Supplementary Material

# Supplementary Table of Advantages and Disadvantages of Various Filtering Techniques

In recent years, diverse methods for processing echo data have emerged with the aim of enhancing data quality in the presence of noise interference. These methods include time-domain filters such as median filtering, mean filtering, Kalman filtering, and matched filtering; low-pass filters in the frequency domain; and wavelet threshold denoising in the wavelet domain. With the development of machine learning, recurrent neural networks (RNNs) have been applied to denoising. Each filtering technique has its own set of advantages and disadvantages, as shown in Supplementary Table 1.

**Supplementary Table 1**

Advantages and disadvantages of various filtering techniques.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Median filtering** | **Mean filtering** | **Kalman filtering** | **Matched filtering** | **Low-pass filtering** | **Wavelet threshold denoising** | **RNN denoising** |
| Advantages | Removing impulse noise while preserving image edge clarity [1]. | Effectively filtering out abrupt noise [2]. | High real-time performance, strong anti-interference capability, and efficiency [3]. | Effectively filters out backscatter noise, thereby enhancing the output signal-to-noise ratio [4]. | Effectively filters out noise [2]. | Variable scale and adaptive matching for signal extraction [5]. | Effective in processing one-dimensional time series [6]. |
| Disadvantages | The window length of the filter affects its filtering performance [7]. | Unable to effectively safeguard local signal changes, resulting in the loss of detailed information [2]. | The algorithm operates under the assumption that the process is influenced by Gaussian white noise [3]. | In scenarios characterized by strong noise, challenges arise in extracting peak target signals [4]. | Less efficient at preserving signal details [8]. | The impact on time series signals with low signal-to-noise ratios is not optimal [5]. | When implemented on complex data, they are susceptible to issues of gradient vanishing and explosion [6]. |

# Supplementary Data on Device Parameters

Figure 1 shows a laser with a wavelength of 532 nm and a repetition rate of 5 MHz. The beam-expanding collimating system, comprised of lenses *L*1 and *L*2 as shown in the diagram, differs from the lenses used in actual experiments. During experiments, the distance between lenses *L*1 and *L*2 can be set to the sum of their focal lengths, by aligning the object-side focal point of *L*2 with the image-side focal point of *L*1 to achieve a collimated output beam. Different magnification ratios of the beam spot can be obtained by selecting different focal lengths for *L*1 and *L*2. In this study, we utilize a ‘THORLABS ZC618FC-A’ variable-focus fiber collimator, which allows focal length adjustments between 6-18 mm, connects directly to a fiber, and includes an anti-reflection coating for the 400-650 nm range. By adjusting the focal length, different magnification levels of beam expansion and collimation are achieved. The distance between lens *L*2 and the quarter-wave plate is 15 cm, and the distance from the quarter-wave plate to the Dammann grating is 17 cm. The Dammann grating splits the beam into a 17×17 array with a diffusion angle of 2°. The focal length of lens *L*3 is 20 cm, and the distance between the detector and lens *L*3 is 20 cm.

# Supplementary Figure of Denoising Efficacy of the MF-LSTM Algorithm Under Near-Zero Visibility Conditions.

When lasers transmit through cloud and fog, the beam's scattering is enhanced as visibility decreases, resulting in a decrease in the proportion of target signal peaks in the echo signal. Therefore, under extremely low visibility conditions, the echo signal is significantly affected. This study investigated the denoising performance of the MF-LSTM algorithm on echo data under extremely low visibility. Initially, matched filtering was applied to preliminarily filter the signal based on the differences between cloud and fog scattering noise and the target signal. Subsequently, the advantages of the LSTM algorithm in processing complex temporal signals were utilized to further extract the target signal. Since zero visibility conditions are extremely rare in real-world scenarios, Monte Carlo simulation was employed to simulate near-zero visibility cloud and fog environments with a visibility of 0.5 m and a target position set at 6 m. The MF-LSTM algorithm was then applied to denoise the data in this extremely low visibility environment, with the results shown in Supplementary Figure 1.



**Supplementary Figure 1**

(A) Histogram of photon count time distribution under near-zero visibility; (B) Denoising results obtained using the MF-LSTM algorithm.

From Supplementary Figure 1(A), it can be observed that at a visibility of 0.5 m, photons are significantly affected by cloud and fog scattering. In the echo data, the backscatter noise far exceeds the peak amplitude of the target signal, making it difficult to extract the target position using the peak value method. From Supplementary Figure 1(B), it can be concluded that after denoising with the MF-LSTM algorithm, the backscatter noise caused by near-zero visibility of cloud and fog is effectively removed. The signal-to-noise ratio (SNR) increases from 1.040 dB to 12.24 dB after denoising, indicating that the MF-LSTM algorithm proposed in this study maintains effectiveness under near-zero visibility conditions.

# Supplementary Table of Denoising Efficacy of the MF-LSTM Algorithm Under Different Cloud and Fog Particle Sizes.

When lasers transmit through cloud and fog, various sizes of particles in the cloud and fog lead to different levels of scattering, resulting in varying proportions of noise in the echo signal. Therefore, this study investigated the denoising efficacy of the MF-LSTM algorithm under different cloud and fog particle sizes. Cloud and fog particles typically have radii concentrated within the range of a few micrometers to tens of micrometers, with a predominant distribution towards smaller sizes. We used Monte Carlo to simulate the cloud and fog environment. As real cloud and fog contain particles of different sizes, particles within a certain size range were selected each time, and the impact of particle sizes on the echo was obtained by adjusting different ranges. The MF-LSTM algorithm was then utilized to denoise the data obtained within different size ranges, thereby evaluating the denoising performance under different particle sizes. The distribution of echo signals for particle sizes ranging from 6 – 10 μm, 11 – 15 μm, and 16 – 20 μm was obtained at a visibility of 6 m and a target distance of 6 m. The denoising effectiveness of MF-LSTM algorithm was assessed by comparing SNR before and after denoising, as shown in Supplementary Table 2.

**Supplementary Table 2**

SNR of the MF-LSTM algorithm under different cloud and fog particle sizes (dB).

|  |  |  |  |
| --- | --- | --- | --- |
| **Range of Particle sizes (μm)** | **6 – 10** | **11 – 15** | **16 – 20** |
| Noisy | 1.207 | 0.7070 | 0.5985 |
| MF-LSTM | 14.86 | 14.33 | 14.16 |

From Supplementary Table 2, it can be observed that as the size of cloud and fog particles increases, the scattering of the beam intensifies, resulting in an increased proportion of scattered noise in the echo signal and consequently reducing the SNR. After denoising with the MF-LSTM algorithm, the SNR is effectively enhanced under different cloud and fog particle sizes. Particularly, within the 16 - 20 μm range, where particles have a more pronounced impact on the echo data, resulting in an SNR improvement of up to 22.66 times after denoising. This indicates that the MF-LSTM algorithm demonstrates robust restoration performance under different cloud and fog particle size conditions.

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