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# Supplementary Material

## 1 LASER-PLASMA X-RAY GENERATION MECHANISMS

### 1.1 Laser-wakefield electron acceleration

The subsequent three acceleration mechanisms result from the laser-wakefield electron acceleration. This is calculated according to the scaling presented by Lu *et al.* Lu et al. (2005, 2007). The first equation of note is the energy gain:

$$\Delta E[\text{GeV}] = 1.7 \left( \frac{P}{100} \right)^{\frac{1}{3}} \left( \frac{10^{18}}{n_e} \right)^{\frac{2}{3}} \left( \frac{0.8}{\lambda} \right)^{\frac{4}{3}} \quad (\text{S1})$$

where  $P$  is the laser power expressed in TW,  $n_e$  the electron density of the plasma in  $\text{cm}^{-3}$  and  $\lambda$  the wavelength of the laser in  $\mu\text{m}$ . The second is the charge of the electron beam:

$$C = 2.5 \times 10^9 \left( \frac{\lambda}{0.8} \sqrt{\frac{P}{100}} \right) \quad (\text{S2})$$

For the scaling outlined in Figure 1 of the main text we consider a laser power of  $P = 100, 1000$  and  $4000$  TW with a laser wavelength of  $0.8 \mu\text{m}$  and a plasma density of  $10^{18} \text{cm}^{-3}$ .

### 1.2 Betatron Radiation

Betatron radiation scales as a function of the electron energy, and observed angle theta. For the scalings presented in Figure 1 of the main text we calculated the peak electron energy from Eqn S1 and the total charge to normalise the following spectra from Eqn S2. The spectral form for the betatron emission is then given by Wood (2017); Albert et al. (2018):

$$\frac{d^2 I}{d\Omega d\omega} = \frac{3e^2}{16\pi^3 \hbar c \epsilon_0} \gamma^2 \left( \frac{E}{E_c} \right)^2 \left[ K_{2/3}^2(\zeta) + \frac{\gamma^2 \theta^2}{1 + \gamma^2 \theta^2} K_{1/3}^2(\zeta) \right] \quad (\text{S3})$$

Where  $e, \pi, \hbar, c$ , and  $\epsilon_0$  have their normal definitions,  $\gamma$  is the Lorentz factor for the electron calculated as  $\gamma = E_e/m_e c^2 + 1$ ,  $\theta$  is the observation angle for the radiation and  $K_{1/3}$  and  $K_{2/3}$  are modified Bessel functions.  $\zeta = E/2E_c(1 + \gamma^2 \theta^2)^{3/2}$  where  $E_c$  is the critical energy of emission calculated from:

$$E_c = \hbar \frac{3}{4c} \gamma^2 \omega_p^2 r_\beta \quad (\text{S4})$$

Where  $\omega_p$  is the plasma frequency (related to the plasma density  $n_e$ ) and  $r_\beta$  is the orbital oscillation radius of the electrons within the plasma, here calculated as  $0.2 \mu\text{m}$  in line with existing literature Wood (2017). The beam divergence, important for the calculations in Figure 7 from the main text, is calculated as  $\theta_e = 1/\gamma$  Albert et al. (2018).

### 1.3 Inverse-Compton Scattering

By contrast this is relatively simple to calculate, the scaling from Tsai *et al.* Tsai et al. (2015) demonstrates a conversion efficiency to inverse compton scattered x-ray photons of  $\% \approx 10^{-6}$ . The resultant x-ray is calculated from the electron energy by the Compton scaling factor as:

$$E_x = 4\gamma^2 E_\ell \quad (\text{S5})$$

Where  $E_\ell$  is the energy of a laser photon  $\approx 1.5$  eV. This results in an x-ray distribution that matches the bandwidth of the electron beam converted by Eqn S5. In Figure 1 we assume a bandwidth in the electron beam of 2, 5, and 10% and a peak electron energy of 0.1, 1, and 10 GeV.

### 1.4 LWFA and LPI Bremsstrahlung

For both Bremsstrahlung spectra we assume the same Maxwell-Boltzmann distributions:

$$dN/dE = N_\gamma \sqrt{\frac{E}{k_B T_x^3}} \exp\left(\frac{-E}{k_B T_x}\right) \quad (\text{S6})$$

where  $E$  is the x-ray energy and  $k_B T_x$  the effective x-ray temperature Giulietti and Gizzi (1998), the Maxwellian distribution rather than a single exponential better represents the electron distribution in solid-target interactions and approximates the self-screening of lower energy electrons Giulietti and Gizzi (1998); Armstrong et al. (2019); Cardarelli et al. (2018); Brenner et al. (2015); Borm et al. (2019). For LWFA driven Bremsstrahlung the electron charge is calculated from Eqn S2 as before, the effective x-ray temperature and conversion efficiency is taken from Underwood *et al.* Underwood et al. (2020). For laser-solid driven Bremsstrahlung the scaling for conversion efficiency is instead taken from Armstrong *et al.* Armstrong et al. (2019), with the x-ray energy scaling from 0.1 MeV to 10 MeV to account for the lower peak acceleration expected in laser-solid interactions.

