Supplementary Material

1 ENERGY SYSTEM CONSTRAINTS

We model the part-load behavior for each unit using piecewise affine segments $s \in S$ parametrized based on Voll (2014). The relative inputs and outputs for the boilers and the combined-heat-and-power units are shown in Tab. S1. Each unit $u \in U$ supplies or demands one or several products $p \in P$ in time step $t \in T$ and request scenario $\omega \in \Omega$. The variable $P_{u,p,t,\omega}$ considers the supply/demand of a unit u for a product p in time step t and request scenario ω: a positive value of the variable $P_{u,p,t,\omega}$ means supply of the product p; a negative value of the variable $P_{u,p,t,\omega}$ is an endogenous demand for the product p.

Eq. (S1) models the supply/demand as a function of the load $LOAD_{u,t,\omega,s}$:

$$P_{u,p,t,\omega} = \sum_{s \in S} (b_{u,p,s} \cdot \gamma_{u,t,\omega,s} + m_{u,p,s} \cdot LOAD_{u,t,\omega,s}) \quad \forall\, u \in U, p \in P, t \in T, \omega \in \Omega. \tag{S1}$$

Thereby, the operation of each unit u is linearized with $s \in S$ linear segments. Each segment is connected to a fixed input/output $b_{u,p,s}$ that is chosen with the binary variable $\gamma_{u,t,\omega,s}$. The binary variable $\gamma_{u,t,\omega,s}$ indicates if segment s is chosen for unit u in time step t and scenario ω ($\gamma_{u,t,\omega,s} = 1$) or not ($\gamma_{u,t,\omega,s} = 0$). Additionally, the load $LOAD_{u,t,\omega,s}$ is multiplied with the marginal input/output $m_{u,p,s}$ which is the slope between two consecutive fixed inputs/outputs.

Eq. (S2) ensures that at the maximum, only one segment is chosen for each time step, scenario, and unit

$$\sum_{s \in S} \gamma_{u,t,\omega,s} \leq 1 \quad \forall\, u \in U, t \in T, \omega \in \Omega. \tag{S2}$$

Finally, Eq. (S3) and (S4) restrict the $LOAD_{u,t,\omega,s}$ for each time step $t \in T$, unit $u \in U$, scenario $\omega \in \Omega$ and segment $s \in S$ with

$$LOAD_{u,t,\omega,s} \leq \overline{bnd}_{u,s} \cdot \gamma_{u,t,\omega,s} \quad \forall\, u \in U, t \in T, \omega \in \Omega, s \in S, \tag{S3}$$

$$LOAD_{u,t,\omega,s} \geq \underline{bnd}_{u,s} \cdot \gamma_{u,t,\omega,s} \quad \forall\, u \in U, t \in T, \omega \in \Omega, s \in S \tag{S4}$$

If a segment s is chosen for unit u via the binary variable $\gamma_{u,t,\omega,s}$, the load $LOAD_{u,t,\omega,s}$ is restricted between the upper bound $\overline{bnd}_{u,s}$ and lower bound $\underline{bnd}_{u,s}$ of the segment.

Table S1. Relative inputs (negative) and outputs (positive) at minimum part-load and full-load for the boiler (B) and the combined-heat-and-power units (CHP).

		Natural Gas	Heat	Electricity
B	Minimum part-load	-0.243	0.2	
	Full-load	-1.112	1.0	
CHP	Minimum part-load	-1.189	0.5	0.476
	Full load	-2.436	1.0	1.110

2 FORMULATION *GRAPH*

The storage formulation *graph* is based on a recombining scenario tree that models the operation of the energy system, including storage (Fig. S1). In the recombining scenario tree, the nodes $n \in N$ of the tree model the respective storage level, while the arcs $(n_1, n_2) \in A$ of the tree model the operation of the energy system, i.e., the operation of the production and storage units.

To prevent the exponential growth of possible storage levels, the recombination takes place over two time steps each: Starting from the same node, two times steps no request (none, none) or positive and negative request (pos, neg) or negative and positive request (neg, pos) of balancing power lead to the same storage level two time steps later.

In the optimization problem, each arc (n_1, n_2) is weighted in the objective function with the probability π_a to reach the respective arc. The constraints from the presented optimization problem from section 2 are, thus, written for all arcs $a \in A$ instead of all scenarios $\omega \in \Omega$ and time steps $t \in T$. In addition, coupling constraints are introduced for the arcs of each time step since the sale or purchase of electricity and the provision of balancing power must be the same in each time step.

