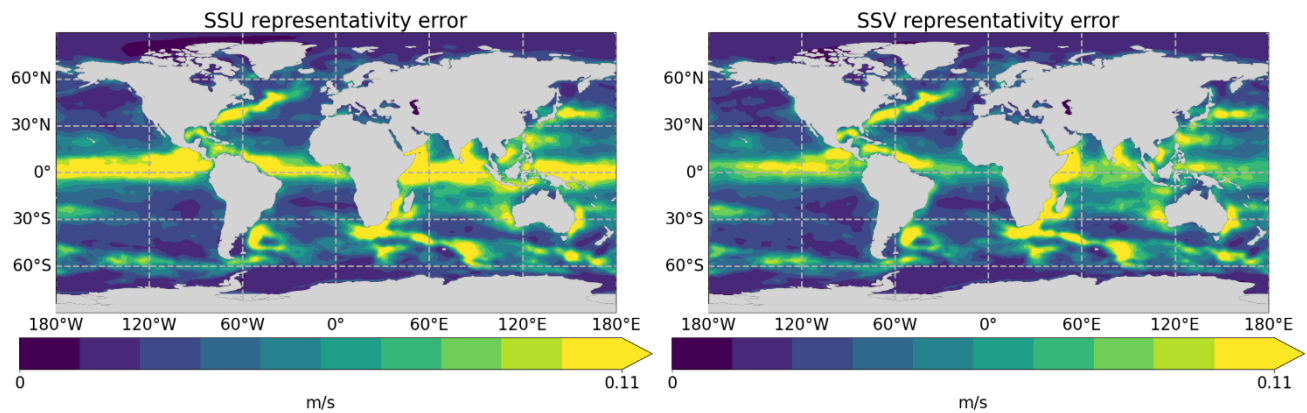**Supplementary materials**

Figure 1. Observation representation error and mapping error for surface zonal velocity (left) and surface meridional velocity (right) used in all the experiments assimilating TSCV data.
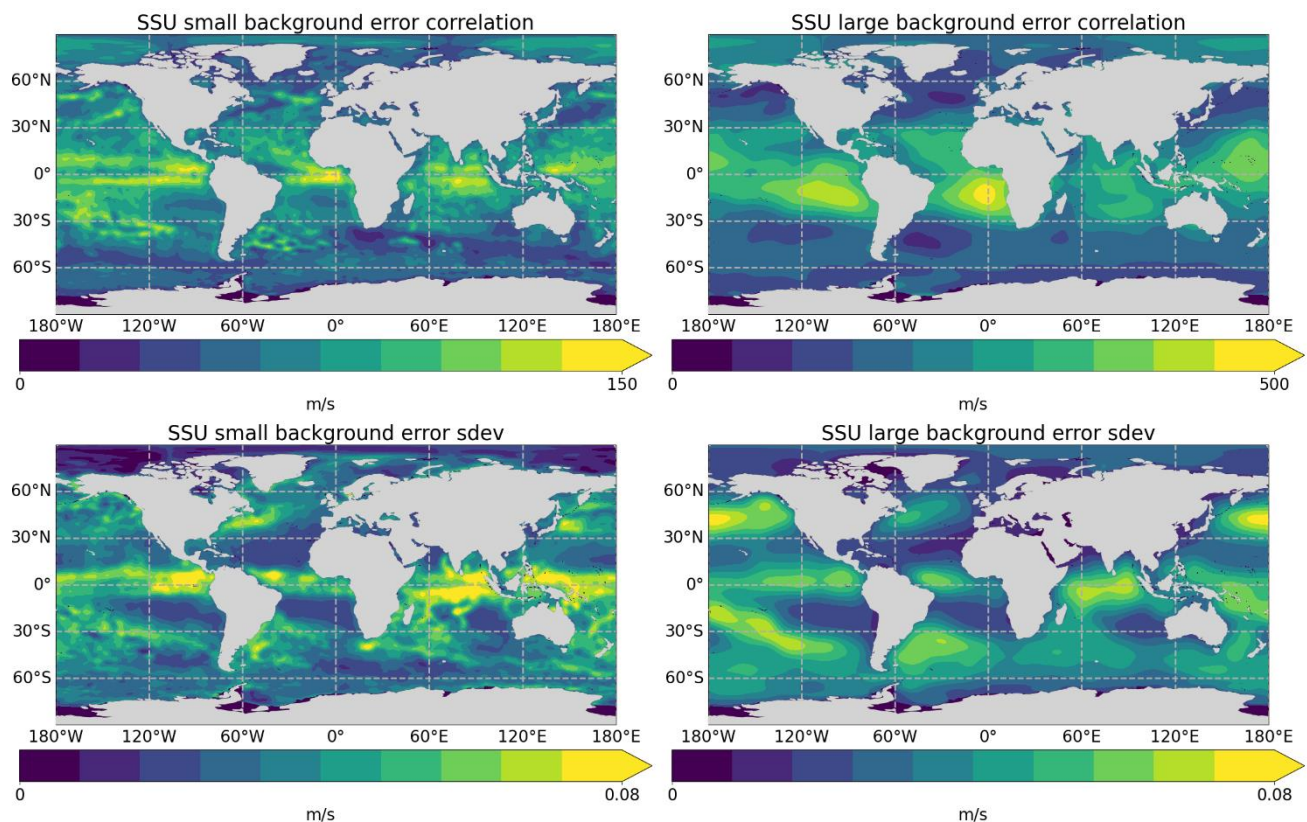

Figure 2. An example of the horizontal background error covariances for surface zonal velocity used in the experiments assimilating TSCV. The top plots show the short (left) and long (right) annual horizontal background error