2)	Edge	Best-fitting Copula	Par (Par2)	Edge	Best-fitting Copula	Par (Par2)
3- dimensional	Tree 1	**2,1**	**G**	**2.17**	2,1	G	2.17	2,1	G	2.17
		3,2	**C**	**1.31**	3,2	C	1.31	3,2	C	1.31
	Tree 2	**3,1;2**	**N**	**-0.11**	3,1;2	N	-0.11	3,1;2	N	-0.11
4- dimensional	Tree 1	**2,3**	**C270**	**-1.74**	2,3	C270	-1.74	3,1	G270	-1.51
		2,1	**G**	**2.17**	2,1	G	2.17	2,3	C270	-1.74
		4,2	**C**	**1.31**	4,2	C	1.31	4,2	C	1.31
	Tree 2	**1,3;2**	**J90**	**-1.11**	1,3;2	J90	-1.11	2,1;3	SC	1.33
		4,1;2	**N**	**-0.11**	4,1;2	N	-0.11	4,3;2	SC	0.19
	Tree 3	**4,3;1,2**	**SC**	**0.21**	4,3;1,2	SC	0.21	4,1;2,3	C90	-0.18
5-dimensional	Tree 1	**3,4**	**C270**	**-1.74**	3,4	C270	-1.74	5,2	F	32.72
		3,1	**G**	**2.17**	3,1	G	2.17	4,5	C90	-0.73
		5,2	**F**	**32.72**	3,2	C	1.06	3,4	C270	-1.74
	Tree 2	**5,3**	**C**	**1.31**	5,3	C	1.31	1,3	G	2.17
		1,4;3	**J90**	**-1.11**	2,4;3	J	1.15	4,2;5	SC	0.48
		5,1;3	**N**	**-0.11**	2,1;3	N	-0.10	3,5;4	C	0.63
		3,2;5	**N**	**-0.38**	5,2;3	F	26.11	1,4;3	J90	-1.11
	Tree 3	**5,4;1,3**	**SC**	**0.21**	1,4;2,3	J90	-1.14	3,2;4,5	N	-0.28
		2,1;5,3	**C90**	**-0.09**	5,1;2,3	F	0.17	1,5;3,4	N	-0.10
	Tree 4	**2,4;5,1,3**	**SC**	**0.25**	5,4;1,2,3	N	0.04	1,2;3,4,5	C270	-0.10

Notes: 1—ACD, 2—WT, 3—V, 4—DO, 5—Q.

Table S4

Decomposition structure and parameter test of three Vine Copula functions in different dimensions of downstream.

Dimension	Tree	R-Vine			C-Vine			D-Vine		
		Edge	Best-fitting Copula	Par (Par2)	Edge	Best-fitting Copula	Par (Par2)	Edge	Best-fitting Copula	Par (Par2)
3- dimensional	Tree 1	**2,1**	**F**	**5.68**	2,1	F	5.68	2,1	F	5.68
		3,2	**SJ**	**2.06**	3,2	SJ	2.06	3,2	SJ	2.06
	Tree 2	**3,1;2**	**J**	**1.11**	3,1;2	J	1.11	3,1;2	J	1.11
4- dimensional	Tree 1	**2,1**	**F**	**5.68**	2,1	F	5.68	3,4	F	-2.70
		2,3	**F**	**-9.18**	2,3	F	-9.18	2,3	F	-9.18
		4,2	**SJ**	**2.06**	4,2	SJ	2.06	1,2	F	5.68
	Tree 2	**4,1;2**	**J**	**1.11**	4,1;2	J	1.11	2,4;3	SJ	1.60
		4,3;2	**SJ**	**1.11**	4,3;2	SJ	1.11	1,3;2	J	1.06
	Tree 3	**3,1;4,2**	**F**	**0.28**	3,1;4,2	F	0.28	1,4;2,3	J	1.05
5-dimensional	Tree 1	**3,4**	**F**	**-9.18**	3,2	F	3.87	5,2	C	4.44
		3,1	**F**	**5.68**	3,1	F	5.68	1,5	SJ	1.79
		2,3	**F**	**3.87**	3,4	F	-9.18	3,1	F	5.68
		5,2	**C**	**4.44**	5,3	SJ	2.06	4,3	F	-9.18
	Tree 2	**1,4;3**	**J**	**1.06**	5,2;3	C	2.95	1,2;5	J90	-1.10
		2,1;3	**G**	**1.00**	5,1;3	J	1.11	3,5;1	F	2.57
		5,3;2	**C270**	**-0.22**	5,4;3	SJ	1.11	4,1;3	J	1.06
	Tree 3	**2,4;1,3**	**SJ**	**1.13**	1,2;5,3	t	0.08 (3.29)	3,2;1,5	SC	0.40
		5,1;2,3	**J90**	**-1.11**	4,1;5,3	F	0.28	4,5;3,1	J270	-1.22
	Tree 4	**5,4;2,1,3**	**J**	**1.09**	4,2;1,5,3	N	0.01	4,2;3,1,5	C	0.10

Notes: 1—ACD, 2—WT, 3—V, 4—DO, 5—Q.

Table S5

The Copula function selects the best result and the corresponding parameters.

Variates	The best fitting copula	Kendall (τ)	Parameter
ACD_{Up}/ACD_{Mid}	SG	0.47	1.89
ACD_{Mid}/ACD_{Down}	SG	0.66	2.92

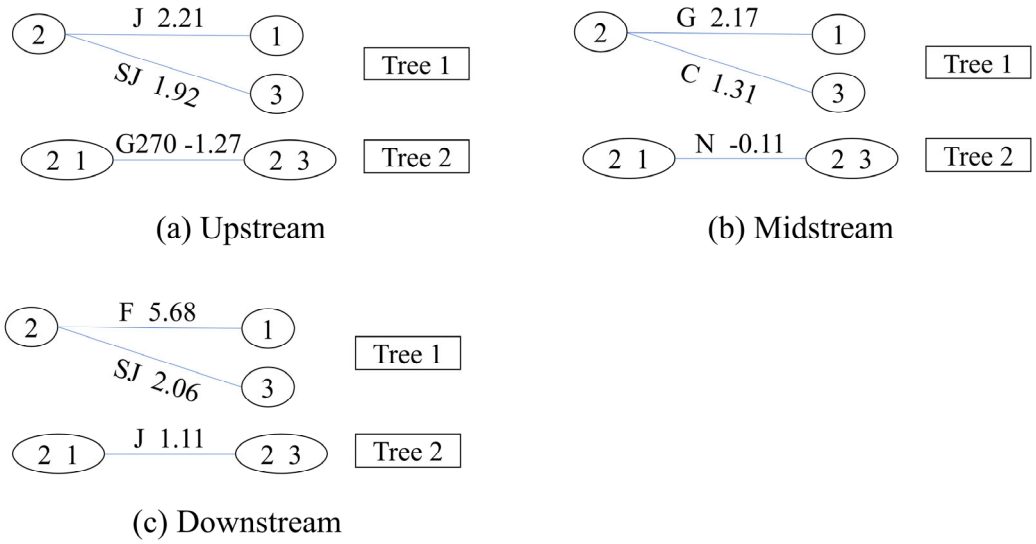


Fig. S1. Three-dimensional Vine Copula structure in this study (Notes: 1—ACD, 2—WT, 3—V).