## 2 LENS COLLECTION EFFICIENCY

Equation 2 from the main text outlines a heuristic approach to calculating the signal expected per pixel in a given system. For the benefit of the reader the equation is delineated in three parts, the energy deposited per area, the collection efficiency of the lens system, and the spreading over multiple pixels. The second term here is the least similar to other approaches Koch et al. (1998); Cardarelli et al. (2018) who use different approximations to calculate the lens efficiency term. The term we use is relatively straight forward to define, first we take the area of a spherical cap relative to surface area of a sphere (i.e. a  $4\pi$  emitting volume):

$$\frac{A_{lens}}{A_{sphere}} = \frac{1 - \cos \theta}{2} \quad (\text{S7})$$

The numerical aperture of a lens system is dependent on the scintillators refractive index  $\eta$  and the subtended angle  $\theta$ , better known as Snell's Law:  $NA = \eta \sin(\theta)$ . Therefore we can re-express the above equation as:

$$\frac{A_{lens}}{A_{sphere}} = \frac{1 - \cos \sin^{-1}(\text{NA}/\eta)}{2} \quad (\text{S8})$$

And as the trigonometric identity,  $\cos \sin^{-1} x = \sqrt{1 - x^2}$  can be simplified by a Taylor expansion to  $\approx 1 - x^2/2$  the above equation can be simplified further to:

$$\frac{A_{lens}}{A_{sphere}} \approx \frac{\text{NA}^2}{4\eta^2} \quad (\text{S9})$$

### 3 COMMERCIAL AVAILABLE LENS/OBJECTIVE PARAMETERS

The parameters used to calculate Figure 2d) are taken from the manufacturer values presented by retailers of the lenses - this is by no means an exhaustive list 3. For objectives numerical aperture and magnification are directly quoted, for machine vision lenses the numerical aperture is calculated from the minimum working distance and maximum aperture size. The magnification is calculated assuming a thin lens system  $M_o \approx f/f - d_o$ .

	Model	NA	Mo	D	f	$d_o$	Reference
Mitutoyo	1x	0.025	1	-	-	-	[A]
Mitutoyo	2x	0.055	2	-	-	-	[A]
Mitutoyo	5x	0.140	5	-	-	-	[A]
Mitutoyo	5x HR	0.210	5	-	-	-	[A]
Mitutoyo	7.5x	0.210	7.5	-	-	-	[A]
Mitutoyo	10x	0.280	10	-	-	-	[A]
Mitutoyo	10x HR	0.420	10	-	-	-	[A]
Mitutoyo	20x	0.280	20	-	-	-	[A]
Mitutoyo	20x HR	0.420	20	-	-	-	[A]
EHD	25085	0.162	0.172	55	25	170	[B]
EHD	50085	0.135	0.263	65	50	240	[B]
Edmund Optics	25mm C Series	0.071	0.333	-	25	100	[C]
Edmund Optics	35mm C Series	0.056	0.269	-	35	165	[C]
Edmund Optics	50mm C Series	0.044	0.250	-	50	250	[C]
Edmund Optics	100mm C Series	0.023	0.154	-	100	750	[C]
Hamamatsu	Tapered FOP	$\approx 1$	0.500	-	-	-	[D]

**Table S1.** Lens/objective parameters used for Figure 2 in the main text. Parameters herein are taken from manufacturer and retailer websites; [A] <https://www.edmundoptics.co.uk/f/mitutoyo-infinity-corrected-long-working-distance-objectives/12298/>, [B] <https://www.ehd.de/products/objectives/EHD-F085-Lenses.pdf>, [C] <https://www.edmundoptics.co.uk/f/c-series-fixed-focal-length-lenses/13679/>, [D] <https://www.hamamatsu.com/eu/en/product/optical-components/fop.html>.

Paired with these lenses and objectives is then a camera sensor with a defined pixel size and quantum efficiency at the wavelength of the scintillator 550 nm 3. The scintillator used for the modelling was CsI:TI with thicknesses of 5, 50, 500, and 5000  $\mu\text{m}$  with the attenuation to a 100 keV Betatron distribution used to determined the energy deposited.

	Model	dx	QE(550 nm)	Reference
AVT	Manta 235b	5.86	0.65	[E]
AVT	Manta 033b	6.5	0.65	[E]
Andor	Zyla 5.5	6.5	0.85	[F]
Andor	iXon Ultra 888	13	0.85	[G]
Hamamatsu	Orca Quest	4.85	0.95	[H]

**Table S2.** Sensor parameters used for Figure 2 in the main text. Parameters herein are taken from manufacturer and retailer websites; [E] <https://www.alliedvision.com/en/products/camera-series/manta/>, [F] <https://andor.oxinst.com/products/zyla-zl41-scmos-cameras>, [G] <https://andor.oxinst.com/products/ixon-emccd-cameras-for-physical-science>, [H] [https://www.hamamatsu.com/content/dam/hamamatsu-photonics/sites/documents/99\\_SALES\\_LIBRARY/sys/SCAS0151\\_C15550-20UP.pdf](https://www.hamamatsu.com/content/dam/hamamatsu-photonics/sites/documents/99_SALES_LIBRARY/sys/SCAS0151_C15550-20UP.pdf)

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