Supplementary Material

**Generalized Three-Cornered Hat Method**

The TCH method offers a means to estimate the noise level within time series by comparing them to each other. This involves making certain assumptions regarding the correlations between the observed noises. In our investigation, we employed a generalized TCH approach that doesn't presume zero correlation between the tested series (Koot et al., 2006; Quinn et al., 2019). Here, the calculation of noise variance in combined HAM excitation series using the TCH method relies on the discrepancies between individual series and one designated series treated as a reference, with efforts focused on minimizing the overall correlation among the noises of the individual time series. We opt for a randomly chosen solution as the reference within each group since the selection of a reference series should not affect the resulting combined excitation series (Koot et al., 2006).

Mathematically speaking, when we regard the time series stored as , where corresponds to each solution of hydrological excitations time series (N), the TCH method divides them into two components:

|  |  |
| --- | --- |
| , | (1) |

where is the true value of the series and is common to all of them; is a noise remaining in each hydrological excitations time series. With this specification, discerning the disparities among these series enables us to gather information concerning the noise level of each hydrological excitation time series. Below is the explanation of the generalized TCH method principle.

We take the difference between each series and one of them in group arbitrarily chosen as a reference:

|  |  |
| --- | --- |
| , | (2) |

where is a reference time series. The samples of the solution centres’ differences are concatenated in an matrix as:

|  |  |
| --- | --- |
|  | (3) |

The covariance matrix of the residual time series of their differences is computed as:

|  |  |
| --- | --- |
|  | (4) |

We introduce the ) Allan covariance matrix of the individual noises . Its elements, which are the unknowns in the problem, will be determined in relation to S as follows:

|  |  |
| --- | --- |
| , with , | (5) |

Equation (5) can be rewritten as:

|  |  |
| --- | --- |
| , | (6) |

where is the identity matrix and is the vector, is the submatrix, and is the vector grouping the covariance estimates that involve the th time series, and is the variance of the th reference series.

Next, we isolated the free parameters of Equation (6) by the minimization of the global correlation among the noises of the individual time series using objective function, according to the Kuhn–Tücker theorem:

|  |  |
| --- | --- |
| , | (7) |

with a constraint function (Galindo and Palacio 1999):

|  |  |
| --- | --- |
| . | (8) |

The initial conditions were selected to provide that the initial values achieve the constraints (Torcaso et al. 1998):

|  |  |
| --- | --- |
| . | (9) |

After determining the free parameters, the remaining unknown elements of matrix are determined as follows:

|  |  |
| --- | --- |
| . | (10) |

We assume that the noise level of each HAM series based CMIP6 data may vary significantly, and the most accurate representation of the hydrological signal in polar motion excitation entails a combination of multiple time series. We determined a combination of the HAM series derived from each grouping of CMIP6 climate models, considering their respective qualities, with the aim of ensuring that the merged time series possesses minimal noise levels. We computed the combined hydrological excitation using the noise level of each HAM series as follows:

|  |  |
| --- | --- |
| , | (11) |

where is the weight associated with the and equatorial components of PM excitation.

The requirement for minimal noise variance in the merged time series yields the subsequent normalized weights as a solution:

|  |  |
| --- | --- |
|  | (12) |

**References:**

Galindo, F.J., Palacio, J. (1999). Estimating the instabilities of N correlated clocks. 31st Annual Precise Time and Time Interval Meeting, 285–296

Koot, L., de Viron, O., Dehant, V. (2006). Atmospheric angular momentum time-series: Characterization of their internal noise and creation of a combined series. J Geodesy 79, 663–674. <https://doi.org/10.1007/s00190-005-0019-3>

Quinn K.J., Ponte R.M., Heimbach P., Fukumori I., Campin J-M. (2019). Ocean angular momentum from a recent global state estimate, with assessment of uncertainties, Geophysical Journal International, Volume 216, Issue 1, 584–597, <https://doi.org/10.1093/gji/ggy452>