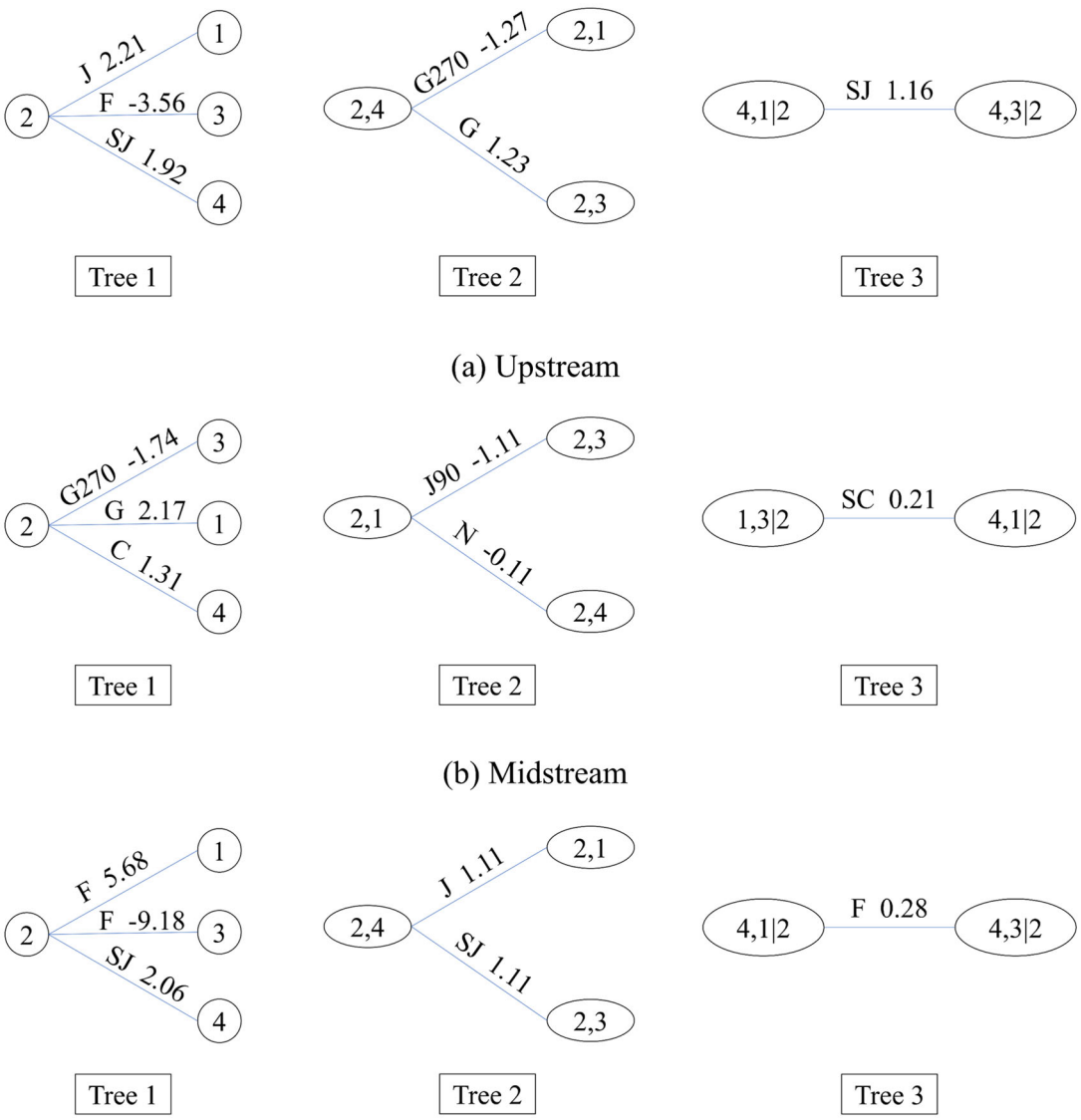
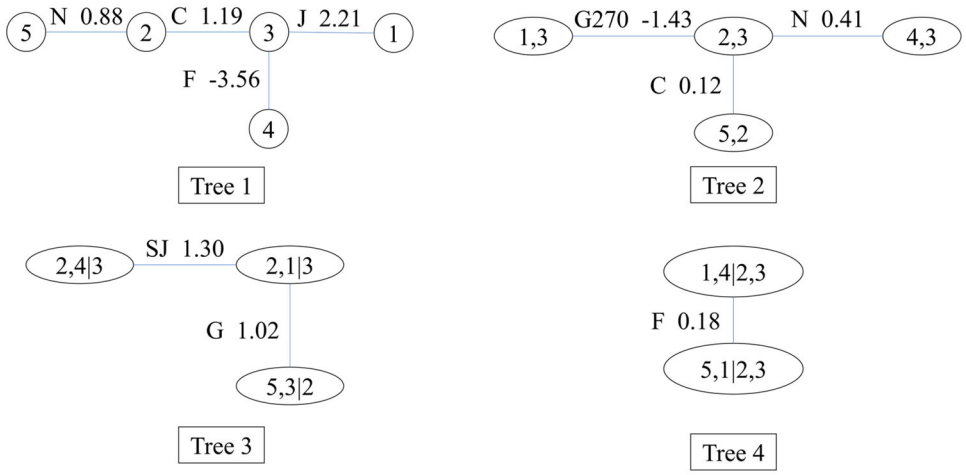
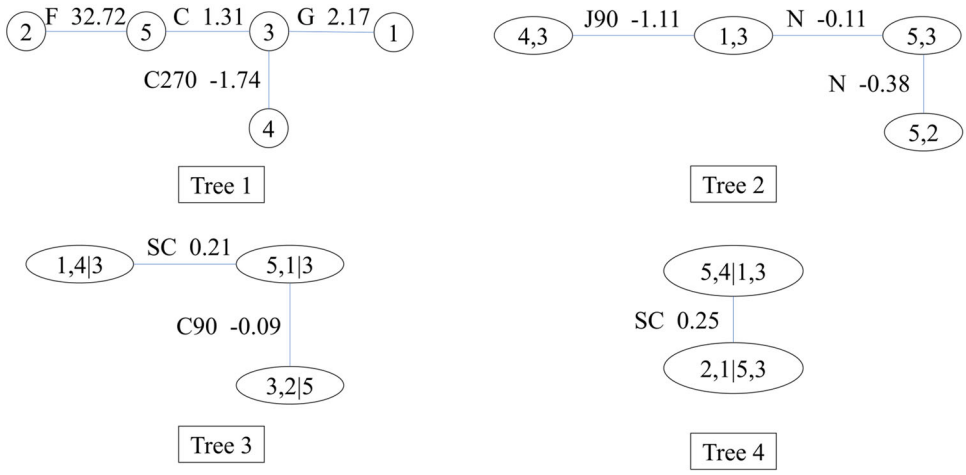


