Supplementary Material

1 SUPPLEMENTARY DATA

1.1 Model state variables

The tracer conservation equation is:

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial z}(k_z \frac{\partial C}{\partial z}) + S(C), \quad \text{(S1)}$$

where k_z is the turbulent diffusivity and $S(C)$ is the source term of tracer C explained as follows.

1.1.1 Core model

The state variables in the core model [1] are phytoplankton (P), zooplankton (Z), nitrate (NO_3), ammonium (NH_4), semi-labile dissolved organic matter (DOM), small particulate organic matter (sPOM), and big particulate organic matter (bPOM), expressed in terms of their nitrogen content (mmolN m^{-3}), as well as their carbon compartments: DOC, sPOC and bPOC in mmolC m^{-3}. The source terms of these variables are as follows.

1.1.1.1 Phytoplankton

The source term of phytoplankton can be described as:

$$S(P) = (1-\gamma)\mu_P L_{\mathrm{PAR}} \left(L_{\mathrm{NO_3}} + L_{\mathrm{NH_4}}\right) P - G_P - m_P P^2, \quad \text{(S2)}$$

where γ is the phytoplankton exudation rate, μ_P is the phytoplankton maximal growth rate, G_P is the grazing of phytoplankton, and m_P is phytoplankton mortality rate. The growth of phytoplankton is limited by photosynthetically available radiation (PAR) and nutrients in the form of nitrate ($\mathrm{NO_3}$) and ammonium ($\mathrm{NH_4}$). The limit equations are given by:

$$L_{\mathrm{PAR}} = 1 - e^{-\mathrm{PAR}/k_{\mathrm{PAR}}}, \quad \text{(S3)}$$

$$L_{\mathrm{NO_3}} = \frac{\mathrm{NO_3}}{\mathrm{NO_3} + k_{\mathrm{NO_3}}} e^{-\psi \mathrm{NH_4}}, \quad \text{(S4)}$$

$$L_{\mathrm{NH_4}} = \frac{\mathrm{NH_4}}{\mathrm{NH_4} + k_{\mathrm{NH_4}}}, \quad \text{(S5)}$$

where k_{PAR}, $k_{\mathrm{NO_3}}$, and $k_{\mathrm{NH_4}}$ are the half-saturation values of PAR, $\mathrm{NO_3}$, and $\mathrm{NH_4}$, respectively, and ψ represents inhibition of nitrate uptake by ammonium.

The PAR at each depth $\mathrm{PAR}(z)$ is computed from a two-wavelength light absorption model(Levy Marina, 2001) as follows:

$$\mathrm{PAR}(z) = \frac{\mathrm{PAR}(0)}{2} \sum_{i\in[r,b]} \left(\exp\left(-k_i^w z - \int_0^z \chi_i (\mathrm{Chl}/r_{pig})^{e_i} dz \right) \right). \quad \text{(S6)}$$

[1] https://oceanbiome.github.io/OceanBioME.jl/stable/model_components/biogeochemical/LOBSTER/

At the sea surface, it is assumed that the PAR is evenly distributed between red and blue light. The light is attenuated throughout depth by water and chlorophyll inferred from the phytoplankton concentration. k_i^w is water attenuation coefficient, χ_i chlorophyll attenuation coefficient, r_{pig} is the pigment ratio, and e_i is chlorophyll exponent. $\mathrm{Chl} = R_{\mathrm{Chl}} P$ is the chlorophyll concentration (in mg Chl m^{-3}) in the water and R_{Chl} is the chlorophyll/phytoplankton ratio.

1.1.1.2 Zooplankton

The source term of zooplankton can be described as:

$$S(Z) = a_z \left(G_P + G_{\mathrm{sPOM}}\right) - m_Z Z^2 - \mu_Z Z, \tag{S7}$$

where a_z is assimilated food fraction by zooplankton, m_Z is zooplankton mortality rate, and μ_Z is the zooplankton excretion rate. The growth of zooplankton occurs from grazing of both phytoplankton and small particulate organic matter. The grazing equations are given by:

$$G_P = g_z \frac{pP}{k_z + pP + (1-p)\mathrm{sPOM}} Z, \tag{S8}$$

$$G_{\mathrm{sPOM}} = g_z \frac{(1-p)\mathrm{sPOM}}{k_z + pP + (1-p)\mathrm{sPOM}} Z, \tag{S9}$$

where g_z is zooplankton maximal grazing rate, k_z is half-saturation value of grazing, and $p = \frac{\tilde{p}P}{\tilde{p}P + (1-\tilde{p})\mathrm{sPOM}}$ is the grazing preference for phytoplankton where $\tilde{p}$ is the preference parameter.