Torcaso, F., Ekstrom, C., Burt, E., Matsakis, D. (1998). Estimating frequency stability and cross–correlations, Proceedings of the 30th Annual Precise Time and Time Interval Systems and Applications Meeting, Reston, Virginia, December 1998, pp. 69-82

**Table S1.** Standard deviation (STD) of χ1 and χ2 components of GAO and HAM computed from GRACE, LSDM, and grouped CMIP6 models in different spectral ranges. The model count shows the number of models used as input for computing combined series

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| No. | Model  provider | Model count | STD  Overall series (mas) | | STD  Seasonal (mas) | | STD  Non-seasonal short-term (mas) | | STD  Non-seasonal long-term (mas) | |
| χ1 | χ2 | χ1 | χ2 | χ1 | χ2 | χ1 | χ2 |
| 1 | ACCESS TCH | 5 | 4.06 | 5.88 | 3.49 | 5.27 | 1.43 | 2.01 | 1.23 | 0.85 |
| 2 | BCC TCH | 6 | 2.52 | 1.87 | 2.21 | 1.47 | 0.99 | 0.88 | 0.51 | 0.44 |
| 3 | CanESM5 TCH | 5 | 6.62 | 3.90 | 6.38 | 3.59 | 1.24 | 1.33 | 0.99 | 0.42 |
| 4 | GISS TCH | 52 | 3.73 | 1.77 | 3.69 | 1.60 | 0.45 | 0.51 | 0.22 | 0.47 |
| 5 | MIROC TCH | 12 | 4.43 | 5.59 | 4.05 | 5.19 | 1.23 | 1.36 | 1.05 | 0.68 |
| 6 | MPI TCH | 12 | 6.39 | 4.11 | 6.32 | 3.70 | 0.82 | 0.89 | 0.31 | 1.30 |
| 7 | MRI TCH | 6 | 3.75 | 3.00 | 3.52 | 2.48 | 1.07 | 1.42 | 0.31 | 0.73 |
| 8 | ACCESS M | 5 | 3.78 | 6.16 | 3.25 | 5.80 | 1.22 | 1.56 | 1.29 | 0.68 |
| 9 | BCC M | 6 | 2.49 | 1.92 | 2.17 | 1.48 | 1.00 | 0.90 | 0.55 | 0.52 |
| 10 | CanESM5 M | 5 | 6.37 | 3.70 | 6.29 | 3.56 | 0.88 | 0.83 | 0.33 | 0.45 |
| 11 | GISS M | 52 | 3.72 | 1.80 | 3.69 | 1.61 | 0.44 | 0.52 | 0.08 | 0.50 |
| 12 | MIROC M | 12 | 4.47 | 5.49 | 4.09 | 5.31 | 1.14 | 1.04 | 1.25 | 0.54 |
| 13 | MPI M | 12 | 6.03 | 4.16 | 5.95 | 3.69 | 0.86 | 1.02 | 0.43 | 1.40 |
| 14 | MRI M | 6 | 3.78 | 2.88 | 3.59 | 2.46 | 0.98 | 1.29 | 0.30 | 0.60 |
| 15 | ALL M | 99 | 3.78 | 1.95 | 3.75 | 1.88 | 0.37 | 0.37 | 0.22 | 0.28 |
| 16 | ALL WM | 99 | 3.83 | 2.03 | 3.80 | 1.96 | 0.37 | 0.38 | 0.20 | 0.27 |
| 17 | ALL TWS M | 99 | 3.77 | 1.91 | 3.73 | 1.84 | 0.41 | 0.40 | 0.26 | 0.21 |
| 18 | ALL TCH | 99 | 3.35 | 1.87 | 3.32 | 1.76 | 0.43 | 0.49 | 0.18 | 0.28 |
| 19 | GFDL | 1 | 6.03 | 11.22 | 3.76 | 9.08 | 2.76 | 3.55 | 3.28 | 4.31 |
| 20 | LSDM | 1 | 5.14 | 11.81 | 3.94 | 7.46 | 2.47 | 3.87 | 1.45 | 6.39 |
| 21 | GRACE | 1 | 5.32 | 7.57 | 4.14 | 4.18 | 2.70 | 4.10 | 1.65 | 3.73 |
| 22 | GAO | 1 | 9.31 | 15.10 | 5.92 | 10.04 | 5.37 | 8.81 | 4.16 | 3.23 |