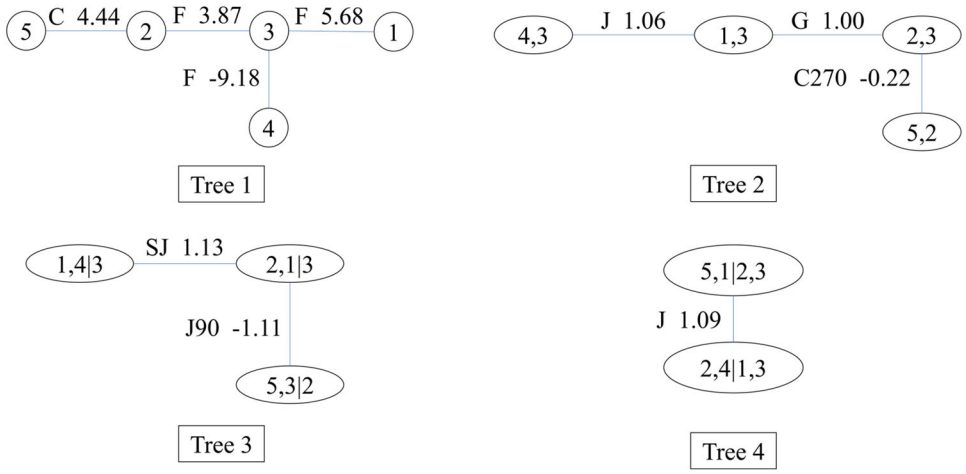
Fig. S2. Four-dimensional Vine Copula structure in this study (Notes: 1—ACD, 2—WT, 3—V, 4—DO).



(a) Upstream



(b) Midstream



(c) Downstream

Fig. S3. Five-dimensional Vine Copula structure in this study (Notes: 1—ACD, 2—WT, 3—V, 4—DO, 5—Q).

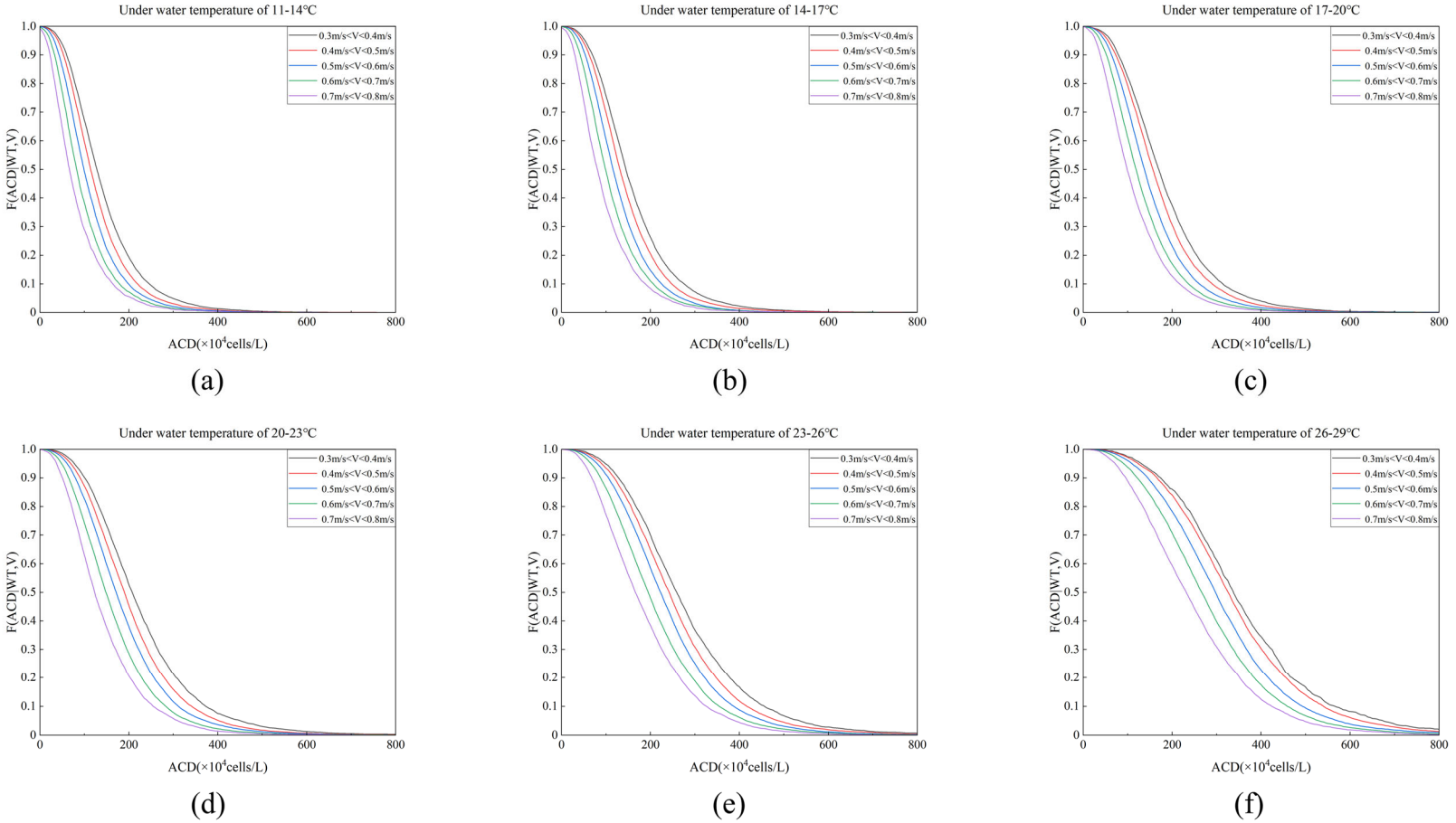


Fig. S4. Conditional probability of ACD at different water temperatures and flow velocity in upstream via 3D Vine Copula.

