load(ctensor);

C:/maxima−5.46.0/share/maxima/5.46.0/share/tensor/ctensor.mac

csetup();

Enter the dimension of the coordinate system: 4;

Do you wish to change the coordinate names? y;

Enter a list containing the names of the coordinates in order [t,r,φ,z];

Do you want to 1. Enter a new metric? 2. Enter a metric from a file? 3. Approximate a metric with a Taylor series?
1;

Is the matrix 1. Diagonal 2. Symmetric 3. Antisymmetric 4. General

2;
c^2·%e^(2·χ);
0;
N·c·%e^(2·χ);
0;
−%e^(−2·χ−2·A);
0;
0;
N^2·%e^(2·χ)−r^2·%e^(−2·χ);
0;
−%e^(−2·χ−2·A);

Matrix entered.

Enter functional dependencies with DEPENDS or 'N' if none

depends(χ,r);depends(A,r);depends(N,r);

Do you wish to see the metric?

Please reply YES or NO.

Do you wish to see the metric?

Please reply YES or NO.

Do you wish to see the metric? y;

$$\begin{pmatrix} c^2\ \%e^{2\chi} & 0 & N\,c\ \%e^{2\chi} & 0 \\ 0 & -\%e^{-2\chi-2A} & 0 & 0 \\ N\,c\ \%e^{2\chi} & 0 & N^2\ \%e^{2\chi}-r^2\ \%e^{-2\chi} & 0 \\ 0 & 0 & 0 & -\%e^{-2\chi-2A} \end{pmatrix}$$

done

einstein(true);

$$ein_{1,1}=(8\,r^3\ \%e^{2\chi+2A}\,(\chi_{rr})-4\,r^3\ \%e^{2\chi+2A}\,(\chi_r)^2+\left(8\,N\,(N_r)\,r\ \%e^{6\chi+2A}+8\,r^2\ \%e^{2\chi+2A}\right)(\chi_r)+4\,(A_{rr})\,r^3\ \%e^{2\chi+2A}+\left(\left(2\ \%e^{2A}\,N\,(N_{rr})+3\ \%e^{2A}\,(N_r)^2\right)r-2\ \%e^{2A}\,N\,(N_r)\right)\%e^{6\chi})/(4\,r^3)$$

$$ein_{1,3}=-\frac{4\,(N_r)\,c\,r\ \%e^{6\chi+2A}\,(\chi_r)+\left(\%e^{2A}\,(N_{rr})\,c\,r-\%e^{2A}\,(N_r)\,c\right)\%e^{6\chi}}{2\,r^3}$$

$$ein_{2,2}=\frac{4\,r^2\ \%e^{2\chi+2A}\,(\chi_r)^2-(N_r)^2\ \%e^{6\chi+2A}+4\,(A_r)\,r\ \%e^{2\chi+2A}}{4\,r^2}$$

$$ein_{3,1}=(4\,N\,r^3\ \%e^{2\chi+2A}\,(\chi_{rr})+\left(4\,N^2\,(N_r)\,r\ \%e^{6\chi+2A}+\left(4\ \%e^{2A}\,(N_r)\,r^3+4\ \%e^{2A}\,N\,r^2\right)\%e^{2\chi}\right)(\chi_r)+\left(\left(\%e^{2A}\,N^2\,(N_{rr})+2\ \%e^{2A}\,N\,(N_r)^2\right)r-\%e^{2A}\,N^2\,(N_r)\right)\%e^{6\chi}+\left(\%e^{2A}\,(N_{rr})\,r^3-\%e^{2A}\,(N_r)\,r^2\right)\%e^{2\chi})/(2\,c\,r^3)$$

$$ein_{3,3}=-(4\,r^3\ \%e^{2\chi+2A}\,(\chi_r)^2+8\,N\,(N_r)\,r\ \%e^{6\chi+2A}\,(\chi_r)-4\,(A_{rr})\,r^3\ \%e^{2\chi+2A}+\left(\left(2\ \%e^{2A}\,N\,(N_{rr})+\%e^{2A}\,(N_r)^2\right)r-2\ \%e^{2A}\,N\,(N_r)\right)\%e^{6\chi})/(4\,r^3)$$