**Table S2.** Correlation coefficients (Corr) between GAO and HAM computed from GRACE, LSDM, and grouped CMIP6 models in different spectral ranges. The model count shows the number of models used as input for computing combined series

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| No. | Model  provider | Model count | Corr  Seasonal | | Corr  Non-seasonal  short-term | | Corr  Non-seasonal  long-term | |
| χ1 | χ2 | χ1 | χ2 | χ1 | χ2 |
| 1 | ACCESS TCH | 5 | –0.24 | 0.97 | –0.01 | 0.12 | –0.72 | –0.54 |
| 2 | BCC TCH | 6 | 0.54 | 0.67 | 0.31 | –0.35 | 0.66 | 0.07 |
| 3 | CanESM5 TCH | 5 | 0.64 | 0.94 | 0.11 | –0.13 | –0.12 | –0.74 |
| 4 | GISS TCH | 52 | 0.78 | –0.23 | 0.19 | 0.08 | 0.66 | –0.77 |
| 5 | MIROC TCH | 12 | 0.87 | 0.88 | 0.10 | 0.28 | –0.37 | 0.66 |
| 6 | MPI TCH | 12 | 0.91 | 0.79 | –0.18 | –0.11 | 0.69 | 0.54 |
| 7 | MRI TCH | 6 | 0.63 | –0.09 | –0.16 | –0.38 | 0.18 | –0.39 |
| 8 | ACCESS M | 5 | –0.19 | 0.98 | 0.00 | 0.02 | –0.35 | –0.75 |
| 9 | BCC M | 6 | 0.53 | 0.66 | 0.29 | –0.36 | 0.67 | 0.10 |
| 10 | CanESM5 M | 5 | 0.62 | 0.94 | 0.13 | –0.14 | 0.46 | –0.71 |
| 11 | GISS M | 52 | 0.76 | –0.23 | 0.15 | 0.04 | –0.47 | –0.66 |
| 12 | MIROC M | 12 | 0.87 | 0.88 | 0.12 | 0.37 | –0.20 | 0.61 |
| 13 | MPI M | 12 | 0.90 | 0.77 | –0.09 | –0.12 | 0.68 | 0.60 |
| 14 | MRI M | 6 | 0.63 | –0.09 | –0.09 | –0.29 | 0.59 | –0.22 |
| 15 | ALL M | 99 | 0.78 | 0.64 | 0.19 | –0.23 | –0.04 | –0.91 |
| 16 | ALL WM | 99 | 0.79 | 0.67 | 0.20 | –0.22 | 0.14 | –0.91 |
| 17 | ALL TWS M | 99 | 0.82 | 0.60 | 0.17 | –0.17 | –0.14 | –0.82 |
| 18 | ALL TCH | 99 | 0.77 | 0.72 | 0.20 | –0.33 | 0.11 | –0.82 |
| 19 | GFDL | 1 | 0.62 | 0.91 | 0.08 | –0.34 | –0.30 | –0.83 |
| 20 | LSDM | 1 | 0.74 | 0.94 | 0.43 | 0.46 | 0.25 | 0.81 |
| 21 | GRACE | 1 | 0.82 | 0.96 | 0.53 | 0.54 | 0.98 | 0.88 |

**Table S3.** Normalized root mean square error (NRMSE) of HAM computed from GRACE, LSDM, and grouped CMIP6 models in different spectral ranges. The model count shows the number of models used as input for computing combined series