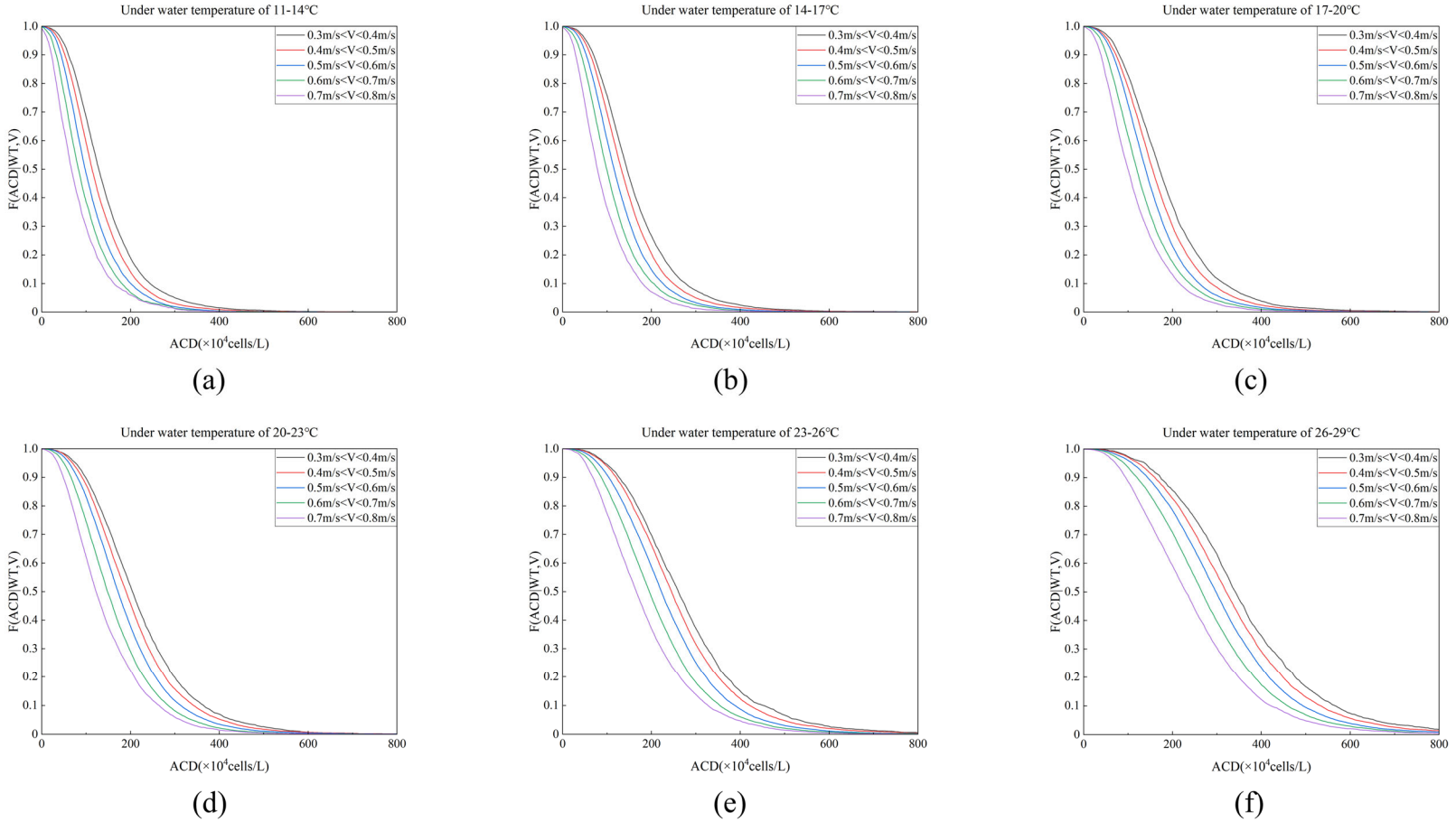


Fig. S5. Conditional probability of ACD at different water temperatures and flow velocity in upstream via 4D Vine Copula.

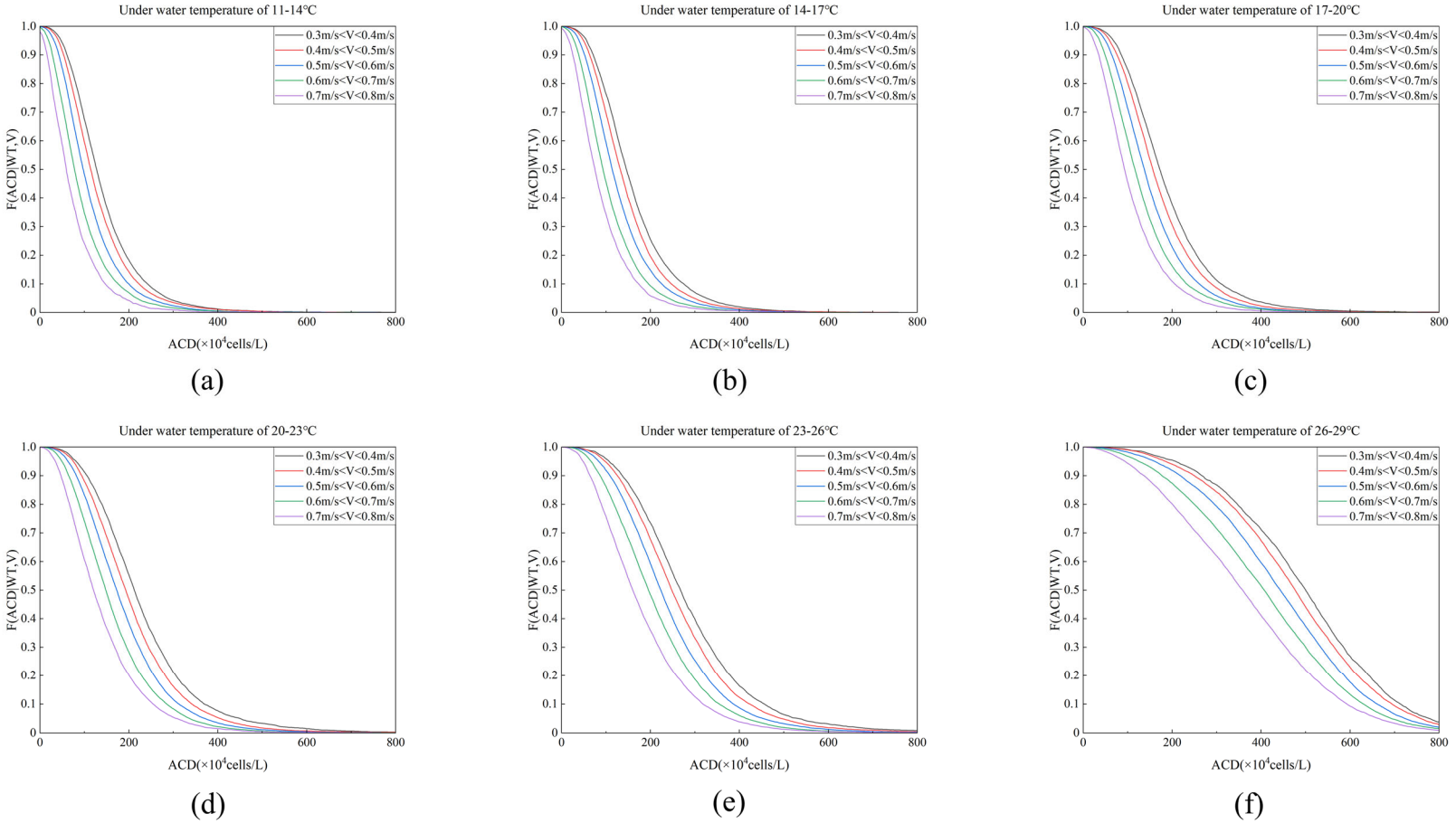


Fig. S6. Conditional probability of ACD at different water temperatures and flow velocity in upstream via 5D Vine Copula.