1.1.1.3 Nitrate

The source term of nitrate can be described as:

$$S(\mathrm{NO_3}) = -\mu_P L_{\mathrm{PAR}} L_{\mathrm{NO_3}} P + \mu_n \mathrm{NH_4}, \tag{S10}$$

where μ_n is the nitrification rate.

1.1.1.4 Ammonium

The source term of ammonium can be described as:

$$\begin{aligned} S(\mathrm{NH_4}) = {} & -\mu_P L_{\mathrm{PAR}} L_{\mathrm{NH_4}} P - \mu_n \mathrm{NH_4} + \alpha_P \gamma \mu_P L_{\mathrm{PAR}} \left(L_{\mathrm{NO_3}} + L_{\mathrm{NH_4}}\right) P \\ & + \alpha_Z \mu_Z Z + \alpha_D \mu_{\mathrm{sPOM}} \mathrm{sPOM} + \alpha_D \mu_{\mathrm{bPOM}} \mathrm{bPOM} + \mu_{\mathrm{DOM}} \mathrm{DOM}, \end{aligned} \tag{S11}$$

where α_P, α_Z, α_D are $\mathrm{NH_4}$/DOM redistribution ratios from P, Z and POM respectively, and μ_{sPOM}, μ_{bPOM} and μ_{DOM} are remineralization rates of sPOM, bPOM, and DOM respectively.

1.1.1.5 Semi-labile dissolved organic matter

The source term of semi-labile dissolved organic matter can be described as:

$$\begin{aligned} S(\mathrm{DOM}) = {} & (1-\alpha_P)\gamma \mu_P L_{\mathrm{PAR}} \left(L_{\mathrm{NO_3}} + L_{\mathrm{NH_4}}\right) P + (1-\alpha_Z)\mu_Z Z \\ & + (1-\alpha_D)\mu_{\mathrm{sPOM}} \mathrm{sPOM} + (1-\alpha_{\mathrm{D}})\mu_{\mathrm{bPOM}} \mathrm{bPOM} - \mu_{\mathrm{DOM}} \mathrm{DOM}. \end{aligned} \tag{S12}$$

The carbon compartment DOC is given as follows:

$$S(\mathrm{DOC}) = (1-\alpha_P)\gamma\mu_P L_{\mathrm{PAR}}\left(L_{\mathrm{NO_3}} + L_{\mathrm{NH_4}}\right) R_P P + (1-\alpha_Z)\mu_Z R_P Z \\ + (1-\alpha_D)\mu_{\mathrm{sPOM}}\mathrm{sPOC} + (1-\alpha_{\mathrm{D}})\mu_{\mathrm{bPOM}}\mathrm{bPOC} - \mu_{\mathrm{DOM}}\mathrm{DOC} \quad \text{(S13)}$$

where R_P is the Redfield ratio (C:N) for phytoplankton.

1.1.1.6 Particulate organic matter

The source term of particulate organic matter can be described as:

$$S(\mathrm{sPOM}) = f_s\big[\big(1-a_Z\big)\left(G_P + G_{\mathrm{sPOM}}\right) + m_P P^2 + m_Z Z^2\big] \\ - G_{\mathrm{sPOM}} - \mu_{\mathrm{sPOM}}\mathrm{sPOM} - \frac{\partial}{\partial z}(\mathrm{sPOM} w_s), \quad \text{(S14)}$$

and

$$S(\mathrm{bPOM}) = (1-f_s)\left[(1-a_Z)\left(G_P + G_{\mathrm{sPOM}}\right) + m_P P^2 + m_Z Z^2\right] \\ - \mu_{\mathrm{bPOM}}\mathrm{bPOM} - \frac{\partial}{\partial z}(\mathrm{bPOM} w_b), \quad \text{(S15)}$$

where f_s is the fraction of sPOM contributed by phytoplankton/zooplankton mortality and zooplankton sloppy grazing, and w_s and w_b are sinking speed of sPOM and bPOM respectively.

