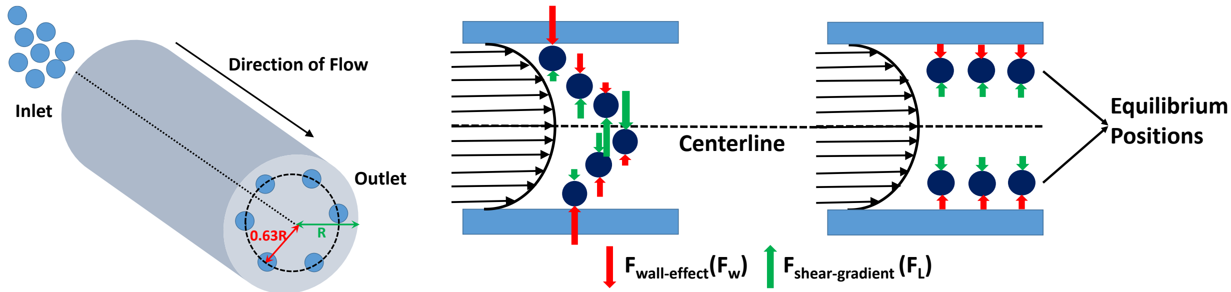
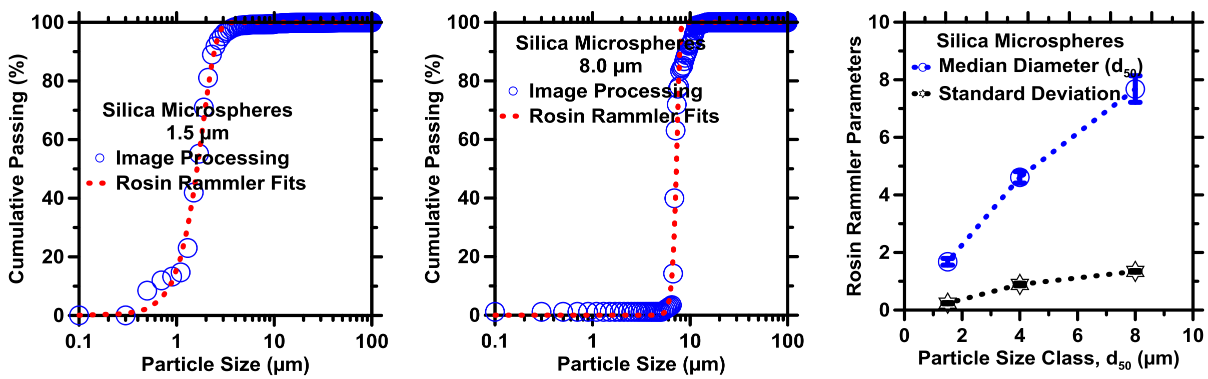
On the Application of Inertial Microfluidics for the Size-Based Separation of Polydisperse Cementitious Particulates

Supplementary Information



(a) (b)

Figure S1: (a) Inertial focusing of randomly distributed particles flowing through a cylindrical channel. (b) The schematic shows the two inertial lift forces acting on the particles perpendicular to the flow direction. The particles experience a ‘‘wall-effect’’ lift that is directed away from the wall and decays with distance from the wall. In addition, a shear gradient lift arises from the viscous nature of fluid that acts to transport the particles down the shear gradient. The interaction of the two forces leads to particles attaining defined equilibrium positions in confined microchannel flows.



(a) (b) (c)

Figure S2: Rosin-Rammler (RR) fits to the measured PSDs of silica microspheres with median diameters of (a) 1.5 μm and (b) 8 μm. (c) shows the median diameter (d_{50}) and standard deviation (σ) of the RR fits, plotted against the size class. For (a and b), the highest uncertainty the particle size, and cumulative passing based on replicate ICA is on the order of 13.5%. In (c), the error bars represent the (minimum-to-maximum) variation in the RR parameters as computed from multiple images.

