Appendix SA: Reservoir flow model

Considering the fracture as a superposition of several point sources, we can easily write the point source solution according to Gringarten's source function method. x' and y' denote the locations of the point sources, x and y denote the pressure drop at a point on the ground surface.

$$p_j = p_{ij} - \frac{1}{4\pi\phi_j c_f \chi h_j} \int_0^t \frac{q_j(\tau)}{t-\tau} exp\left[-\frac{(x-x')^2+(y-y')^2}{4\chi(t-\tau)}\right] d\tau,\ \chi = \frac{k_j}{\phi_j \mu c_f} \tag{A-1}$$

where $\chi = \frac{k_j}{\phi_j \mu c_f}$ is the layer *j* diffusivity, *j* represents the number of layers.

Integrating the point source along the fracture direction yields:

$$p_j = p_{ij} - \frac{1}{4\pi\phi_j c_f \chi h_j} \int_0^t \int_{-x_{fj}}^{x_{fj}} \frac{q_j(\tau)}{t-\tau} exp\left[-\frac{(x-x')^2+(y-y')^2}{4\chi(t-\tau)}\right] dx'\, d\tau \tag{A-2}$$

After dimensionless,

$$p_{jD} = \frac{1}{4\alpha_j\lambda_j} \int_0^{t_D} \int_{-\alpha_j}^{\alpha_j} \frac{q_{fDj}(\tau)}{t_D-\tau} exp\left[-\frac{\omega_j}{\lambda_j}\frac{(x_D-x')^2}{4(t_D-\tau)}\right] dx'\, d\tau \tag{A-3}$$

The above equation is transformed by Laplace into:

$$\bar{p}_{jD} = \frac{1}{2\alpha_j\lambda_j} \int_{-\alpha_j}^{\alpha_j} \bar{q}_{fDj} \cdot K_0\left[(x_D-x')\sqrt{\frac{\omega_j}{\lambda_j}s}\right] dx' \tag{A-4}$$

where q_{fDj} is defined as:

$$q_{fDj} = \frac{2q_{fj}x_{fj}}{q_j} \tag{A-5}$$

Next, the point source solutions of the constant pressure boundary and the closed boundary are given. The first is the line source solution of the constant pressure boundary:

$$\bar{p}_{jD} = \frac{1}{2s\alpha_j\lambda_j}\left\{\int_{-\alpha_j}^{\alpha_j} K_0\left[(x_D-x')\sqrt{\frac{\omega_j}{\lambda_j}s}\right]dx' - \frac{K_0(r_{eDj}\sqrt{\frac{\omega_j}{\lambda_j}s})}{I_0(r_{eDj}\sqrt{\frac{\omega_j}{\lambda_j}s})}\int_{-\alpha_j}^{\alpha_j} I_0\left[(x_D-x')\sqrt{\frac{\omega_j}{\lambda_j}s}\right]dx'\right\} \tag{A-6}$$

The line source solution of the closed boundary:

$$\bar{p}_{jD} = \frac{1}{2s\alpha_j\lambda_j}\left\{\int_{-\alpha_j}^{\alpha_j} K_0\left[(x_D - x')\sqrt{\frac{\omega_j}{\lambda_j}s}\right]dx' + \frac{K_1(r_{eDj}\sqrt{\frac{\omega_j}{\lambda_j}s})}{I_1(r_{eDj}\sqrt{\frac{\omega_j}{\lambda_j}s})}\int_{-\alpha_j}^{\alpha_j} I_0\left[(x_D - x')\sqrt{\frac{\omega_j}{\lambda_j}s}\right]dx'\right\} \quad \text{(A-7)}$$

If it is an infinite conductivity fracture, we can get it directly (Note 0.732 times the length).

$$\bar{p}_{jD} = \frac{1}{2s\alpha_j\lambda_j}\int_{-\alpha_j}^{\alpha_j} K_0\left[(0.732\alpha_j - x')\sqrt{\frac{\omega_j}{\lambda_j}s}\right]dx' \quad \text{(A-8)}$$

Infinite conductivity fracture + constant pressure boundary line source solution:

$$\bar{p}_{jD} = \frac{1}{2s\alpha_j\lambda_j}\left\{\int_{-\alpha_j}^{\alpha_j} K_0\left[(0.732\alpha_j - x')\sqrt{\frac{\omega_j}{\lambda_j}s}\right]dx' - \frac{K_0(r_{eDj}\sqrt{\frac{\omega_j}{\lambda_j}s})}{I_0(r_{eDj}\sqrt{\frac{\omega_j}{\lambda_j}s})}\int_{-\alpha_j}^{\alpha_j} I_0\left[(0.732\alpha_j - x')\sqrt{\frac{\omega_j}{\lambda_j}s}\right]dx'\right\}$$