The carbon compartment sPOC and bPOC are given as:

$$S(\mathrm{sPOC}) = f_s\big[\big(1-a_Z\big)\left(G_P + G_{\mathrm{sPOM}}\right) + m_P P^2 + m_Z Z^2\big] R_P \\ - G_{\mathrm{sPOM}} R_P - \mu_{\mathrm{sPOM}}\mathrm{sPOC} - \frac{\partial}{\partial z}(\mathrm{sPOC} w_s), \quad \text{(S16)}$$

and

$$S(\mathrm{bPOC}) = (1-f_s)\left[(1-a_Z)\left(G_P + G_{\mathrm{sPOM}}\right) + m_P P^2 + m_Z Z^2\right] R_P \\ + (G_P(1-\eta) + m_P P^2) R_P \rho_{\mathrm{CaCO_3}} - \mu_{\mathrm{sPOM}}\mathrm{sPOC} - \frac{\partial}{\partial z}(\mathrm{sPOC} w_b) \quad \text{(S17)}$$

where $\rho_{\mathrm{CaCO_3}}$ is calcium carbonate/organic carbon ratio and η is fraction of calcium carbonate dissolved in zooplankton gut.

1.1.2 Carbonate chemistry model

The carbonate chemistry model consists two state variables: dissolved inorganic carbon (DIC) in mmolC m^{-3} and alkalinity (Alk) in meq m^{-3}, given as follows:

$$S(\text{DIC}) = -\mu_P L_{\text{PAR}} \left(L_{\text{NO}_3} + L_{\text{NH}_4}\right)(1 + \rho_{\text{CaCO}_3}(1-\gamma))PR_P \\ + \alpha_Z \mu_Z Z R_P + \alpha_D \mu_{\text{sPOM}} \text{sPOC} + \alpha_D \mu_{\text{bPOM}} \text{bPOC} + \mu_{\text{DOM}} \text{DOC} \\ + \alpha_P \gamma \mu_P L_{\text{PAR}} \left(L_{\text{NO}_3} + L_{\text{NH}_4}\right) PR_P + G_P \eta R_P \rho_{\text{CaCO}_3} + \text{air} - \text{sea CO}_2 \text{ flux}, \quad \text{(S18)}$$

and

$$S(\text{Alk}) = \mu_P L_{\text{PAR}} L_{\text{NO}_3} P - 2\rho_{\text{CaCO}_3} \mu_P L_{\text{PAR}} \left(L_{\text{NO}_3} + L_{\text{NH}_4}\right) PR_P. \quad \text{(S19)}$$

The first term on the right of $S(\text{Alk})$ is due to the consumption of nitrate by phytoplankton which increases the alkalinity, and the second is due to the assimilation of calcium carbonate by phytoplankton which reduces alkalinity.

1.1.2.1 *Air-sea* CO_2 *flux*

The air-sea CO_2 flux is expressed as follows(Wanninkhof, 2014):

$$F = k\text{K}_0 \rho_o (p\text{CO}_{2w} - p\text{CO}_{2a}), \quad \text{(S20)}$$

where F is the flux ($\text{mol s}^{-1}\text{ m}^{-2}$), k is the gas transfer velocity (m s^{-1}), K_0 is the solubility ($\text{mol kg}^{-1}\text{ atm}^{-1}$) dependent on temperature (T) and salinity (S) (Weiss, 1974), ρ_o is the seawater density (kg m^{-3}), and $p\text{CO}_{2w}$and $p\text{CO}_{2a}$ are partial pressure of CO_2 (atm) in surface seawater and in the above-lying air respectively. $p\text{CO}_{2w}$, dependent on DIC, Alk, temperature (T) and salinity (S), is calculated based on simplified carbonate equilibria including dissolved CO_2, bicarbonate ion HCO_3^-, carbonate ion CO_3^{2-}, hydrogen ion H^+, and borate HBO_3^-(Follows et al., 2006; Millero, 1995). The gas transfer velocity is given by(Wanninkhof, 1992):

$$k = 1.08 \times 10^{-6} u^2 \left(\frac{\text{Sc}}{660}\right)^{-1/2}, \quad \text{(S21)}$$

where u is the wind speed at 10 m above the sea surface and Sc is the Schmidt number dependent on temperature.