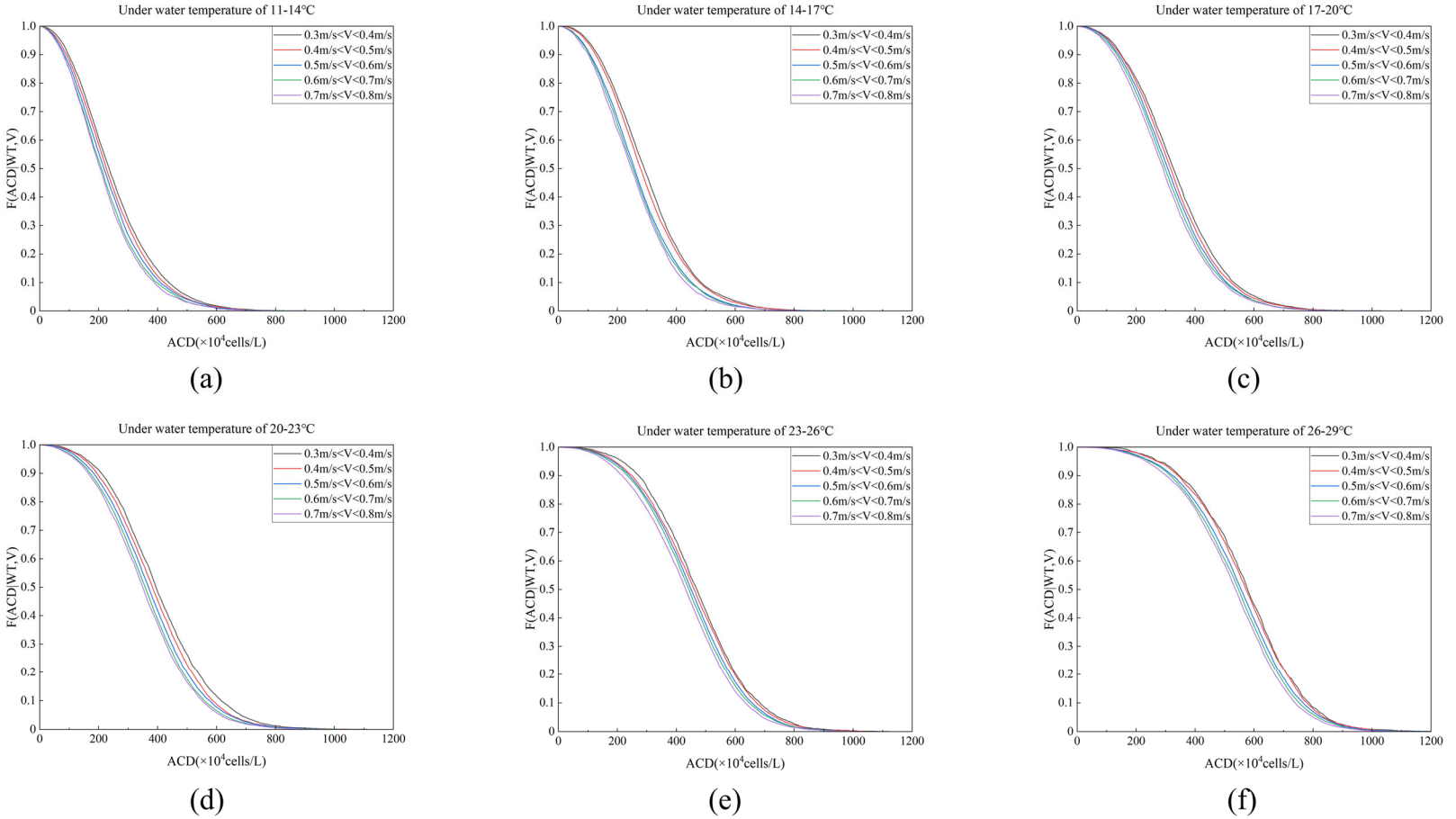


Fig. S7. Conditional probability of ACD at different water temperatures and flow velocity in midstream via 3D Vine Copula.