correlation length scales, the bottom plots show the corresponding short (left) and long (right) seasonal background error standard deviations for December-January-

February. The background error standard deviations provide the respective weighting for the small and large background error correlations.
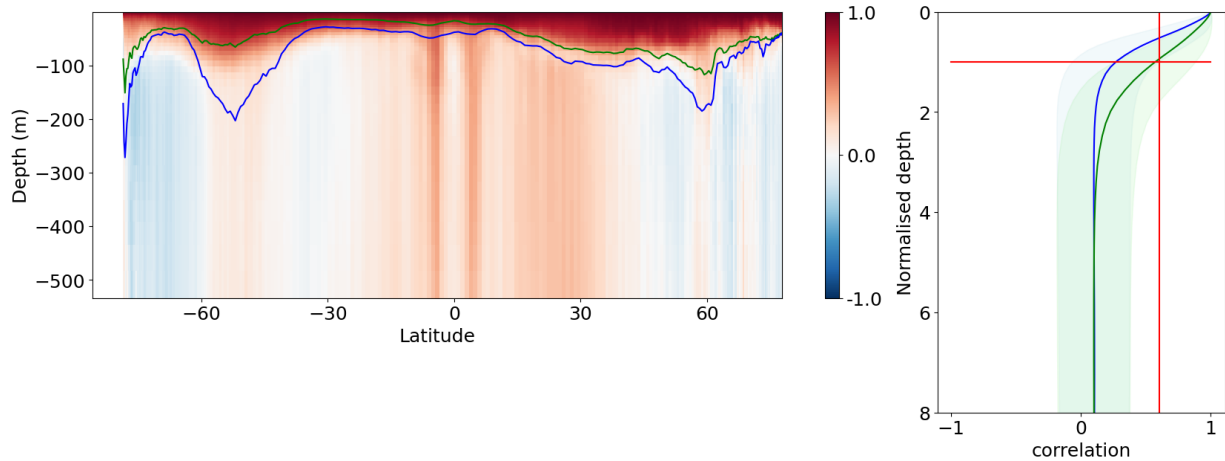

Figure 3. These plots show comparisons of two mixed layer depths and the unbalanced U velocity forecast error vertical correlations with the surface calculated using the NMC.
