Appendices

Appendix A: Derivation of the radial diffusion signal

To solve Eq. (6), we define the initial spin distribution $P(\rho',\theta')$ as a uniform probability distribution on a circle/cylinder with radius a:

$$P(\rho',\theta') = \frac{1}{2\pi a}\delta(\rho'-a), \tag{31}$$

where $\delta(x)$ is a Dirac delta function: it is 1 for x =0 and 0 otherwise. Substituting Eq. (31) into Eq. (6) and integrating over $d\rho'$, we obtain

$$\frac{E_\perp(\mathbf{q}_{xy},t)}{E_\perp(\mathbf{q}_{xy}=0,t)} = \frac{1}{2\pi}\int_0^\infty\int_0^{2\pi}\int_0^{2\pi} P(\rho,\theta|,a,\theta',t)e^{iq_{xy}\rho\cos(\varphi+\theta)}e^{-iq_{xy}a\cos(\varphi+\theta')}\rho d\rho d\theta d\theta'. \tag{32}$$

Since the displacement of the particles is confined to the circle's circumference, the probability $P(\rho,\theta|,a,\theta',t) = P(\theta|\theta',a,t)P(\rho|a,t)$ can be written as the product of the normalized angular distribution $P(\theta|\theta',a,t)$ for moving from angle θ' to θ in time t on the circle with radius a, and a delta function prohibiting any movement in the radial coordinate $P(\rho|a,t) = \delta(\rho-a)/a$ (which guarantees that $\int_{-\infty}^{\infty} P(\rho|a,t)\rho d\rho = 1$) that is appropriate for impermeable cylinders.

After plugging these equations into Eq. (32), and integrating over $d\rho$ we obtain,

$$\begin{aligned}\frac{E_\perp(\mathbf{q}_{xy},t)}{E_\perp(\mathbf{q}_{xy}=0,t)} &= \frac{1}{2\pi}\int_0^{2\pi}\int_0^{2\pi} P(\theta|\theta',a,t)e^{iq_{xy}a\cos(\varphi+\theta)}e^{-iq_{xy}a\cos(\varphi+\theta')}d\theta d\theta', \\ &= \frac{1}{2\pi}\int_0^{2\pi}\int_0^{2\pi} P(\Phi|,a,t)e^{iq_{xy}a\cos(\psi)}e^{-iq_{xy}a\cos(\psi-\Phi)}d\psi d\Phi,\end{aligned} \tag{33}$$

where we used the change of variables $\psi = \varphi + \theta$ and $\Phi = \theta - \theta'$ in the second equation.

Substituting Eq. (8) into Eq. (33), and using the following Jacobi-Anger expansions [118]

$$e^{iq_{xy}a\cos(\psi)} = J_0\left(aq_{xy}\right) + 2\sum_{n=1}^{\infty} i^n J_n\left(aq_{xy}\right)\cos\left(n\psi\right),$$
$$e^{-iq_{xy}a\cos(\psi-\Phi)} = J_0\left(aq_{xy}\right) + 2\sum_{m=1}^{\infty} (-i)^m J_m\left(aq_{xy}\right)\cos\left(m\left(\psi-\Phi\right)\right), \tag{34}$$

we obtain

$$\begin{aligned}
\frac{E_\perp\left(\mathbf{q}_{xy},t\right)}{E_\perp\left(\mathbf{q}_{xy}=0,t\right)} &= \frac{1}{(2\pi)^2}\int_0^{2\pi}\int_0^{2\pi}\left[1+2\sum_{p=1}^{\infty} e^{-p^2\frac{Dt}{a^2}}\cos\left(p\Phi\right)\right]\times \\
&\left[J_0\left(aq_{xy}\right)+2\sum_{m=1}^{\infty}(-i)^m J_m\left(aq_{xy}\right)\cos\left(m\left(\psi-\Phi\right)\right)\right]\times \\
&\left[J_0\left(aq_{xy}\right)+2\sum_{n=1}^{\infty} i^n J_n\left(aq_{xy}\right)\cos\left(n\psi\right)\right]d\psi\, d\Phi, \\
&= \frac{1}{(2\pi)^2}\int_0^{2\pi}\int_0^{2\pi}\left[J_0^2\left(aq_{xy}\right)+2J_0\left(aq_{xy}\right)\sum_{n=1}^{\infty} i^n J_n\left(aq_{xy}\right)\cos\left(n\psi\right)+\right. \\
&2J_0\left(aq_{xy}\right)\sum_{m=1}^{\infty}(-i)^m J_m\left(aq_{xy}\right)\cos\left(m\left(\psi-\Phi\right)\right)+ \\
&\left.4\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} i^n(-i)^m J_m\left(aq_{xy}\right)J_n\left(aq_{xy}\right)\cos\left(m\left(\psi-\Phi\right)\right)\cos\left(n\psi\right)\right]\times \\
&\left[1+2\sum_{p=1}^{\infty} e^{-p^2\frac{Dt}{a^2}}\cos\left(p\Phi\right)\right]d\psi\, d\Phi
\end{aligned} \tag{35}$$

