

Annex 3. Chance calculations

The chance of species A or B in 1 mL draws, without replacement, from a 10 mL (top) or 100 mL (bottom) dilution with 5 (top) and 50 (bottom) cells of A and B using the hypergeometric formula.

Hypergeometric formula:

Combinations formula (e.g. (K over k) = C (n, r)):

$$P(X = k) = \frac{\binom{K}{k} \binom{N-K}{n-k}}{\binom{N}{n}}$$

$$C(n, r) = \frac{n!}{r!(n-r)!}$$

A	B	No cell	N	K	n	k	N-K	n-k	[K k]	[N-K n-k]	[N n]	P	
30	20	50	100	50	5	0	50	5	1	2118760	75287520	0.028	=No A or B's
30	20	50	100	50	5	1	50	4	50	230300	75287520	0.153	=1A or 1B
30	20	50	100	50	5	2	50	3	1225	19600	75287520	0.319	=2 A's+B's
30	20	50	100	50	5	3	50	2	19600	1225	75287520	0.319	=3 A's+B's
30	20	50	100	50	5	4	50	1	230300	50	75287520	0.153	=4 A's+B's
30	20	50	100	50	5	5	50	0	2118760	1	75287520	0.028	=All A+B's
											sum:	1.000	

Table 1. The chances (P) of drawing 0 to 5 cells (k) in five 1 mL draws (n) without replacement, if 50 cells (A and B) are present (K) in a 100 mL volume (N). , as in a 100x dilution of an MPN sample with 50 c/mL. Note that the distribution of cells between A and B is not relevant for the calculation of P.

S1		S11	No cell	N	K	n	k	N-K	n-k	[K k]	[N-K n-k]	[N n]	P	
5		5	45	100	55	5	0	45	5	1	1221759	75287520	0.016	=No
5		5	45	100	55	5	1	45	4	55	148995	75287520	0.109	=1
5		5	45	100	55	5	2	45	3	1485	14190	75287520	0.280	=2
5		5	45	100	55	5	3	45	2	26235	990	75287520	0.345	=3
5		5	45	100	55	5	4	45	1	341055	45	75287520	0.204	=4
5		5	45	100	55	5	5	45	0	3478761	1	75287520	0.046	=5
												sum:	1.000	

Table 2. The chances (P) of drawing 0 to 5 cells (k) in five 1 mL draws (n) without replacement, if 11 species with 5 cells each (S1....S11) are present (K) in a 100 mL volume (N).

A	B	C	No cell	N	K	n	k	N-K	n-k	[K k]	[N-K n-k]	[N n]	P	
24	24	6	46	100	5	5	0	95	5	1	57940519	75287520	0.770	no C
24	24	6	46	100	5	5	1	95	4	5	3183545	75287520	0.211	1C
24	24	6	46	100	5	5	2	95	3	10	138415	75287520	0.018	2C
24	24	6	46	100	5	5	3	95	2	10	4465	75287520	0.001	3C
24	24	6	46	100	5	5	4	95	1	5	95	75287520	0.000	4C
24	24	6	46	100	5	5	5	95	0	1	1	75287520	0.000	5C
												sum:	1.000	

Table 3. The chances (P) of drawing 0 to 5 cells (k) of species C from a mixture with A and B in five 1 mL draws (n) without replacement, with 5 C cells present (K) in a 100 mL volume (N).

The formula for the number of permutations with n objects (also known as permutations with repetition) with n = 5 (total number of objects), n1 = 1x species A, n2 = 1x species B, and n3 = 3x no cell.:

$$P = n! / (n1! \times n2! \times n3!) = 20$$