$$ein_{4,4}=-\frac{4\,r^2\ \%e^{2\chi+2A}\,(\chi_r)^2-(N_r)^2\ \%e^{6\chi+2A}+4\,(A_r)\,r\ \%e^{2\chi+2A}}{4\,r^2}$$

done

```
factor(%t3);factor(%t4);factor(%t5);factor(%t6);factor(%t7);factor(%t8);
```

$$ein_{1,1}=(\%e^{2\chi+2A}\,(8\,r^3\,(\chi_{rr})-4\,r^3\,(\chi_r)^2+8\,N\,(N_r)\,r\,\%e^{4\chi}\,(\chi_r)+8\,r^2\,(\chi_r)$$
$$+2\,N\,(N_{rr})\,r\,\%e^{4\chi}+3\,(N_r)^2\,r\,\%e^{4\chi}-2\,N\,(N_r)\,\%e^{4\chi}+4\,(A_{rr})\,r^3))/(4\,r^3)$$

$$ein_{1,3}=-\frac{c\,\%e^{6\chi+2A}\,(4\,(N_r)\,r\,(\chi_r)+(N_{rr})\,r-N_r)}{2\,r^3}$$

$$ein_{2,2}=\frac{\%e^{2\chi+2A}\left(4\,r^2\,(\chi_r)^2-(N_r)^2\,\%e^{4\chi}+4\,(A_r)\,r\right)}{4\,r^2}$$

$$ein_{3,1}=(\%e^{2\chi+2A}\,(4\,N\,r^3\,(\chi_{rr})+4\,N^2\,(N_r)\,r\,\%e^{4\chi}\,(\chi_r)+4\,(N_r)\,r^3\,(\chi_r)+4$$
$$N\,r^2\,(\chi_r)+N^2\,(N_{rr})\,r\,\%e^{4\chi}+2\,N\,(N_r)^2\,r\,\%e^{4\chi}-N^2\,(N_r)\,\%e^{4\chi}+(N_{rr})\,r^3-(N_r)\,r^2))$$
$$/(2\,c\,r^3)$$

$$ein_{3,3}=-(\%e^{2\chi+2A}\,(4\,r^3\,(\chi_r)^2+8\,N\,(N_r)\,r\,\%e^{4\chi}\,(\chi_r)+2\,N\,(N_{rr})\,r\,\%e^{4\chi}+$$
$$(N_r)^2\,r\,\%e^{4\chi}-2\,N\,(N_r)\,\%e^{4\chi}-4\,(A_{rr})\,r^3))/(4\,r^3)$$

$$ein_{4,4}=-\frac{\%e^{2\chi+2A}\left(4\,r^2\,(\chi_r)^2-(N_r)^2\,\%e^{4\chi}+4\,(A_r)\,r\right)}{4\,r^2}$$

```
ar(r):=((diff(N,r,1))^2·%e^(4·χ)−4·r^2·(diff(χ,r,1))^2)/(4·r);
```

$$\mathrm{ar}(r):=\frac{(N_r)^2\,\%e^{4\chi}-4\,r^2\,(\chi_r)^2}{4\,r}$$

```
/*we will substitude Ar by eq. 22 */;
subst(ar(r),diff(A,r,1),%o9);subst(ar(r),diff(A,r,1),%o10);subst(ar(r),diff(A,r,1),%o11);subst(ar(r),diff(A,r,1),%o12);subst(ar(r),diff(A,r,1),%o13);subst(ar(r),diff(A,r,1),%o14);
```

$$ein_{1,1}=(\%e^{2\chi+2A}\,(8\,r^3\,(\chi_{rr})-4\,r^3\,(\chi_r)^2+8\,N\,(N_r)\,r\,\%e^{4\chi}\,(\chi_r)+8\,r^2\,(\chi_r)$$
$$+2\,N\,(N_{rr})\,r\,\%e^{4\chi}+3\,(N_r)^2\,r\,\%e^{4\chi}-2\,N\,(N_r)\,\%e^{4\chi}+4\,(A_{rr})\,r^3))/(4\,r^3)$$