1.1.3 Kelp growth model

The state variables of the kelp growth model[2] (Broch and Slagstad, 2012; Broch et al., 2013) include frond area A in dm^2, nitrogen reserves N in gram N per gram structural mass (sw) (g N $(\text{g sw})^{-1}$), and carbon reserves C in g C $(\text{g sw})^{-1}$. The main model equations are as follows:

$$\frac{dA}{dt} = [\mu(A, N, C, T, t) - \nu(A)]A, \quad \text{(S22)}$$

$$\frac{dN}{dt} = k_A^{-1} J - \mu(A, N, C, T, t)(N + N_{\text{struct}}), \quad \text{(S23)}$$

$$\frac{dC}{dt} = k_A^{-1}[P(I, T)(1 - E(C)) - R(T)] - \mu(A, N, C, T, t)(C + C_{\text{struct}}), \quad \text{(S24)}$$

[2] https://oceanbiome.github.io/OceanBioME.jl/stable/model_components/individuals/slatissima/

where μ is the specific growth rate (area) as a function of A, N, C, temperature T, and time t. The relative rate of frond loss, ν, accounts for frond erosion and is modeled as

$$\nu(A) = \frac{10^{-6}\exp(\epsilon \mathrm{A})}{1 + 10^{-6}(\exp(\epsilon \mathrm{A}) - 1)}, \tag{S25}$$

following Broch and Slagstad (2012), where the number 10^{-6} indicates the rate at which frond is lost when $A = 0$, while ϵ controls the sensitivity of the erosion rate to the frond area, A. As discussed in Broch and Slagstad (2012), this function is inspired by the observation of Sjøtun (1993) that longer laminae (with larger areas) are more easily eroded than shorter ones.

To test the sensitivity of our results to the parameterized erosion rate, we varied the parameter ϵ by $\pm 50\%$ about its baseline value (see table S1) for the simulation at the kelp density of 1.1 fronds m^{-3}. Figure S2 shows the timeseries of the gravitational pump and the air-sea CO_2 flux for the baseline simulation and the simulations where ϵ was varied. Both timeseries are qualitatively similar when varying ϵ. When ϵ is reduced by 50%, the maximum amplitude of the gravitational pump decreases by 13.7% while the maximum amplitude of the air-sea CO_2 flux increases by 2.4%. Similarly, increasing ϵ by 50% decreases the maximum amplitude of the gravitational pump and the air-sea CO_2 flux by 4.9% and 2.1%, respectively. The 'sensitivity' for an output variable y to changes in control parameter (in this case ϵ) can be defined as:

$$S = \frac{\triangle y / y}{\triangle \epsilon / \epsilon}. \tag{S26}$$

If $|S| < 1$, the percentage change in the output variable is smaller than the percentage change in ϵ. Table S2 shows the sensitivity of the carbon flux associated with the gravitational pump and the air-sea CO_2 flux to changes in the erosion parameter, ϵ. In all cases $|S| < 1$, implying that our results are not highly sensitive to the erosion parameter.

Additionally, k_A is the structural dry weight per unit area, J is the nitrogen uptake rate per unit area, N_{struct} is the amount of nitrogen per unit dry weight of structural mass, P is the gross photosynthesis as a function of irradiance I and T, E is the exudation rate, R is the respiration rate, and C_{struct} is the amount of carbon per unit dry weight of structural mass. The model has been modified to take into account the uptake of both nitrate and ammonium(Fossberg et al., 2018) and the ability of sugar kelp to remove inorganic carbon from seawater(Maberly, 1990). The kelp growth model is coupled with the biogeochemical model through uptake of $\mathrm{NO_3}$, $\mathrm{NH_4}$, and DIC, release of DOM, and the erosion of frond as bPOM.