Simplifying, the optimization problem from section 2 is, thus, written using the *graph* formulation as:

$$\begin{aligned} \min\ & TAC = CAPEX + \sum_{a \in A} \pi_a \cdot OPEX_a \\ \text{s.t.}\ & \quad market\ participation \quad \forall\, a \in A, \\ & \quad product\ balance \quad \forall\, a \in A, \\ & \quad system\ operations \quad \forall\, a \in A, \\ & \quad storage\ level \quad \forall\, n \in N, \\ & \quad coupling\ constraints. \end{aligned}$$

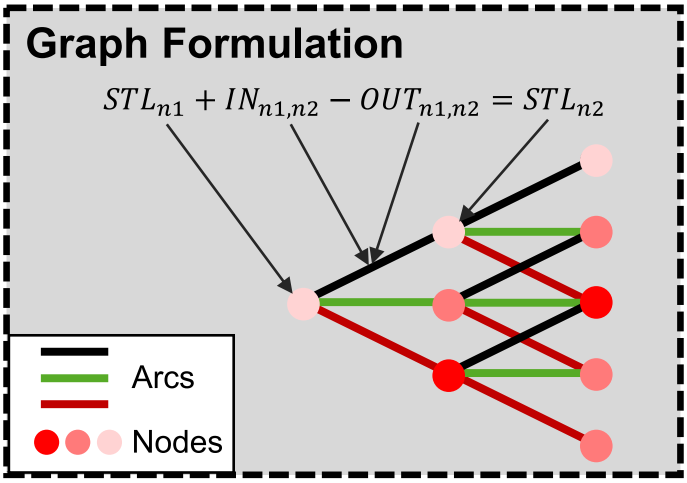

Figure S1. Recombining scenario tree of the formulation graph comprising nodes $n \in N$ and arcs $(n_1, n_2) \in A$. As exemplarily shown with ideal storage equations, the arcs model the energy system operation, e.g., charging IN and discharging OUT, while the nodes track the storage level STL.

3 TIME-VARYING DEMANDS AND PRICES

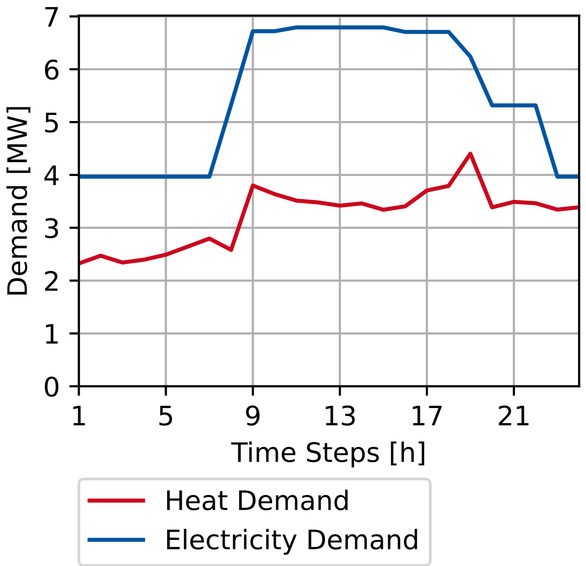

Figure S2. Demand profiles for electricity and heat. The demand profiles show a pattern typical for industrial energy systems: While heat and electricity demands are low during nighttime hours, they increase during business hours - with local peaks in heat demands at 8 am and 6 pm.

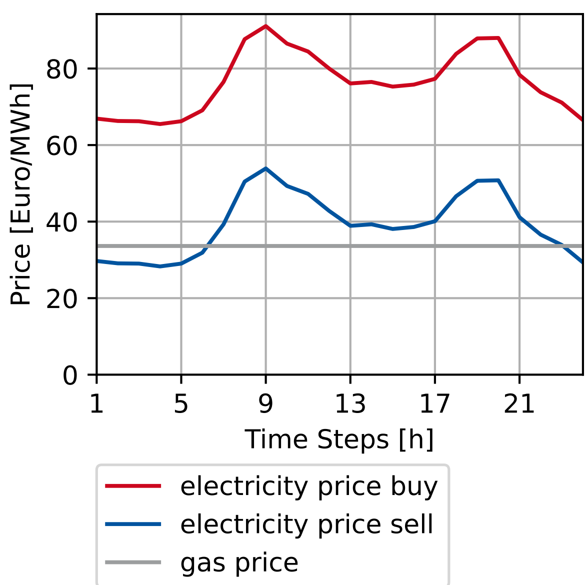

Figure S3. Prices for the purchase and sale of electricity as well as natural gas.

Table S2. Capacity and energy prices for each time slice. Note that the capacity prices are per four-hour time slice, and, thus, need to be divided by 4 to be used in the optimization problem.

	Time slice	**1**	**2**	**3**	**4**	**5**	**6**
Capacity price [Euro/MW/4h]	positive	4.4	15.4	14.2	14.0	19.0	11.4
	negative	8.2	8.2	8.2	8.2	8.2	8.2
Energy price [Euro/MWh]	positive	168.7	177.0	324.3	171.5	202.8	204.9
	negative	118.6	179.0	182.6	327.3	155.5	62.5

4 CASE STUDY RESULT’S USING FOUR TYPICAL DAYS

Here, the optimization of the case study is performed for four typical days. Using more typical days allows for greater variability in the time-dependent parameters, e.g., in the electricity and heat demands (Fig. S5). Still, using four typical days leads to similar results as presented in section 3.