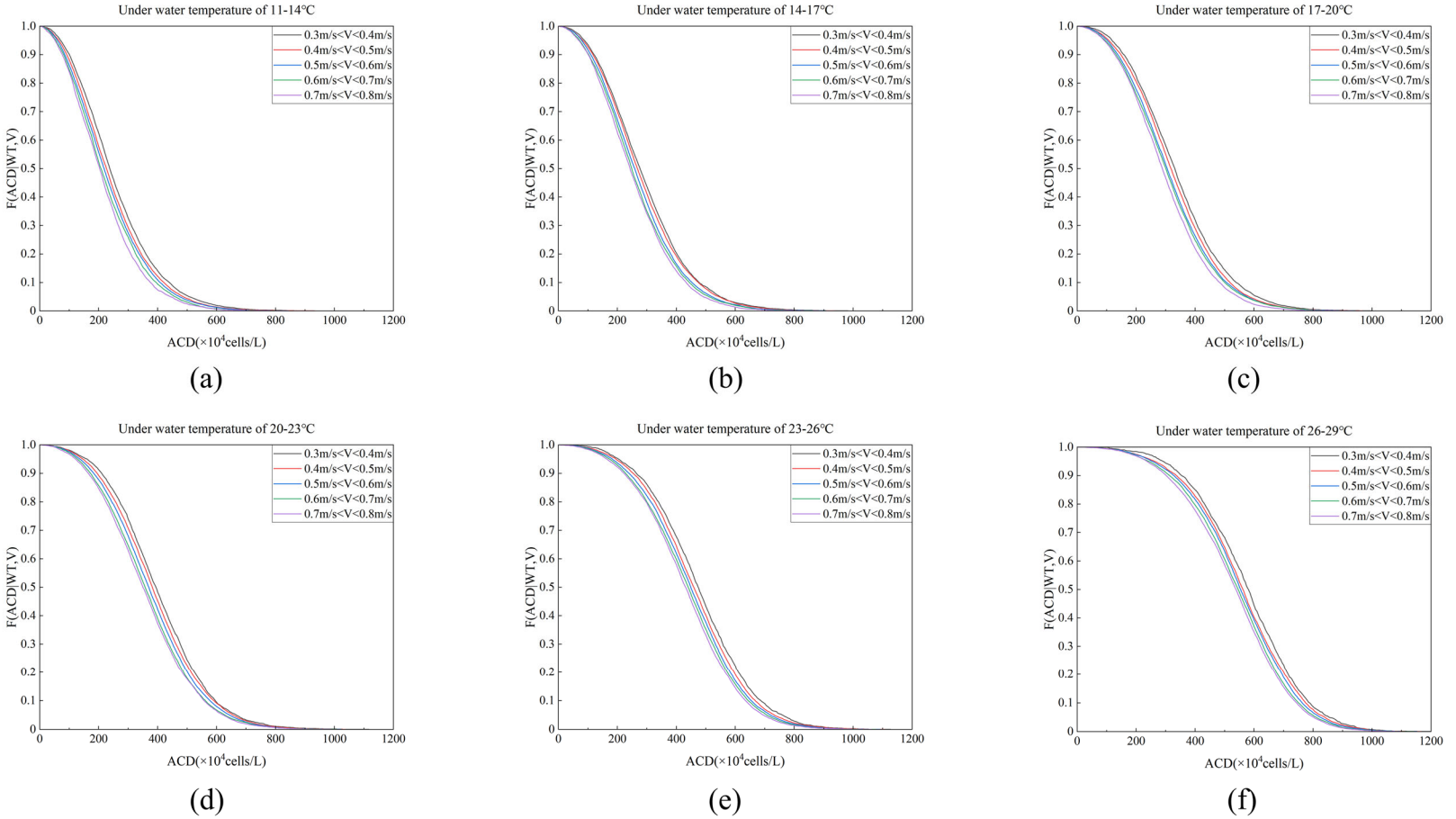


Fig. S8. Conditional probability of ACD at different water temperatures and flow velocity in midstream via 4D Vine Copula.

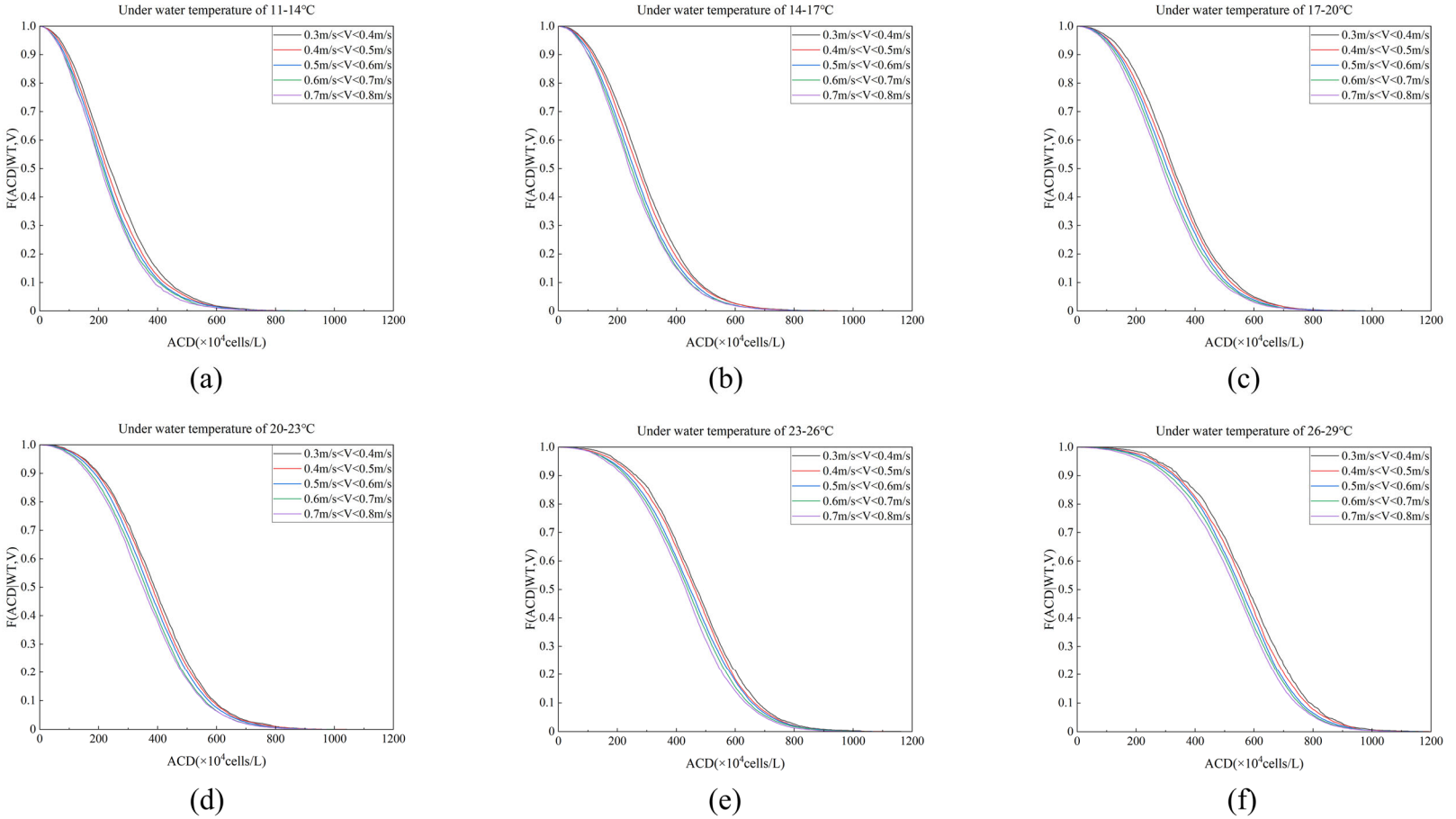


Fig. S9. Conditional probability of ACD at different water temperatures and flow velocity in midstream via 5D Vine Copula.