It is convenient to integrate over $d\psi$,

$$\begin{aligned}
\frac{E_\perp\left(\mathbf{q}_{xy},t\right)}{E_\perp\left(\mathbf{q}_{xy}=0,t\right)} &= \frac{1}{(2\pi)^2}\int_0^{2\pi}\left\{\int_0^{2\pi} J_0^2\left(aq_{xy}\right)d\psi + 2J_0\left(aq_{xy}\right)\sum_{n=1}^{\infty} i^n J_n\left(aq_{xy}\right)\int_0^{2\pi}\cos\left(n\psi\right)d\psi+\right. \\
&2J_0\left(aq_{xy}\right)\sum_{m=1}^{\infty}(-i)^m J_m\left(aq_{xy}\right)\int_0^{2\pi}\cos\left(m\left(\psi-\Phi\right)\right)d\psi+ \\
&\left.4\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} i^n(-i)^m J_m\left(aq_{xy}\right)J_n\left(aq_{xy}\right)\int_0^{2\pi}\cos\left(m\left(\psi-\Phi\right)\right)\cos\left(n\psi\right)d\psi\right]\times \\
&\left.\left[1+2\sum_{p=1}^{\infty} e^{-p^2\frac{Dt}{a^2}}\cos\left(p\Phi\right)\right]\right\}d\Phi, \\
&= \frac{1}{2\pi}\int_0^{2\pi}\left[J_0^2\left(aq_{xy}\right)+2\sum_{n=1}^{\infty} J_n^2\left(aq_{xy}\right)\cos\left(n\Phi\right)\right]\times \\
&\left.\left[1+2\sum_{p=1}^{\infty} e^{-p^2\frac{Dt}{a^2}}\cos\left(p\Phi\right)\right]\right\}d\Phi,
\end{aligned} \tag{36}$$

where we used the following identities,

$$\begin{aligned}
&\int_0^{2\pi}\cos\left(m\left(\psi-\Phi\right)\right)d\psi=\frac{\sin\left(m\Phi\right)-\sin\left(m\left(\Phi-2\pi\right)\right)}{m}=0,\\
&\int_0^{2\pi}\cos\left(n\psi\right)d\psi=\frac{\sin\left(2\pi n\right)}{n}=0,\\
&\int_0^{2\pi}\cos\left(n\psi\right)\cos\left(m\left(\psi-\Phi\right)\right)d\psi=\pi\cos\left(n\Phi\right)\delta\left(n-m\right).
\end{aligned} \tag{37}$$

The second and third identities are also helpful in integrating Eq. (36) over $d\Phi$

$$\begin{aligned}
\frac{E_\perp\left(\mathbf{q}_{xy},t\right)}{E_\perp\left(\mathbf{q}_{xy}=0,t\right)}=\frac{1}{2\pi}\Bigg[&J_0^2\left(aq_{xy}\right)\int_0^{2\pi}d\Phi+2\sum_{n=1}^{\infty}J_n^2\left(aq_{xy}\right)\int_0^{2\pi}\cos\left(n\Phi\right)d\Phi+\\
&2J_0^2\left(aq_{xy}\right)\sum_{p=1}^{\infty}e^{-p^2\frac{Dt}{a^2}}\int_0^{2\pi}\cos\left(p\Phi\right)d\Phi\\
&+4\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}J_n^2\left(aq_{xy}\right)e^{-p^2\frac{Dt}{a^2}}\int_0^{2\pi}\cos\left(p\Phi\right)\cos\left(n\Phi\right)d\Phi\Bigg],\\
=&J_0^2\left(aq_{xy}\right)+2\sum_{p=1}^{\infty}J_p^2\left(aq_{xy}\right)e^{-p^2\frac{Dt}{a^2}}.
\end{aligned} \tag{38}$$