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| No. | Model  provider | Model count | NRMSE  Seasonal | | NRMSE  Non-seasonal  short-term | | NRMSE  Non-seasonal  long-term | |
| χ1 | χ2 | χ1 | χ2 | χ1 | χ2 |
| 1 | ACCESS TCH | 5 | 2.16 | 0.97 | 3.89 | 4.36 | 4.16 | 4.43 |
| 2 | BCC TCH | 6 | 2.29 | 6.19 | 5.17 | 10.37 | 7.56 | 7.32 |
| 3 | CanESM5 TCH | 5 | 0.82 | 1.88 | 4.33 | 6.81 | 4.44 | 8.36 |
| 4 | GISS TCH | 52 | 1.03 | 6.57 | 11.67 | 17.13 | 18.24 | 7.70 |
| 5 | MIROC TCH | 12 | 0.76 | 1.15 | 4.37 | 6.26 | 4.43 | 4.12 |
| 6 | MPI TCH | 12 | 0.42 | 2.01 | 6.80 | 9.97 | 12.67 | 2.13 |
| 7 | MRI TCH | 6 | 1.30 | 4.24 | 5.25 | 6.61 | 13.03 | 4.90 |
| 8 | ACCESS M | 5 | 2.23 | 0.77 | 4.50 | 5.69 | 3.68 | 5.57 |
| 9 | BCC M | 6 | 2.36 | 6.14 | 5.15 | 10.15 | 6.92 | 6.16 |
| 10 | CanESM5 M | 5 | 0.85 | 1.90 | 6.02 | 10.81 | 12.31 | 7.82 |
| 11 | GISS M | 52 | 1.07 | 6.52 | 11.96 | 16.80 | 51.36 | 7.15 |
| 12 | MIROC M | 12 | 0.75 | 1.11 | 4.67 | 8.16 | 3.66 | 5.45 |
| 13 | MPI M | 12 | 0.44 | 2.05 | 6.37 | 8.81 | 8.95 | 1.91 |
| 14 | MRI M | 6 | 1.28 | 4.28 | 5.62 | 7.14 | 13.13 | 5.73 |
| 15 | ALL M | 99 | 1.01 | 4.64 | 14.47 | 23.77 | 18.93 | 11.88 |
| 16 | ALL WM | 99 | 0.99 | 4.39 | 14.44 | 23.12 | 20.28 | 12.32 |
| 17 | ALL TWS M | 99 | 0.96 | 4.76 | 12.87 | 21.81 | 16.10 | 15.66 |
| 18 | ALL TCH | 99 | 1.19 | 4.98 | 12.39 | 18.15 | 22.99 | 11.93 |
| 19 | GFDL | 1 | 1.23 | 0.45 | 2.11 | 2.96 | 1.83 | 1.70 |
| 20 | LSDM | 1 | 1.02 | 0.52 | 1.96 | 2.02 | 2.81 | 0.66 |
| 21 | GRACE | 1 | 0.83 | 1.46 | 1.69 | 1.81 | 1.54 | 0.47 |

**Table S4.** Absolute values of differences between GAO and HAM computed from GRACE, LSDM, and grouped CMIP6 models in terms of amplitude of annual prograde and retrograde oscillation. The results are sorted from the smallest to the largest value. STD of differences received for all models (STDdifferences) are also provided