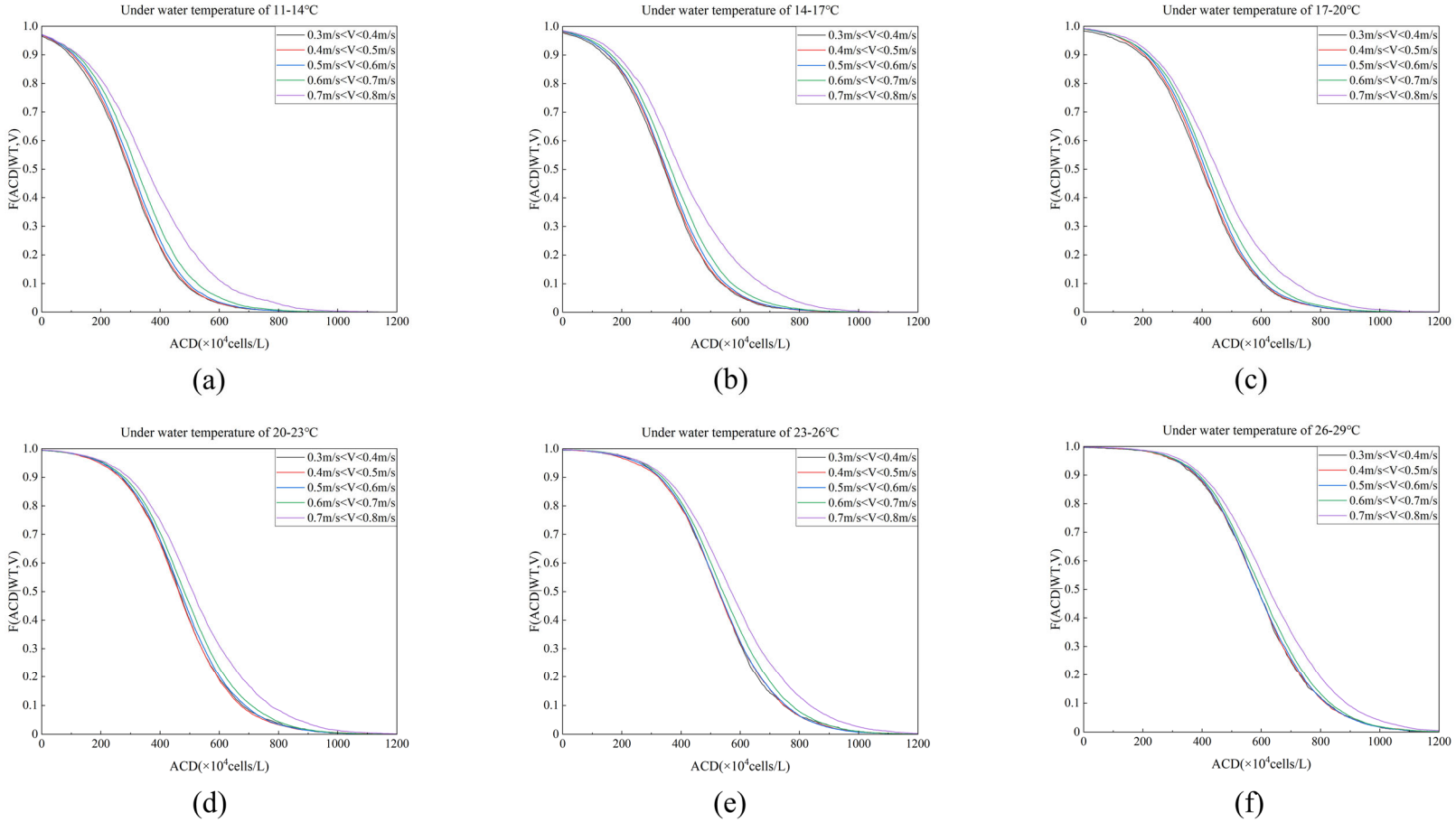


Fig. S10. Conditional probability of ACD at different water temperatures and flow velocity in downstream via 3D Vine Copula.

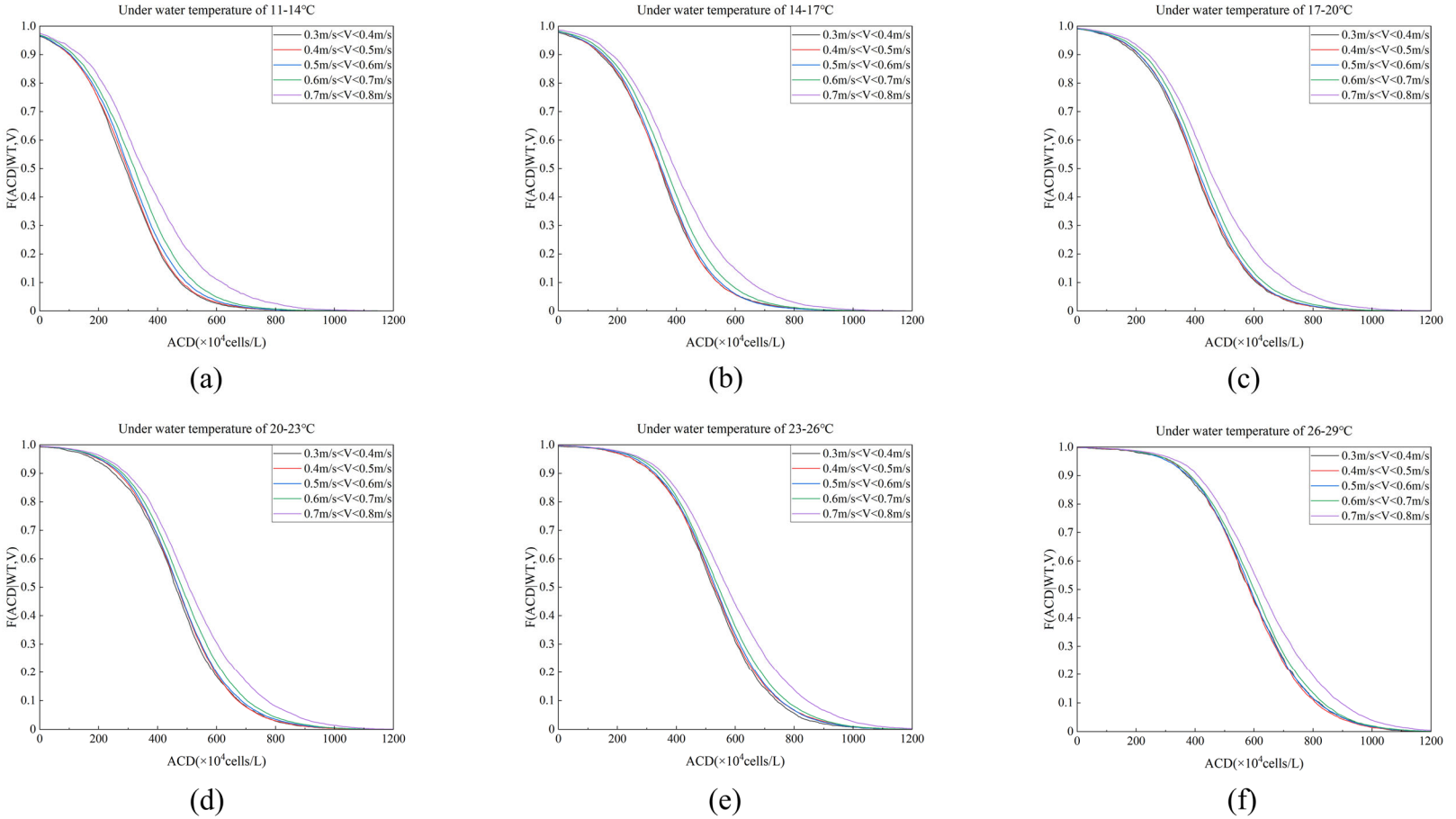


Fig. S11. Conditional probability of ACD at different water temperatures and flow velocity in downstream via 4D Vine Copula.