$$ein_{1,3}=-\frac{c\,\%e^{6\chi+2A}\,(4\,(N_r)\,r\,(\chi_r)+(N_{rr})\,r-N_r)}{2\,r^3}$$

$$ein_{2,2}=0$$

$$ein_{3,1}=(\%e^{2\chi+2A}\,(4\,N\,r^3\,(\chi_{rr})+4\,N^2\,(N_r)\,r\,\%e^{4\chi}\,(\chi_r)+4\,(N_r)\,r^3\,(\chi_r)+4$$
$$N\,r^2\,(\chi_r)+N^2\,(N_{rr})\,r\,\%e^{4\chi}+2\,N\,(N_r)^2\,r\,\%e^{4\chi}-N^2\,(N_r)\,\%e^{4\chi}+(N_{rr})\,r^3-(N_r)\,r^2))$$
$$/(2\,c\,r^3)$$

$$ein_{3,3}=-(\%e^{2\chi+2A}\,(4\,r^3\,(\chi_r)^2+8\,N\,(N_r)\,r\,\%e^{4\chi}\,(\chi_r)+2\,N\,(N_{rr})\,r\,\%e^{4\chi}+$$
$$(N_r)^2\,r\,\%e^{4\chi}-2\,N\,(N_r)\,\%e^{4\chi}-4\,(A_{rr})\,r^3))/(4\,r^3)$$

$$ein_{4,4}=0$$

```
subst(diff(ar(r),r,1),diff(A,r,2),%o16);subst(diff(ar(r),r,1),diff(A,r,2),%o17);subst(diff(ar(r),r,1),diff(A,r,2),%o19);subst(diff(ar(r),r,1),diff(A,r,2),%o20);
```

$$ein_{1,1}=(\%e^{2\chi+2A}\,(4\,r^3\,($$
$$\frac{-8\,r^2\,(\chi_r)\,(\chi_{rr})-8\,r\,(\chi_r)^2+4\,(N_r)^2\,\%e^{4\chi}\,(\chi_r)+2\,(N_r)\,(N_{rr})\,\%e^{4\chi}}{4\,r}-$$
$$\frac{(N_r)^2\,\%e^{4\chi}-4\,r^2\,(\chi_r)^2}{4\,r^2})+8\,r^3\,(\chi_{rr})-4\,r^3\,(\chi_r)^2+8\,N\,(N_r)\,r\,\%e^{4\chi}\,(\chi_r)+8\,r^2\,(\chi_r)+$$
$$2\,N\,(N_{rr})\,r\,\%e^{4\chi}+3\,(N_r)^2\,r\,\%e^{4\chi}-2\,N\,(N_r)\,\%e^{4\chi}))/(4\,r^3)$$

$$ein_{1,3}=-\frac{c\,\%e^{6\chi+2A}\,(4\,(N_r)\,r\,(\chi_r)+(N_{rr})\,r-N_r)}{2\,r^3}$$

$$ein_{3,1}=(\%e^{2\chi+2A}\,(4\,N\,r^3\,(\chi_{rr})+4\,N^2\,(N_r)\,r\,\%e^{4\chi}\,(\chi_r)+4\,(N_r)\,r^3\,(\chi_r)+4$$
$$N\,r^2\,(\chi_r)+N^2\,(N_{rr})\,r\,\%e^{4\chi}+2\,N\,(N_r)^2\,r\,\%e^{4\chi}-N^2\,(N_r)\,\%e^{4\chi}+(N_{rr})\,r^3-(N_r)\,r^2))$$
$$/(2\,c\,r^3)$$