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| No. | Annual oscillation amplitude differences | | | |
| Prograde | | Retrograde | |
| Model | Difference (mas) | Model | Difference (mas) |
| 1 | ACCESS M | 0.59 | LSDM | 1.00 |
| 2 | GFDL | 0.61 | GFDL | 2.53 |
| 3 | CanESM5 TCH | 0.61 | GRACE | 4.06 |
| 4 | CanESM5 M | 0.67 | MPI TCH | 4.45 |
| 5 | ACCESS TCH | 0.85 | MPI M | 4.83 |
| 6 | MPI TCH | 1.10 | MIROC M | 4.84 |
| 7 | MPI M | 1.27 | MIROC TCH | 4.92 |
| 8 | MIROC M | 1.52 | CanESM5 TCH | 5.31 |
| 9 | MIROC TCH | 1.62 | CanESM5 M | 5.40 |
| 10 | ALL WM | 3.10 | GISS M | 6.60 |
| 11 | ALL M | 3.17 | GISS TCH | 6.62 |
| 12 | ALL TWS M | 3.27 | ALL TWS M | 7.03 |
| 13 | ALL TCH | 3.58 | ALL WM | 7.08 |
| 14 | MRI M | 3.73 | ACCESS M | 7.10 |
| 15 | MRI TCH | 3.76 | ALL M | 7.12 |
| 16 | GISS TCH | 3.83 | ALL TCH | 7.41 |
| 17 | GISS M | 3.84 | MRI M | 7.47 |
| 18 | LSDM | 3.88 | ACCESS TCH | 7.48 |
| 19 | GRACE | 4.14 | MRI TCH | 7.53 |
| 20 | BCC TCH | 4.83 | BCC TCH | 8.47 |
| 21 | BCC M | 4.89 | BCC M | 8.49 |
| STDdifferences | 1.53 | | 1.93 | |
| min+1STD | 2.12 | | 2.92 | |
| min+2STD | 3.65 | | 4.86 | |

**Table S5.** Absolute values of differences between GAO and HAM computed from GRACE, LSDM, and grouped CMIP6 models in terms of the phase of annual prograde and retrograde oscillation. The results are sorted from smallest to largest value. STD of differences received for all models (STDdifferences) are also provided

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| No. | Annual oscillation phase differences | | | |
| Prograde | | Retrograde | |
| Model | Difference (°) | Model | Difference (°) |
| 1 | CanESM5 M | 13.49 | LSDM | 0.13 |
| 2 | BCC M | 15.26 | GRACE | 0.28 |
| 3 | CanESM5 TCH | 16.12 | MPI M | 9.35 |
| 4 | BCC TCH | 16.62 | MIROC M | 9.44 |
| 5 | ACCESS TCH | 23.50 | MIROC TCH | 9.45 |
| 6 | MIROC M | 24.46 | MPI TCH | 10.40 |
| 7 | ACCESS M | 25.08 | GFDL | 13.79 |
| 8 | MIROC TCH | 25.11 | ACCESS M | 36.29 |
| 9 | GFDL | 26.47 | BCC M | 39.85 |
| 10 | LSDM | 31.96 | BCC TCH | 42.51 |
| 11 | ALL WM | 32.11 | ALL TCH | 50.02 |
| 12 | ALL TCH | 32.55 | CanESM5 TCH | 52.71 |
| 13 | ALL M | 32.91 | ALL TWS M | 53.58 |
| 14 | ALL TWS M | 35.85 | ALL WM | 54.19 |
| 15 | GRACE | 40.18 | CanESM5 M | 56.07 |
| 16 | GISS M | 49.12 | ALL M | 57.08 |
| 17 | GISS TCH | 50.97 | ACCESS TCH | 57.85 |
| 18 | MPI M | 54.85 | GISS TCH | 98.47 |
| 19 | MPI TCH | 55.74 | GISS M | 100.40 |
| 20 | MRI M | 58.07 | MRI M | 109.15 |
| 21 | MRI TCH | 58.59 | MRI TCH | 110.43 |
| STDdifferences | 13.49 | | 35.40 | |
| min+1STD | 28.44 | | 35.53 | |
| min+2STD | 43.39 | | 70.93 | |

**Table S6.** Absolute values of differences between GAO and HAM computed from GRACE, LSDM, and grouped CMIP6 models in terms of amplitude of semiannual prograde and retrograde oscillation. The results are sorted from smallest to largest value. STD of differences received for all models (STDdifferences) are also provided