2 SUPPLEMENTARY TABLES AND FIGURES

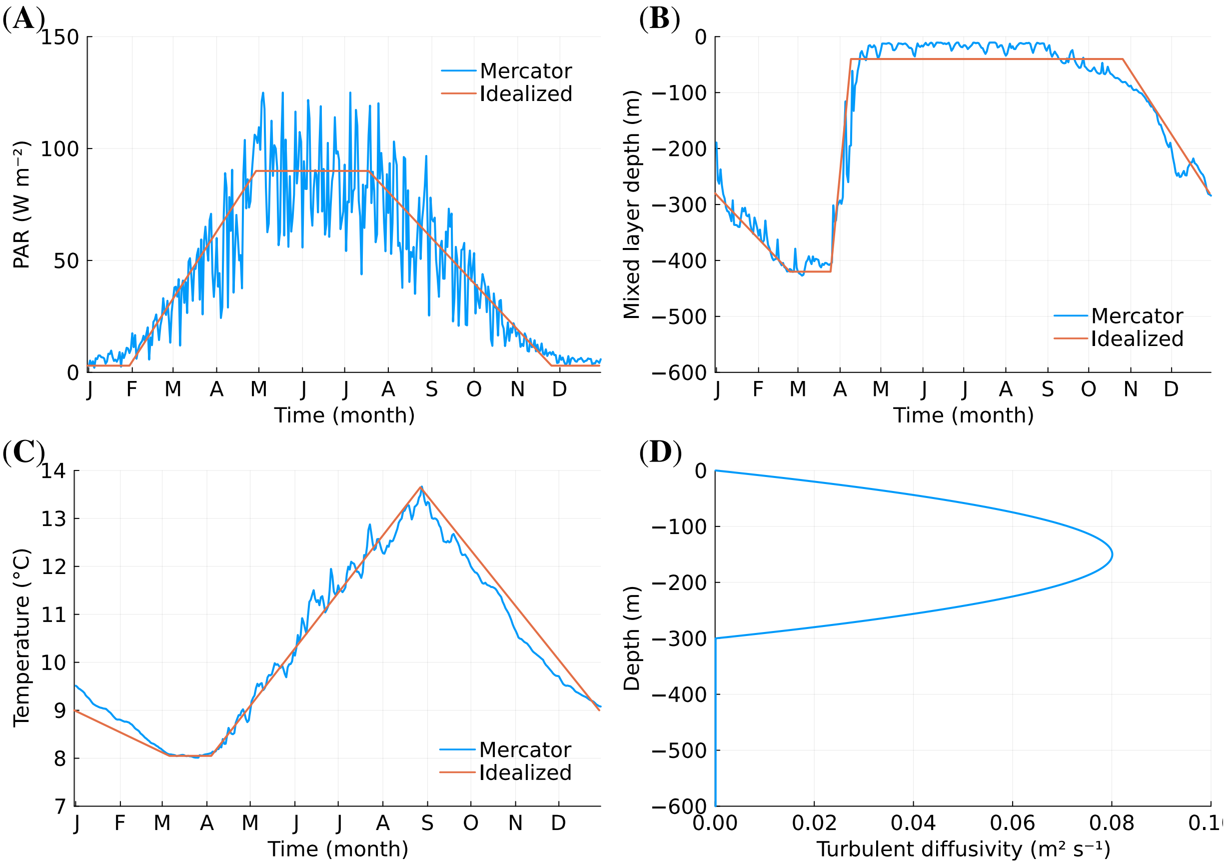

Figure S1. Parameter idealization. (**A**) Idealized annual cycle of PAR. (**B**) Idealized annual cycle of mixed layer depth. (**C**) Idealized annual cycle of temperature. (**D**) Idealized turbulent diffusivity when the mixed layer depth is 300 m.