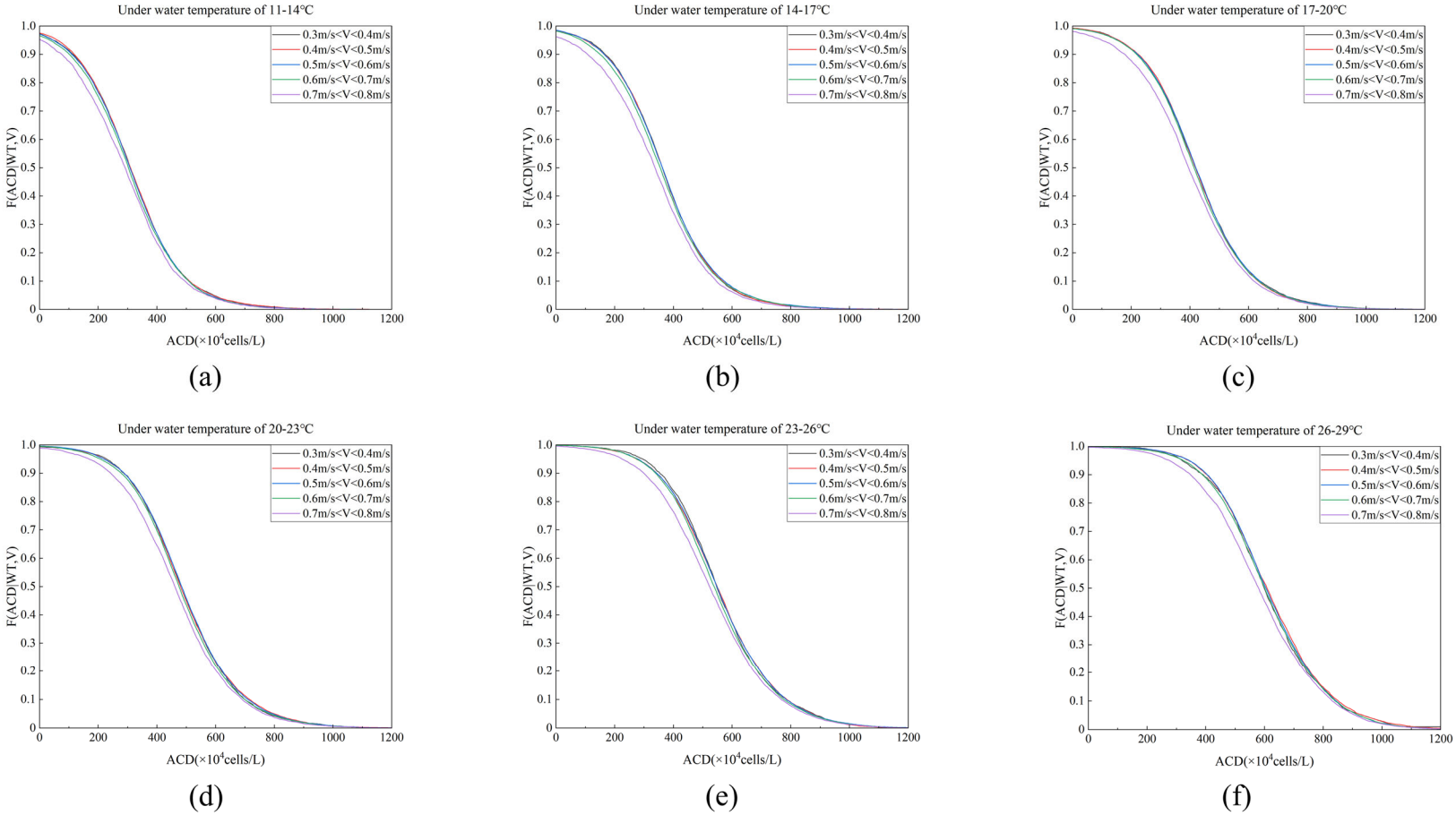
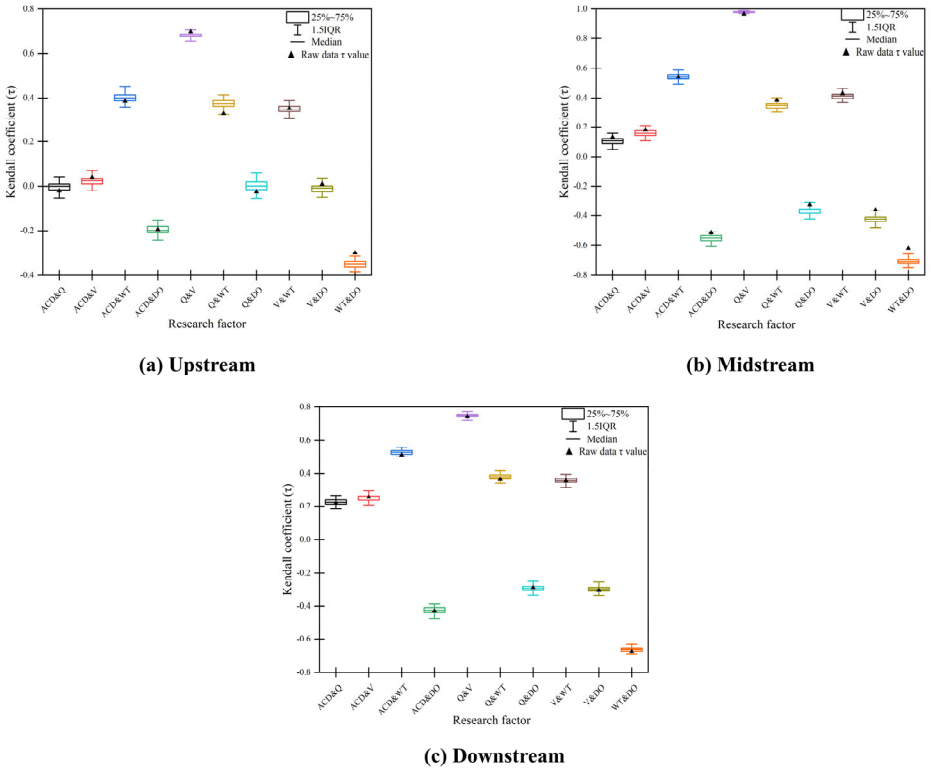


Fig. S12. Conditional probability of ACD at different water temperatures and flow velocity in downstream via 5D Vine Copula.



(a) Upstream **(b) Midstream**

(c) Downstream

Fig. S13. Kendall coefficient model verification.

References

Lee, T., Modarres, R. & Ouarda, T. B. M. J. (2013). Data-based analysis of bivariate copula tail dependence for drought duration and severity. *Hydrol. Process.* 27**,** 1454-1463.

Manstavicius, M. & Leipus, R. (2017). Bounds for the Clayton copula. *Nonlinear Anal-Model.* 22**,** 248-260.