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| No. | Semiannual oscillation amplitude differences | | | |
| Prograde | | Retrograde | |
| Model | Difference (mas) | Model | Difference (mas) |
| 1 | GRACE | 1.75 | BCC M | 0.13 |
| 2 | MRI TCH | 1.77 | BCC TCH | 0.22 |
| 3 | BCC M | 1.78 | LSDM | 0.30 |
| 4 | MRI M | 1.78 | GISS TCH | 0.31 |
| 5 | BCC TCH | 1.83 | GISS M | 0.34 |
| 6 | GFDL | 1.93 | MIROC TCH | 0.35 |
| 7 | GISS TCH | 1.95 | MIROC M | 0.39 |
| 8 | GISS M | 1.97 | ALL TCH | 0.42 |
| 9 | CanESM5 M | 2.11 | ALL TWS M | 0.43 |
| 10 | ACCESS TCH | 2.12 | ALL WM | 0.45 |
| 11 | ACCESS M | 2.13 | ALL M | 0.46 |
| 12 | CanESM5 TCH | 2.13 | GFDL | 0.51 |
| 13 | ALL TCH | 2.14 | MPI TCH | 0.68 |
| 14 | ALL WM | 2.23 | ACCESS M | 0.73 |
| 15 | ALL M | 2.23 | ACCESS TCH | 0.80 |
| 16 | ALL TWS M | 2.25 | CanESM5 M | 0.82 |
| 17 | MPI M | 2.54 | GRACE | 0.82 |
| 18 | MPI TCH | 2.60 | MPI M | 0.83 |
| 19 | LSDM | 2.70 | CanESM5 TCH | 0.84 |
| 20 | MIROC TCH | 2.81 | MRI TCH | 1.35 |
| 21 | MIROC M | 2.88 | MRI M | 1.35 |
| STDdifferences | 0.35 | | 0.33 | |
| min+1STD | 2.10 | | 0.46 | |
| min+2STD | 2.45 | | 0.80 | |

**Table S7.** Absolute values of differences between GAO and HAM computed from GRACE, LSDM, and grouped CMIP6 models in terms of phase of semiannual prograde and retrograde oscillation. The results are sorted from smallest to largest value. STD of differences received for all models (STDdifferences) are also provided

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| No. | Semiannual oscillation phase differences | | | |
| Prograde | | Retrograde | |
| Model | Difference (°) | Model | Difference (°) |
| 1 | LSDM | 12.62 | MIROC M | 0.82 |
| 2 | GISS TCH | 16.55 | MIROC TCH | 3.55 |
| 3 | GISS M | 17.46 | BCC TCH | 10.37 |
| 4 | MPI M | 24.82 | BCC M | 10.82 |
| 5 | MPI TCH | 27.17 | GISS TCH | 12.30 |
| 6 | ALL TWS M | 28.83 | ALL TCH | 13.16 |
| 7 | ALL M | 33.60 | MRI TCH | 13.88 |
| 8 | ALL WM | 33.97 | GISS M | 14.22 |
| 9 | GRACE | 34.86 | MRI M | 14.58 |
| 10 | ACCESS M | 35.61 | ALL TWS M | 17.26 |
| 11 | ACCESS TCH | 40.34 | ALL WM | 18.17 |
| 12 | MRI TCH | 49.41 | ALL M | 18.28 |
| 13 | ALL TCH | 49.81 | MPI TCH | 20.60 |
| 14 | MRI M | 51.35 | MPI M | 20.97 |
| 15 | MIROC M | 57.14 | ACCESS TCH | 23.47 |
| 16 | MIROC TCH | 58.62 | ACCESS M | 30.76 |
| 17 | BCC TCH | 60.00 | CanESM5 TCH | 43.61 |
| 18 | BCC M | 61.08 | CanESM5 M | 44.47 |
| 19 | CanESM5 M | 83.87 | LSDM | 86.17 |
| 20 | CanESM5 TCH | 92.88 | GRACE | 86.56 |
| 21 | GFDL | 95.94 | GFDL | 139.65 |
| STDdifferences | 23.82 | | 33.96 | |
| min+1STD | 36.43 | | 34.78 | |
| min+2STD | 60.25 | | 68.74 | |