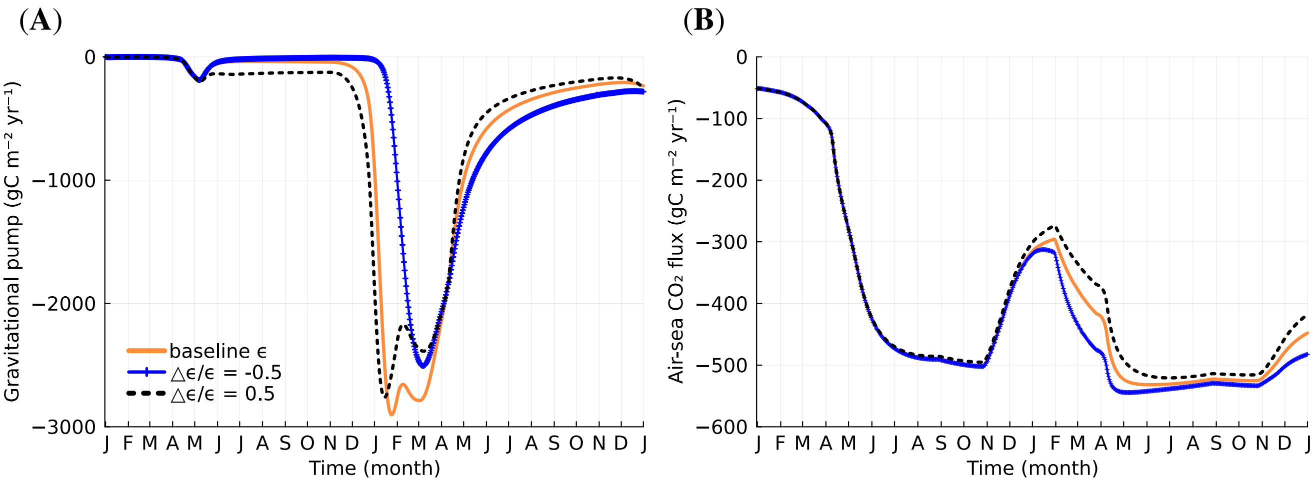

Figure S2. Sensitivity of the gravitational pump (**A**) and the air-sea CO_2 flux (**B**) to changes in the erosion parameter, ϵ, for a kelp density of 1.1 fronds m^{-3}. ϵ was perturbed $\pm 50\%$ about the baseline value.

Table S1. OceaBioME model parameters

Symbol	Parameter	Value	Unit
γ	Phytoplankton exudation rate	0.05	-
μ_P	Phytoplankton maximal growth rate	1.21×10^{-5}	s^{-1}
m_P	Phytoplankton mortality rate	5.80×10^{-7}	m^3 (s mmol N)$^{-1}$
k_{PAR}	Half-saturation values of PAR	33.0	W m^{-2}
k_{NO_3}	Half-saturation values of NO_3	0.7	mmol N m^{-3}
k_{NH_4}	Half-saturation values of NH_4	0.001	mmol N m^{-3}
ψ	Inhibition of NO_3 uptake by NH_4	3.0	-
k_r^w	Water attenuation coefficient in red	0.225	m^{-1}
k_b^w	Water attenuation coefficient in blue	0.0232	m^{-1}
χ_r	Chlorophyll attenuation coefficient in red	0.037	m^{-1}(mg Chl $m^{-3})^{-e_r}$
χ_b	Chlorophyll attenuation coefficient in blue	0.074	m^{-1}(mg Chl $m^{-3})^{-e_b}$
e_r	Chlorophyll red exponent	0.629	-
e_b	Chlorophyll blue exponent	0.674	-
r_{pig}	Pigment ratio	0.7	-
R_{Chl}	Chlorophyll/phytoplankton ratio	1.31	g Chl (mol N)$^{-1}$
a_z	Assimilated food fraction by zooplankton	0.7	-
m_Z	Zooplankton mortality rate	2.31×10^{-6}	m^3 (s mmol N)$^{-1}$
μ_Z	Zooplankton excretion rate	5.80×10^{-7}	s^{-1}
g_z	Zooplankton maximal grazing rate	9.26×10^{-6}	s^{-1}
k_z	Zooplankton half-saturation value of grazing	1.0	mmol N m^{-3}
$\tilde{p}$	Grazing preference parameter for phytoplankton	0.5	-
μ_n	Nitrification rate	5.80×10^{-7}	s^{-1}
α_P	NH_4/DOM redistribution ratio from P	0.75	-
α_Z	NH_4/DOM redistribution ratio from Z	0.5	-
α_D	NH_4/DOM redistribution ratio from POM	0.0	-
μ_{sPOM}	Remineralization rate of sPOM	5.88×10^{-7}	s^{-1}
μ_{bPOM}	Remineralization rate of bPOM	5.88×10^{-7}	s^{-1}
μ_{DOM}	Remineralization rate of DOM	3.86×10^{-7}	s^{-1}
R_P	Redfield ratio (C:N) for phytoplankton	6.56	mol C (mol N)$^{-1}$
f_s	Fraction of sPOM by mortality/sloppy grazing	0.5	-
w_s	Sinking speed of sPOM	3.0	m day^{-1}
w_b	Sinking speed of bPOM	200.0	m day^{-1}
ρ_{CaCO_3}	Calcium carbonate/organic carbon ratio	0.1	mol $CaCO_3$ (mol C)$^{-1}$
η	Fraction of $CaCO_3$ dissolved in zooplankton gut	0.3	-
ρ_o	Seawater density	1027	kg m^{-3}
$p\text{CO}_{2a}$	Partial pressure of CO_2 in the air	400×10^{-6}	atm
u	Wind speed at 10 m above the sea surface	10	m s^{-1}
ϵ	Erosion parameter	$0.22A^{-1}$	day^{-1} A^{-1}
k_A	Structural dry weight per unit area	0.5	g dm^{-2}
N_{struct}	Nitrogen per unit dry weight of structural mass	0.01	g N (g sw)$^{-1}$
C_{struct}	Carbon per unit dry weight of structural mass	0.2	g C (g sw)$^{-1}$