Appendix B: Derivation of the effective radial diffusivity

In the following equation, we first represent the mean squared displacement in polar coordinates and later use Eq. (8) to compute the mean squared displacement on the circle, where we used the second and third identities reported in Eq. (37):

$$
\begin{aligned}
\left\langle \left| \mathbf{r}_{xy} \right|^2 \right\rangle &= \left\langle r_x^2 + r_y^2 \right\rangle, \\
&= \left\langle a^2 \left(\cos(\theta) - \cos(\theta')\right)^2 + a^2 \left(\sin(\theta) - \sin(\theta')\right)^2 \right\rangle, \\
&= \left\langle 2a^2 \left(1 - \cos(\Phi)\right) \right\rangle, \\
&= \int_0^{2\pi} 2a^2 \left(1 - \cos(\Phi)\right) P\left(\Phi \middle|, a, t\right) d\Phi, \\
&= 2a^2 \left(1 - \int_0^{2\pi} P\left(\Phi \middle|, a, t\right) \cos(\Phi) \, d\Phi \right), \\
&= 2a^2 \left(1 - \frac{1}{2\pi} \int_0^{2\pi} \left[\cos(\Phi) + 2\sum_{p=1}^{\infty} e^{-p^2 \frac{Dt}{a^2}} \cos(p\Phi) \cos(\Phi) \right] d\Phi \right), \\
&= 2a^2 \left(1 - e^{-\frac{Dt}{a^2}} \right).
\end{aligned} \tag{39}
$$

Appendix C: Derivation of Lori's correction approach

The precise expression for $\left\langle e^{i\phi} \middle| \mathbf{r} \right\rangle_{\Delta,\delta,\mathbf{g}}$ in Eq. (15) is generally unknown, making it difficult to estimate a closed-form analytical expression for $\left\langle e^{i\phi} \right\rangle_{\Delta,\delta,\mathbf{g}}$ from the diffusion propagator. Fortunately, it can be derived for some particular cases.

It is well known that for a Gaussian anisotropic diffusion process characterized by a diffusion tensor $\mathbf{D}$, the PGSE signal attenuation for a rectangular diffusion gradient is given by [76]

$$
\frac{\left\langle e^{i\phi} \right\rangle_{\Delta,\delta,\mathbf{g}}}{\left\langle e^{i\phi} \right\rangle_{\Delta,\delta,\mathbf{g}=0}} = e^{-\mathbf{q}^T \mathbf{D} \mathbf{q} [\Delta - \delta/3]} \; . \tag{40}
$$

By plugging Eq. (40) into Eq. (15), and substituting the corresponding Gaussian anisotropic distribution of displacements producing such a signal, we note that the following equality must hold:

$$
e^{-\mathbf{q}^T \mathbf{D} \mathbf{q} [\Delta - \delta/3]} = \int_{\mathbb{R}^3} \frac{1}{\sqrt{(2\pi)^3 \left| 2\mathbf{D}(\Delta + \delta) \right|}} e^{-\frac{1}{2} \mathbf{r}^T [2\mathbf{D}(\Delta+\delta)]^{-1} \mathbf{r}} \left\langle e^{i\phi} \middle| \mathbf{r} \right\rangle_{\Delta,\delta,\mathbf{g}} d\mathbf{r} \, . \tag{41}
$$

Notably, $\left\langle e^{i\phi} \middle| \mathbf{r} \right\rangle_{\Delta,\delta,\mathbf{g}}$ can be determined via inverse induction from this relationship. First, let us rewrite Eq. (40) as

$$e^{-\mathbf{q}^T\mathbf{D}\mathbf{q}[\Delta-\delta/3]} = e^{-\frac{1}{2}\mathbf{q}'^T[2\mathbf{D}(\Delta+\delta)]\mathbf{q}'}, \tag{42}$$

where a scaled q-space vector $\mathbf{q}' = \mathbf{q}\sqrt{\frac{\Delta-\delta/3}{\Delta+\delta}}$ was introduced. Next, by applying the Fourier integral theorem to Eq. (42) - stating that if we take the inverse Fourier transform of a function and subsequently take the Fourier transform of the resulting expression, we retrieve the original function - we get:

$$\begin{aligned} e^{-\frac{1}{2}\mathbf{q}'^T[2\mathbf{D}(\Delta+\delta)]\mathbf{q}'} &= \int_{\mathbb{R}^3}\left(\int_{\mathbb{R}^3} e^{-\frac{1}{2}\mathbf{q}'^T[2\mathbf{D}(\Delta+\delta)]\mathbf{q}'} e^{-i\mathbf{q}'\mathbf{r}} d\mathbf{q}'\right) e^{i\mathbf{q}'\mathbf{r}} d\mathbf{r}, \\ &= \int_{\mathbb{R}^3} \frac{1}{\sqrt{(2\pi)^3 \left|2\mathbf{D}(\Delta+\delta)\right|}} e^{-\frac{1}{2}\mathbf{r}^T[2\mathbf{D}(\Delta+\delta)]^{-1}\mathbf{r}} e^{i\mathbf{q}'\mathbf{r}} d\mathbf{r}. \end{aligned} \tag{43}$$

By comparing Eqs. (41), (42) and (43) we obtain that $\left\langle e^{i\phi} \middle| \mathbf{r} \right\rangle_{\Delta,\delta,\mathbf{g}} = e^{i\mathbf{q}'\mathbf{r}}$ is a complex exponential similar to that in the q-space formalism but with the scaled q-vector $\mathbf{q}'$. Substituting this result into Eq. (15), we obtain the following approximation:

$$\frac{\left\langle e^{i\phi}\right\rangle_{\Delta,\delta,\mathbf{g}}}{\left\langle e^{i\phi}\right\rangle_{\Delta,\delta,\mathbf{g}=0}} \approx \int_{\mathbb{R}^3} P(\mathbf{r},\Delta+\delta) e^{i\sqrt{\frac{\Delta-\delta/3}{\Delta+\delta}}\mathbf{q}\mathbf{r}} d\mathbf{r}. \tag{44}$$

A similar result can be obtained for PGSE sequences with trapezoidal gradients by replacing the total diffusion encoding time $t_{\exp} = \Delta+\delta$ with $t_{\exp} = \Delta+\delta+\xi$, and the effective diffusion time $t_{eff} = \Delta-\delta/3$ with $t_{eff} = \Delta-\delta/3+\xi^3/30\delta^2-\xi^2/6\delta$.

Appendix D: Derivation of the spherical mean signal

Since the spherical mean signal is rotationally invariant, its value for a distribution of identical cylinders with arbitrary orientations is equal to that from cylinders oriented along the z-axis. The spherical mean of Eq. (11) is

$$\begin{aligned} \left\langle S\right\rangle &= \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} E(\mathbf{q},t)\sin(\beta)\,d\beta\, d\phi, \\ &= \frac{E(\mathbf{q}=0,t)}{4\pi}\int_0^{2\pi}\int_0^{\pi} e^{-q^2\cos(\beta)^2 D_{\|}t}\left[J_0^2(aq\sin(\beta)) + 2\sum_{p=1}^{\infty} J_p^2(aq\sin(\beta)) e^{-p^2\frac{D_{\|}t}{a^2}}\right]\sin(\beta)\,d\beta\, d\phi, \\ &= \frac{E(\mathbf{q}=0,t)}{2}\int_0^{\pi} e^{-q^2\cos(\beta)^2 D_{\|}t}\left[J_0^2(aq\sin(\beta)) + 2\sum_{p=1}^{\infty} J_p^2(aq\sin(\beta)) e^{-p^2\frac{D_{\|}t}{a^2}}\right]\sin(\beta)\,d\beta, \end{aligned} \tag{45}$$

where the integral by angle ϕ was computed straightforwardly since the signal is antipodal symmetric (i.e., constant) for all angles ϕ.