In the left plot the contours show the zonally averaged NMC unbalanced U vertical correlations with the surface. These are calculated using the NMC method with data from December 2008. The green and blue lines are the zonally averaged monthly mean MldRho and MldZ mixed layer depths, respectively, calculated for December 2008.
The right plot shows the global mean unbalanced U vertical correlations plotted against a normalising depth. For the green, and blue lines the normalising quantity is the global mean MldRho and MldZ mixed layer depth respectively. The horizontal red line shows where the normalised depth is 1 and the vertical red line is the value of a Gaussian function when the depth variable equals the correlation length scale. When the normalising depth is a good approximation to the correlation length scale, the correlation profile passes close to the red line intersect. The shaded region shows the standard deviation of the error correlations.
These plots demonstrate that the MldRho provides a reasonable parameterization of the vertical correlations with the surface.
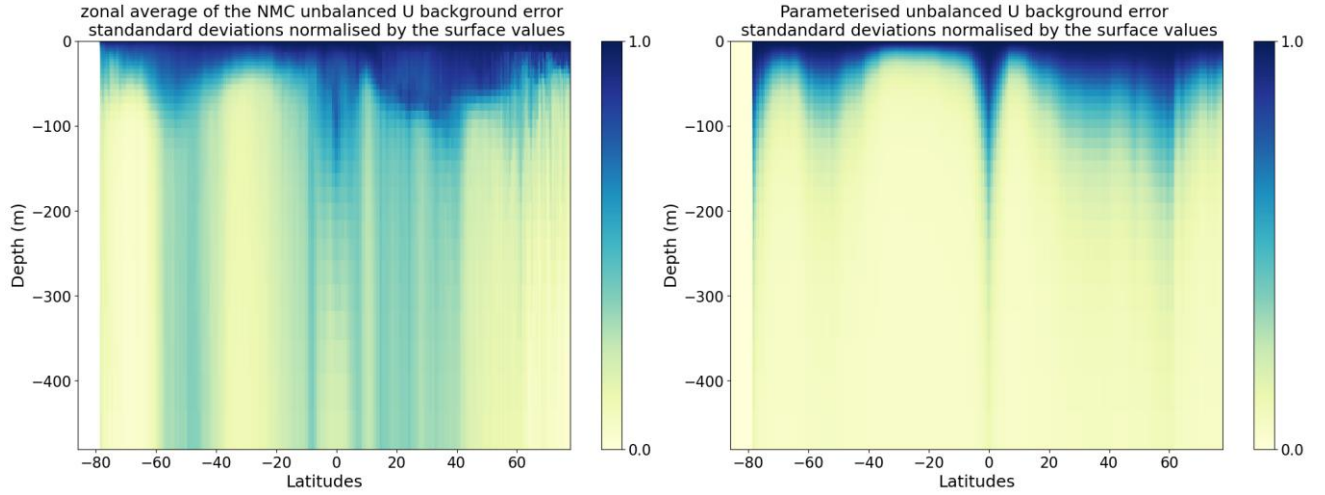
Figure 4. The left plot shows the zonal average of the unbalanced zonal velocity background error standard deviations normalised by the surface values. This was calculated using the NMC method with data from December 2008. The plot on the right shows f(z) from equation (1) where L is the zonally averaged monthly mean mixed layer depth for December 2008. These plots demonstrate that the parametrization for the vertical structure of the background error standard deviations produces a reasonable approximation to the NMC determined background error standard deviations.