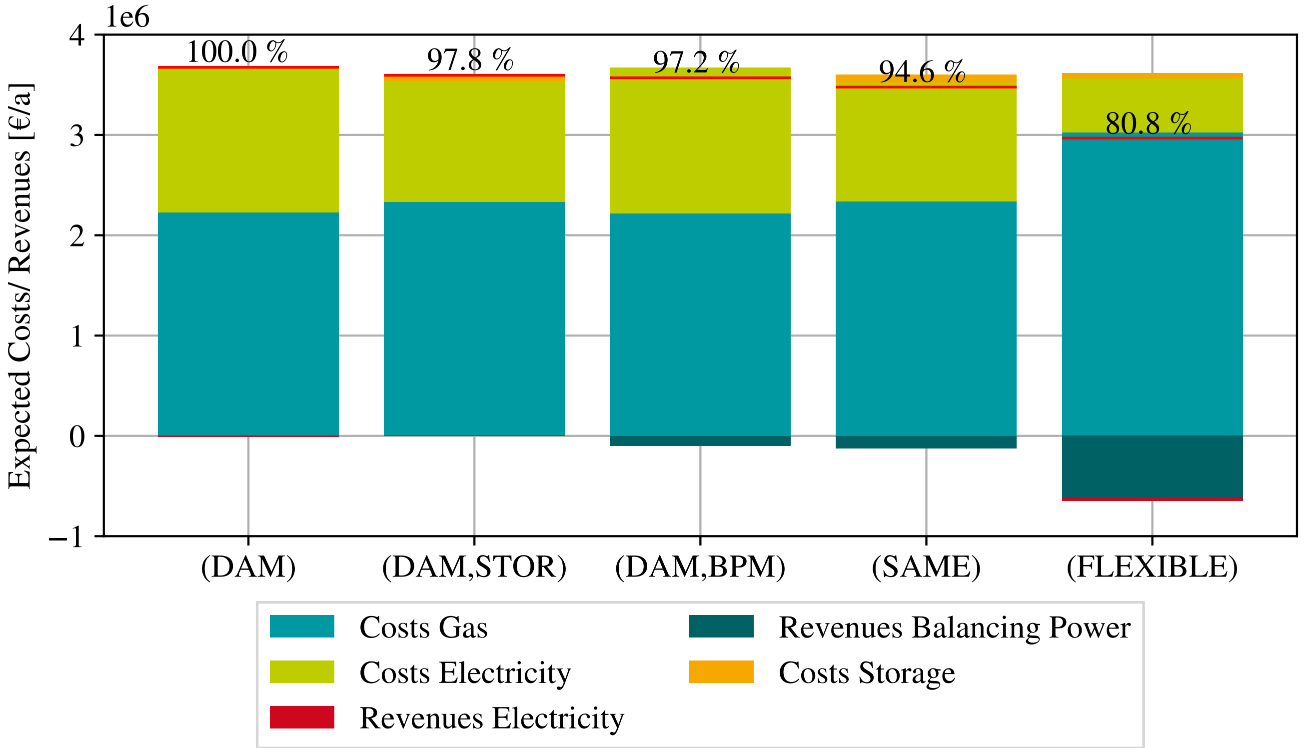

Figure S4. Annual costs and revenues with investment in the heat storage unit and/or additional participation in the balancing-power market using four typical days. The relative total annualized costs, shown as a number above the bar, are referenced to the base case (DAM) with sole participation in the day-ahead market. At each bar, the red line indicates the total costs as the sum of costs and revenues.

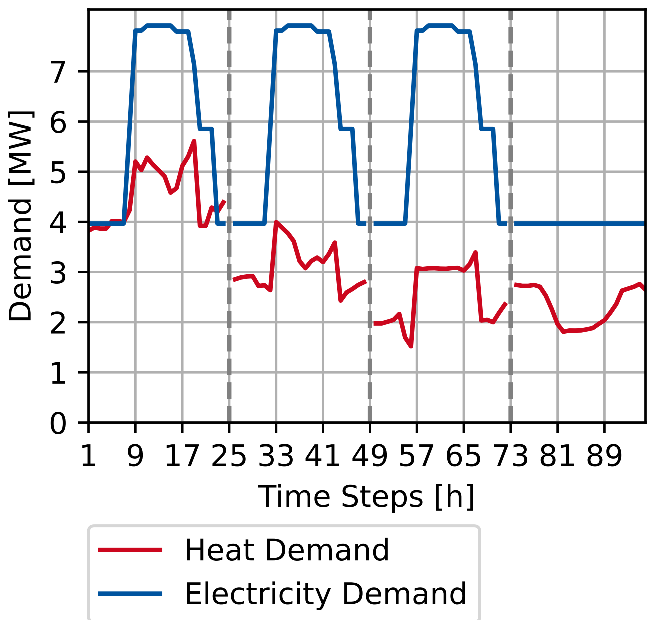

Figure S5. Demand profiles for electricity and heat using four typical days.

REFERENCES

Voll, P. (2014). *Automated optimization-based synthesis of distributed energy supply systems: Aachen, Techn. Hochsch., Diss., 2013*, vol. 1 of *Aachener Beiträge zur Technischen Thermodynamik* (Aachen: Hochschulbibliothek der Rheinisch-Westfälischen Technischen Hochschule Aachen)