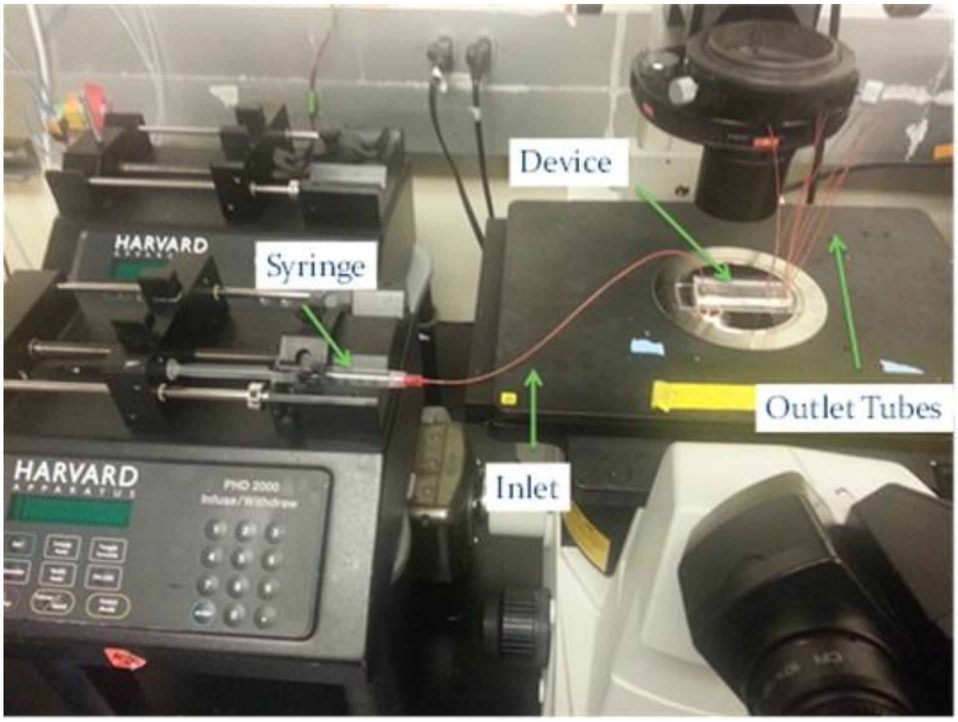


Figure S3: The experimental apparatus for particle separation. The inlet is connected to a syringe that holds the magnetically stirred suspension. Solid particles suspended within the carrier fluid are collected from all seven outlets by means of tubes that connect the outlets to separate collection vials.

Dean Forces: In curved channels, in addition to the viscous drag forces and the inertial lift forces, there are contributions from another category of forces, termed as "Dean drag" forces [1]. Secondary flow, or Dean flow develops in a fluid flowing through a curved channel because of a mismatch of velocity in the downstream direction between the fluid in the center and near-wall regions of a channel. In such cases, fluid elements near the centerline, due to their larger inertia as compared to fluid near the channel walls, tend to flow outward around a curve, creating a pressure gradient in the radial direction of the channel. Due to this centrifugal pressure gradient and for mass conservation, near stagnant fluid near the walls re-circulates inward creating two symmetric and counter-rotating vortices perpendicular to the flow direction (Figure S4). The magnitude of such secondary flows has been quantified by Berger et al. [2] in the dimensionless form shown in Equation (S1), wherein the extent of secondary flow of the fluid depends on the height (H) and the radius of curvature ($R_{curvature}$) of the curvilinear section of the microchannel. Di Carlo et al. [1] proposed an inertial force ratio (R_f, Equation S2), that describes the order of magnitude scaling between the inertial lift forces and the Dean drag forces acting on a particle simultaneously. This ratio can be used to predict particle behavior with respect to values of R_f. At very low values of the force ratio (R_f), particle streams will be predominantly affected by the Dean drag force and, as such, will remained entrained in the secondary flow while neglecting their inertial equilibrium positions. On the other hand, at large values of R_f, particles will migrate to inertial focusing positions regardless of the secondary flow. At intermediate values of R_f, particle streams are affected by both inertial and Dean drag forces and consequently acquire modified inertial equilibrium positions. Since R_f is a complex parameter, and is also a function of particle size simultaneous application of inertial lift and Dean drag forces could have a significant effect on equilibrium positioning of particles, in relation to their size.

To illustrate this consider the example of OPC, in which particle sizes range from 0.1 µm to 1000 µm. Here, in any section of a curvilinear microchannel, the value of R_f would be small for the small particles and large for a large particles. Consequently, small particles would be affected more by the Dean drag forces, compared to the inertial lift forces, and would follow the secondary flow stream. The large particles, on the other hand, would acquire their inertial equilibrium positions regardless of the radius of curvature of the microchannel. Therefore, and especially when it is desirable to separate particulates with broad spans in particle size these aspects of the nature of hydrodynamic forces warrant careful consideration, in relation to device design. It should be noted that, though R_f provides a parameter to evaluate curvilinear microchannel based devices, the exact mechanism and location of superposition of secondary flow and inertial lift effects is complex and thus far, poorly understood.

Dimensionless magnitude of Secondary flow

$$£ = R_P \sqrt{\frac{H}{2R_{curvature}}} \tag{S1}$$

Ratio of Inertial life force to Dean Drag force

$$R_f = \frac{a_P^2 R_{curvature}}{H} \tag{S2}$$

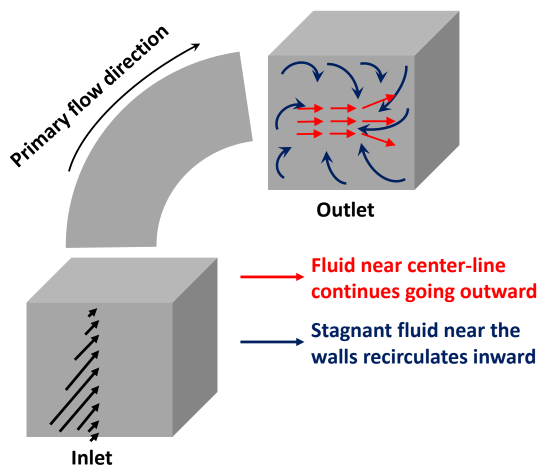


Figure S4: Dean flow: In curved channels, when inertia is important, faster moving fluid near the channel center tends to continue outward, and to conserve mass, more stagnant fluid near the walls re-circulates inward. This creates two counter-rotating vortices perpendicular to the primary flow direction [1].

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[1] Di Carlo, D. Inertial microfluidics. *Lab Chip.* **(2009)**, 9(21), 3038-3046.

[2] Berger, S. A., L. Talbot, and L. S. Yao. "Flow in curved pipes." *Annual Review of Fluid Mechanics* **(1983)**, 15(1) 461-512.