(A-9)

Infinite conductivity fracture + closed boundary line source solution:

$$\bar{p}_{jD} = \frac{1}{2s\alpha_j\lambda_j}\left\{\int_{-\alpha_j}^{\alpha_j} K_0\left[(0.732\alpha_j - x')\sqrt{\frac{\omega_j}{\lambda_j}s}\right]dx' + \frac{K_1(r_{eDj}\sqrt{\frac{\omega_j}{\lambda_j}s})}{I_1(r_{eDj}\sqrt{\frac{\omega_j}{\lambda_j}s})}\int_{-\alpha_j}^{\alpha_j} I_0\left[(0.732\alpha_j - x')\sqrt{\frac{\omega_j}{\lambda_j}s}\right]dx'\right\}$$

(A-10)

After Laplace transform, it can be obtained：

$$\bar{p}_{jD} = \sqrt{\frac{1}{\lambda_j\omega_j\alpha_j^2}}\frac{\pi}{2s\sqrt{s}} \quad \text{(A-11)}$$

Appendix SB: Fracture flow model

Fracture flow equation (without considering the compressibility of fluid in fracture)：

$$\frac{\partial}{\partial x}(\frac{k_{Fj}}{\mu}\frac{\partial p_j}{\partial x}) + \frac{q_{fj}}{w_{Fj}h_j} = 0$$

$$\frac{\partial}{\partial x_D}(F_{cDj}\frac{\partial p_{Dj}}{\partial x_D}) - \frac{\pi q_{fDj}}{\lambda_j \alpha_j^2} = 0, q_{fDj} = \frac{2q_{fj}x_{fj}}{q_j} \tag{B-1}$$

Initial condition:

$$p_j(t_D = 0) = p_{ij}(t_D = 0)$$

$$p_{Dj}(t_D = 0) = 0(t_D = 0) \tag{B-2}$$

Boundary condition:

$$\frac{k_F}{\mu}\frac{\partial p_j}{\partial x}\bigg|_{x=0} \times 2w_{Fj}h_j = q_j$$

$$\frac{\partial p_{1Dj}}{\partial x_D}\bigg|_{x_D=0} = -\frac{\pi q_{Dj}}{F_{cDj(x_D=0)}\alpha_j\lambda_j} \tag{B-3}$$

$$\frac{\partial p_j}{\partial x}\bigg|_{x=x_{fj}} = 0$$

$$\frac{\partial p_{Dj}}{\partial x_D}\bigg|_{x_D=\alpha_j} = 0 \tag{B-4}$$

The integral flow equation is obtained:

$$(F_{cDj}\frac{\partial p_{Dj}}{\partial x_D})\bigg|_0^{x_D} - \int_0^{x_D}\frac{\pi q_{fDj}}{\lambda_j\alpha_j^2}dx_D = 0$$

$$F_{cDj}\frac{\partial p_{Dj}}{\partial x_D} + \frac{\pi}{\alpha_j\lambda_j} - \int_0^{x_D}\frac{\pi q_{fDj}}{\lambda_j\alpha_j^2}dx_D = 0 \tag{B-5}$$

Integrate again to get:

$$(F_{cDj}\frac{\partial p_{Dj}}{\partial x_D})\bigg|_0^{x_D} - \int_0^{x_D}\frac{\pi q_{fDj}}{\lambda_j\alpha_j^2}dx_D = 0$$

$$\int_0^{x_D} F_{cDj}\partial p_{Dj} + \frac{\pi}{\alpha_j\lambda_j}x_D - \int_0^{x_D}\int_0^{x_D}\frac{\pi q_{fDj}}{\lambda_j\alpha_j^2}dx_D dx_D = 0 \tag{B-6}$$

Organize the above equation to get：

$$p_{wDj} - p_{FDj} + \frac{\pi}{F_{cDj(x_D)}\lambda_j\alpha_j^2}\int_0^{x_D}\int_0^{x_D} q_{fDj}dx_D dx_D = \frac{\pi}{F_{cDj(x_D)}\alpha_j\lambda_j}x_D$$

$$\bar{p}_{wDj} - \bar{p}_{FDj} + \frac{\pi}{F_{cDj(x_D)}\lambda_j\alpha_j^2}\int_0^{x_D}\int_0^{x_D} \bar{q}_{fDj}dx_D dx_D = \frac{\pi}{sF_{cDj(x_D)}\alpha_j\lambda_j}x_D \qquad \text{(B-7)}$$