$$ein_{3,3}=-(\%e^{2\chi+2A}\,(-4\,r^3\,($$
$$\frac{-8\,r^2\,(\chi_r)\,(\chi_{rr})-8\,r\,(\chi_r)^2+4\,(N_r)^2\,\%e^{4\chi}\,(\chi_r)+2\,(N_r)\,(N_{rr})\,\%e^{4\chi}}{4\,r}-$$
$$\frac{(N_r)^2\,\%e^{4\chi}-4\,r^2\,(\chi_r)^2}{4\,r^2})+4\,r^3\,(\chi_r)^2+8\,N\,(N_r)\,r\,\%e^{4\chi}\,(\chi_r)+2\,N\,(N_{rr})\,r\,\%e^{4\chi}+$$
$$(N_r)^2\,r\,\%e^{4\chi}-2\,N\,(N_r)\,\%e^{4\chi}))/(4\,r^3)$$

factor(%o22);factor(%o23);factor(%o24);factor(%o25);

$$ein_{1,1}=-(\%e^{2\chi+2A}\,(4\,r^4\,(\chi_r)\,(\chi_{rr})-4\,r^3\,(\chi_{rr})+4\,r^3\,(\chi_r)^2-2\,(N_r)^2\,r^2$$
$$\%e^{4\chi}\,(\chi_r)-4\,N\,(N_r)\,r\,\%e^{4\chi}\,(\chi_r)-4\,r^2\,(\chi_r)-(N_r)\,(N_{rr})\,r^2\,\%e^{4\chi}-N\,(N_{rr})\,r\,\%e^4$$
$$-(N_r)^2\,r\,\%e^{4\chi}+N\,(N_r)\,\%e^{4\chi}))/(2\,r^3)$$

$$ein_{1,3}=-\frac{c\,\%e^{6\chi+2A}\,(4\,(N_r)\,r\,(\chi_r)+(N_{rr})\,r-N_r)}{2\,r^3}$$

$$ein_{3,1}=(\%e^{2\chi+2A}\,(4\,N\,r^3\,(\chi_{rr})+4\,N^2\,(N_r)\,r\,\%e^{4\chi}\,(\chi_r)+4\,(N_r)\,r^3\,(\chi_r)+$$
$$N\,r^2\,(\chi_r)+N^2\,(N_{rr})\,r\,\%e^{4\chi}+2\,N\,(N_r)^2\,r\,\%e^{4\chi}-N^2\,(N_r)\,\%e^{4\chi}+(N_{rr})\,r^3-(N_r)\,r^2$$
$$/(2\,c\,r^3)$$

$$ein_{3,3}=-(\%e^{2\chi+2A}\,(4\,r^4\,(\chi_r)\,(\chi_{rr})+4\,r^3\,(\chi_r)^2-2\,(N_r)^2\,r^2\,\%e^{4\chi}\,(\chi_r)+4$$
$$N\,(N_r)\,r\,\%e^{4\chi}\,(\chi_r)-(N_r)\,(N_{rr})\,r^2\,\%e^{4\chi}+N\,(N_{rr})\,r\,\%e^{4\chi}+(N_r)^2\,r\,\%e^{4\chi}-N\,(N_r$$
$$\%e^{4\chi}))/(2\,r^3)$$

/* for typical galaxy density exponential terms are close to 1 */;
subst(1,%e,%o26);subst(1,%e,%o27);subst(1,%e,%o28);subst(1,%e,%o29);

$$ein_{1,1}=-(4\,r^4\,(\chi_r)\,(\chi_{rr})-4\,r^3\,(\chi_{rr})+4\,r^3\,(\chi_r)^2-2\,(N_r)^2\,r^2\,(\chi_r)-4\,r^2\,(\chi_r$$
$$4\,N\,(N_r)\,r\,(\chi_r)-(N_r)\,(N_{rr})\,r^2-N\,(N_{rr})\,r-(N_r)^2\,r+N\,(N_r))/(2\,r^3)$$