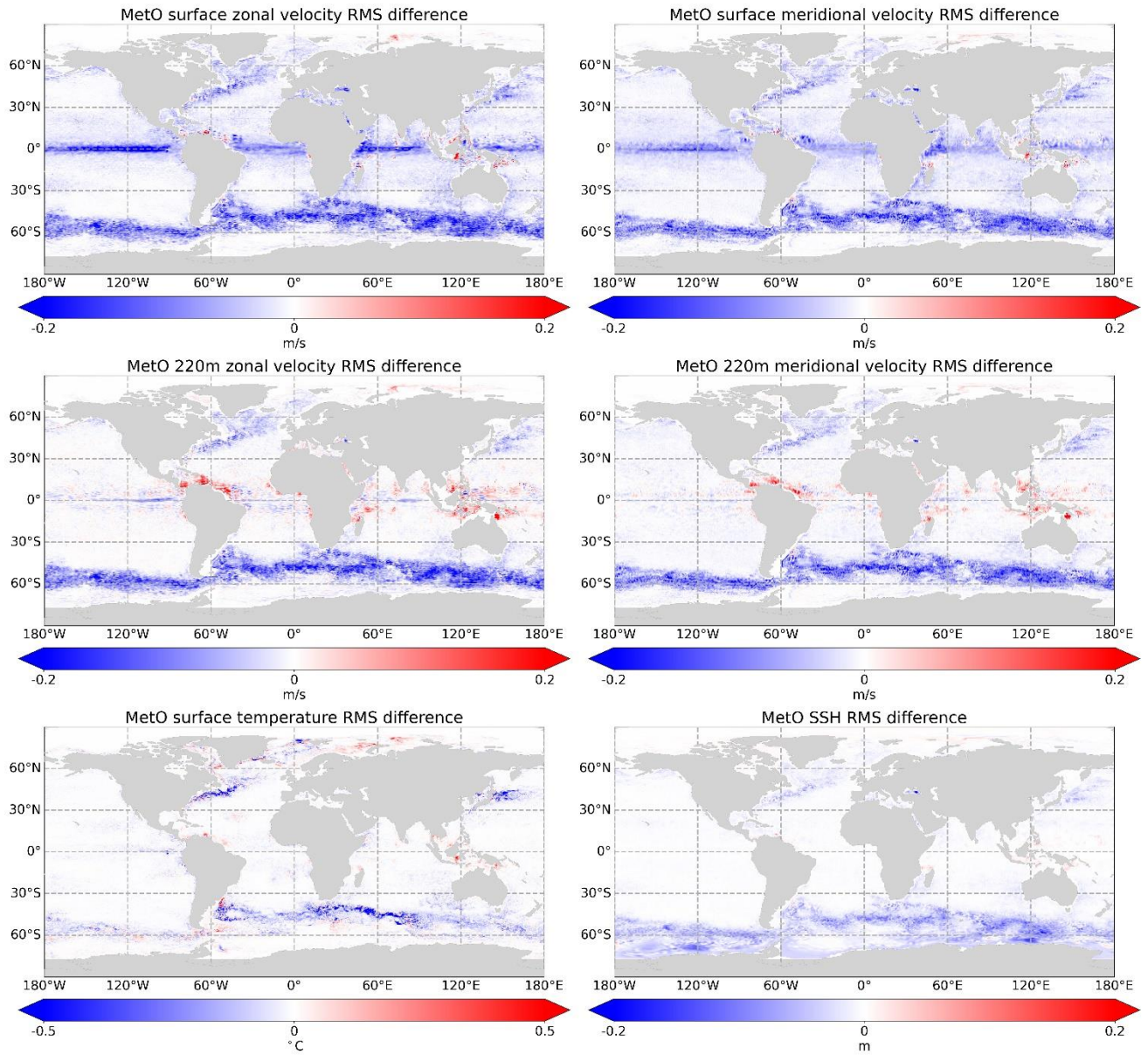
Figure 5. Spatial plot of A-TSCV_No_Err RMSE minus control RMSE calculated between the 25th of February and 30th of December for surface zonal velocity (top left), surface meridional velocity (top right), 220 m zonal velocity (middle left), 220m meridional velocity (middle right), SST (bottom left) and SSH (bottom right). Blue areas indicate regions where the A-TSCV exp has a lower RMSE than the control while red indicates regions where the RMSE is higher.