The above equation is discretized by boundary element method：

$$\bar{p}_{FDj} = \bar{p}_{jD} = \frac{1}{2\alpha_j\lambda_j}\int_{-\alpha_j}^{\alpha_j}\bar{q}_{fDj}\cdot K_0\left[(x_D - x')\sqrt{\frac{\omega_j}{\lambda_j}s}\right]dx'$$

$$= \frac{1}{2\alpha_j\lambda_j}\left\{\int_0^{\alpha_j}\bar{q}_{fDj}\cdot K_0\left[(x_D + x')\sqrt{\frac{\omega_j}{\lambda_j}s}\right]dx' + \int_0^{\alpha_j}\bar{q}_{fD}\cdot K_0\left[(x_D - x')\sqrt{\frac{\omega_j}{\lambda_j}s}\right]dx'\right\} \qquad \text{(B-8)}$$

Where $\Delta x_D = \alpha_j / n, x_{Dk} = (k + 0.5)\Delta x_D, x_{Di} = i\cdot\Delta x_D$.

The discrete form of the above equation:

$$\bar{p}_{FDj} = \frac{1}{2\alpha_j\lambda_j}\sum_{i=1}^{n}\int_{x_{Di}}^{x_{Di+1}}\bar{q}_{fDj(i)}\cdot\left\{K_0\left[(x_D + x')\sqrt{\frac{\omega_j}{\lambda_j}s}\right] + K_0\left[(x_D - x')\sqrt{\frac{\omega_j}{\lambda_j}s}\right]\right\}dx' \qquad \text{(B-9)}$$

The third term of the discrete formula can be obtained (integration by parts):

$$\int_0^{x_D}\int_0^{x_D}\bar{q}_{fDj}dx_D dx_D = x_D\int_0^{x_D}\bar{q}_{fDj}dx_D\Big|_0^{x_{Dk}} - \int_0^{x_{Dk}}\bar{q}_{fDj}x_D dx_D \qquad \text{(B-10)}$$

The first item on the right can be written:

$$x_D\int_0^{x_D}\bar{q}_{fDj}dx_D\Big|_0^{x_{Dk}} = x_{Dk}\int_0^{x_{Dk}}\bar{q}_{fDj}dx_D = (x_{Di} + \frac{\Delta x_D}{2})\left[\sum_{i=1}^{k-1}\bar{q}_{fDji}\Delta x_D + \bar{q}_{fDjk}\frac{\Delta x_D}{2}\right] \qquad \text{(B-11)}$$

The second item on the right can be written:

$$\int_0^{x_{Dk}} \bar{q}_{fDj} x_D dx_D = \sum_{i=1}^{k-1} \bar{q}_{fDji} \int_{x_{Di-1}}^{x_{Di}} x_D dx_D + \bar{q}_{fDjk} \int_{x_{Di=k}}^{x_{Dk}} x_D dx_D$$
$$= \sum_{i=1}^{k-1} \bar{q}_{fDji} (x_{Di} - \frac{\Delta x_D}{2}) \Delta x_D + \bar{q}_{fDjk} \left(x_{Di=k} + \frac{\Delta x_D}{4} \right) \frac{\Delta x_D}{2} \tag{B-12}$$

Finally, the discrete form of double integral is obtained.

$$\int_0^{x_D} \int_0^{x_D} \bar{q}_{fDj} dx_D dx_D = (x_{Di} + \frac{\Delta x_D}{2}) \left[\sum_{i=1}^{k-1} \bar{q}_{fDji} \Delta x_D + \bar{q}_{fDjk} \frac{\Delta x_D}{2} \right]$$
$$- \sum_{i=1}^{k-1} \bar{q}_{fDji} (x_{Di} - \frac{\Delta x_D}{2}) \Delta x_D - \bar{q}_{fDjk} \left(x_{Di=k} + \frac{\Delta x_D}{4} \right) \frac{\Delta x_D}{2} \tag{B-13}$$
$$= \sum_{i=1}^{k-1} \bar{q}_{fDji} (k - i +) \Delta x_D^2 + \bar{q}_{fDjk} \frac{\Delta x_D^2}{8}$$

The flow rate condition is:

$$\sum_{i=1}^{n} q_{fi} \frac{2x_{fj}}{n} = q_j$$
$$\sum_{i=1}^{n} \bar{q}_{fDi} = \frac{n}{s} \tag{B-14}$$

Therefore, $\bar{q}_{fDji}$ and $\bar{p}_{wDj}(s)$ can be obtained by solving the following matrix $(N+1) \times (N+1)$, and using the above method for discretization has many advantages: it can avoid the discretization of time, and the wellbore storage effect and skin effect can be directly added by the Duhamel principle.

$$\begin{bmatrix} & .. & & 1 \\ & A_{ij} & & .. \\ & .. & & 1 \\ 1 & .. & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} .. \\ q_{fDji}(s) \\ .. \\ p_{wDj}(s) \end{bmatrix} = B_j \tag{B-15}$$