$$ein_{1,3}=-\frac{c\,(4\,(N_r)\,r\,(\chi_r)+(N_{rr})\,r-N_r)}{2\,r^3}$$

$$ein_{3,1}=(4\,N\,r^3\,(\chi_{rr})+4\,(N_r)\,r^3\,(\chi_r)+4\,N\,r^2\,(\chi_r)+4\,N^2\,(N_r)\,r\,(\chi_r)+(N_{rr})$$
$$r^3-(N_r)\,r^2+N^2\,(N_{rr})\,r+2\,N\,(N_r)^2\,r-N^2\,(N_r))/(2\,c\,r^3)$$

$$ein_{3,3}=-(4\,r^4\,(\chi_r)\,(\chi_{rr})+4\,r^3\,(\chi_r)^2-2\,(N_r)^2\,r^2\,(\chi_r)+4\,N\,(N_r)\,r\,(\chi_r)-(N$$
$$(N_{rr})\,r^2+N\,(N_{rr})\,r+(N_r)^2\,r-N\,(N_r))/(2\,r^3)$$

%o30+%o33;

$$ein_{3,3}+ein_{1,1}=-(4\,r^4\,(\chi_r)\,(\chi_{rr})-4\,r^3\,(\chi_{rr})+4\,r^3\,(\chi_r)^2-2\,(N_r)^2\,r^2\,(\chi_r)-$$
$$r^2\,(\chi_r)-4\,N\,(N_r)\,r\,(\chi_r)-(N_r)\,(N_{rr})\,r^2-N\,(N_{rr})\,r-(N_r)^2\,r+N\,(N_r))/(2\,r^3)-(4\,r^4$$
$$(\chi_r)\,(\chi_{rr})+4\,r^3\,(\chi_r)^2-2\,(N_r)^2\,r^2\,(\chi_r)+4\,N\,(N_r)\,r\,(\chi_r)-(N_r)\,(N_{rr})\,r^2+N\,(N_{rr})\,r+$$
$$(N_r)^2\,r-N\,(N_r))/(2\,r^3)$$

factor(%o30);factor(%o31);factor(%o32);factor(%o33);factor(%o34);

$$ein_{1,1}=-(4\,r^4\,(\chi_r)\,(\chi_{rr})-4\,r^3\,(\chi_{rr})+4\,r^3\,(\chi_r)^2-2\,(N_r)^2\,r^2\,(\chi_r)-4\,r^2\,(\chi_r)-4\,N\,(N_r)\,r\,(\chi_r)-(N_r)\,(N_{rr})\,r^2-N\,(N_{rr})\,r-(N_r)^2\,r+N\,(N_r))/(2\,r^3)$$

$$ein_{1,3}=-\frac{c\,(4\,(N_r)\,r\,(\chi_r)+(N_{rr})\,r-N_r)}{2\,r^3}$$

$$ein_{3,1}=(4\,N\,r^3\,(\chi_{rr})+4\,(N_r)\,r^3\,(\chi_r)+4\,N\,r^2\,(\chi_r)+4\,N^2\,(N_r)\,r\,(\chi_r)+(N_{rr})\,r^3-(N_r)\,r^2+N^2\,(N_{rr})\,r+2\,N\,(N_r)^2\,r-N^2\,(N_r))/(2\,c\,r^3)$$

$$ein_{3,3}=-(4\,r^4\,(\chi_r)\,(\chi_{rr})+4\,r^3\,(\chi_r)^2-2\,(N_r)^2\,r^2\,(\chi_r)+4\,N\,(N_r)\,r\,(\chi_r)-(N_r)\,(N_{rr})\,r^2+N\,(N_{rr})\,r+(N_r)^2\,r-N\,(N_r))/(2\,r^3)$$

$$ein_{3,3}+ein_{1,1}=-\frac{4\,r^2\,(\chi_r)\,(\chi_{rr})-2\,r\,(\chi_{rr})+4\,r\,(\chi_r)^2-2\,(N_r)^2\,(\chi_r)-2\,(\chi_r)-(N_r)\,(N_{rr})}{r}$$

/* %o44 is eq. 23 in which we regroup the Xrr and Xr terms, we made the same groupement in eqs. 24,25 */;