Table S2. Sensitivity of the carbon flux to the erosion parameter at the kelp density of 1.1 fronds m^{-3}. ϵ was perturbed $\pm 50\%$ about the baseline value. The percentage changes in the output variables were calculated based on a 2-year average.

Variable	$\triangle\epsilon/\epsilon$	
	-0.5	0.5
Gravitational pump	0.38	-0.02
Air-sea CO_2 flux	-0.06	-0.06

REFERENCES

Broch, O. J., Ellingsen, I. H., Forbord, S., Wang, X., Volent, Z., Alver, M. O., et al. (2013). Modelling the cultivation and bioremediation potential of the kelp *Saccharina latissima* in close proximity to an exposed salmon farm in Norway. *Aquaculture Environment Interactions* 4, 187–206. doi:10.3354/AEI00080

Broch, O. J. and Slagstad, D. (2012). Modelling seasonal growth and composition of the kelp *Saccharina latissima*. *Journal of Applied Phycology* 24, 759–776. doi:10.1007/s10811-011-9695-y

Follows, M. J., Ito, T., and Dutkiewicz, S. (2006). On the solution of the carbonate chemistry system in ocean biogeochemistry models. *Ocean Modelling* 12, 290–301. doi:10.1016/j.ocemod.2005.05.004

Fossberg, J., Forbord, S., Broch, O. J., Malzahn, A. M., Jansen, H., Handå, A., et al. (2018). The Potential for Upscaling Kelp (*Saccharina latissima*) Cultivation in Salmon-Driven Integrated Multi-Trophic Aquaculture (IMTA). *Frontiers in Marine Science* 9, 408713. doi:10.3389/FMARS.2018.00418

Levy Marina, T. A.-M., Klein Patrice (2001). Impact of sub-mesoscale physics on production and subduction of phytoplankton in an oligotrophic regime. *Journal of Marine Research* 59, 535–565

Maberly, S. C. (1990). Exogenous Sources of Inorganic Carbon for Photosynthesis by Marine Macroalgae. *Journal of Phycology* 26, 439–449. doi:10.1111/J.0022-3646.1990.00439.X

Millero, F. J. (1995). Thermodynamics of the carbon dioxide system in the oceans. *Geochimica et Cosmochimica Acta* 59, 661–677. doi:10.1016/0016-7037(94)00354-O

Sjøtun, K. (1993). Seasonal Lamina Growth in two Age Groups of *Laminaria saccharina* (L.) Lamour. in Western Norway. *Botanica Marina* 36, 433–442. doi:10.1515/botm.1993.36.5.433

Wanninkhof, R. (1992). Relationship between wind speed and gas exchange over the ocean. *Journal of Geophysical Research: Oceans* 97, 7373–7382. doi:10.1029/92JC00188

Wanninkhof, R. (2014). Relationship between wind speed and gas exchange over the ocean revisited. *Limnology and Oceanography: Methods* 12, 351–362. doi:10.4319/LOM.2014.12.351

Weiss, R. F. (1974). Carbon dioxide in water and seawater: the solubility of a non-ideal gas. *Marine Chemistry* 2, 203–215. doi:10.1016/0304-4203(74)90015-2