# **Supplementary materials**

#### 1. Normalization of Mueller matrices

An unnormalized Mueller matrix  $\tilde{\mathbf{M}}$  relates incident and emerging Stokes vectors  $\mathbf{S}_i$  and  $\mathbf{S}_o$ , respectively, as,

$$\mathbf{S}_{o} = \widetilde{\mathbf{M}}\mathbf{S}_{i} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \mathbf{S}_{i} .$$
(S1)

Normalization is done by dividing all elements of  $\widetilde{\mathbf{M}}$  with  $M_{11}$  resulting in the normalized Mueller matrix  $\mathbf{M}$  according to,

$$\mathbf{M} = \begin{bmatrix} 1 & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix},$$
 (S2)

where  $m_{ij} = M_{ij}/M_{11}$ .

## 2. Comparison with a dielectric mirror



**Figure S1**. Mueller matrix of a dielectric mirror (dotted line) with n = 1.6 calculated at  $\theta = 20^{\circ}$  compared to **M** for *P*. *mirifica* PM1 (solid line).

## 3. Cauchy dispersions

The refractive indices of the biaxial slices (the lamellae) in the chiral structures in the exocuticle as well as the epicuticle and endocuticle were modelled with refractive indices according to,

$$n = A + \frac{B}{\lambda^2},\tag{S3}$$

where A and B are fit parameters. For the endocuticle index  $n_3$ , i.e. normal to the cuticle surface, only A could be fitted due to low sensitivity at  $\theta = 20^{\circ}$ .

## 4. Parameter values for the fit in Fig. 4



Figure S2. Best fit parameter values for fit in Fig. 4.

## 5. Exocuticle refractive indices



Figure S3. Best fit refractive indices for the exocuticle slices.



## 6. Mueller matrix of P. mirifica PM2

Wavelength (nm)

Figure S4. Mueller matrix of *P. mirifica* PM2 measured at  $\theta = 20^{\circ}$  and used in sum decomposition.

## 7. Mueller matrices $M_1 \mbox{ and } M_2$ in sum decomposition



Figure S5. Mueller matrices  $M_1$  and  $M_2$  in sum decomposition of M from beetle PM2.



Figure S6. Sum of  $M_1$  and  $M_2$  in Fig. S5.

#### 8. Blue shift with angle of incidence



**Figure S7.** Mueller matrices measured on beetle PM1 at  $\theta$  from 20° (Exp#1) to 70° (Exp#6) in steps of 10°.



## 9. Differential decomposition

**Figure S8.** Transmission Mueller matrices on an elytron from specimen PM1 measured from inside (Exp#1) and outside (Exp#2). The noise for  $\lambda < 550$  nm is due to low transmission.



**Figure S9.** Circular (CB and CD) and xy linear (LB, LD) and 45°/135° linear (LBp and LDp) birefringent optical properties obtained from a differential decomposition of **M** in Fig. S8. See references [24] and [34] in the main manuscript for details.