The previous equation can be rewritten as

$$\langle S \rangle = \frac{E(\mathbf{q}=0,t)}{2}\left[I(0) + 2\sum_{p=1}^{\infty} e^{-p^2 \frac{D_{\|}t}{a^2}} I(p) \right], \tag{46}$$

where,

$$\begin{aligned} I(p) &= \int_0^{\pi} e^{-q^2\cos(\beta)^2 D_{\|}t} J_p^2(aq\sin(\beta))\sin(\beta)\,d\beta, \\ &= -\int_1^{-1} e^{-q^2 D_{\|}tx^2} J_p^2\left(aq\sqrt{1-x^2}\right)dx, \\ &= \int_{-1}^{1} e^{-q^2 D_{\|}tx^2} J_p^2\left(aq\sqrt{1-x^2}\right)dx, \\ &= 2\int_0^{1} e^{-q^2 D_{\|}tx^2} J_p^2\left(aq\sqrt{1-x^2}\right)dx. \end{aligned} \tag{47}$$

We introduced the change of variables $x = \cos(\beta)$, thus, $\sin(\beta) = \sqrt{1-x^2}$ and $\sin(\beta)\,d\beta = -d\cos(\beta) = -dx$. The integral is symmetric around zero; therefore, we integrate from zero to one.

The above integral does not have a compact closed-form solution. However, it can be solved by expanding the squared Bessel function of the first kind in series [119]:

$$J_p^2(z) = \sum_{k=0}^{\infty} c_{kp} z^{2(p+k)}, \tag{48}$$

where the coefficients c_{kp} are determined by

$$c_{kp} = \frac{(-1)^k}{k!(2p+k)!}\left(\frac{1}{2}\right)^{2(p+k)}\binom{2(p+k)}{p+k}. \tag{49}$$

Inserting Eq. (48) into Eq. (47) we obtain

$$I\left(p\right)=2\sum_{k=0}^{\infty}c_{kp}\left(aq\right)^{2\left(p+k\right)}\int_{0}^{1}e^{-q^2D_{\parallel}tx^2}\left(1-x^2\right)^{\left(p+k\right)}dx. \tag{50}$$

Using the binomial theorem,

$$\left(1-x^2\right)^{\left(p+k\right)}=\sum_{j=0}^{p+k}\binom{p+k}{j}\left(-1\right)^j x^{2j}\ , \tag{51}$$

we obtain,

$$I\left(p\right)=2\sum_{k=0}^{\infty}c_{kp}\left(aq\right)^{2\left(p+k\right)}\sum_{j=0}^{p+k}\binom{p+k}{j}\left(-1\right)^j\int_{0}^{1}e^{-q^2D_{\parallel}tx^2}x^{2j}dx\ . \tag{52}$$

The integral in the previous equation can be solved as

$$\int_{0}^{1}e^{-q^2D_{\parallel}tx^2}x^{2j}dx=\frac{1}{2\left(q^2D_{\parallel}t\right)^{j+1/2}}\left(\Gamma\left(j+\frac{1}{2}\right)-\Gamma\left(j+\frac{1}{2},q^2D_{\parallel}t\right)\right), \tag{53}$$

where $\Gamma\left(j+1/2,bD\right)$ is the upper incomplete Gamma function. It is important to note that this function has been implemented in various libraries using different formats. For example, in *scipy*, it is defined by $\Gamma\left(j+1/2,x\right)=\Gamma\left(j+1/2\right)gammaincc\left(j+1/2,x\right)$, and by $igamma$ in *Matlab*.

By plugging Eq. (53) into Eq. (52) we get

$$I\left(p\right)=\sum_{k=0}^{\infty}c_{kp}\left(aq\right)^{2\left(p+k\right)}\sum_{j=0}^{p+k}\binom{p+k}{j}\left(-1\right)^j\frac{\Gamma\left(j+\frac{1}{2}\right)-\Gamma\left(j+\frac{1}{2},q^2D_{\parallel}t\right)}{\left(q^2D_{\parallel}t\right)^{j+1/2}}. \tag{54}$$

Inserting this result into Eq. (46),

$$\begin{aligned}\left\langle S\right\rangle=\frac{E\left(\mathbf{q}=0,t\right)}{2}\Bigg[&\sum_{k=0}^{\infty}c_{k0}\left(aq\right)^{2k}\sum_{j=0}^{k}\binom{k}{j}\left(-1\right)^j\frac{\Gamma\left(j+\frac{1}{2}\right)-\Gamma\left(j+\frac{1}{2},q^2D_{\parallel}t\right)}{\left(q^2D_{\parallel}t\right)^{j+1/2}}\\&+\sum_{p=1}^{\infty}e^{-p^2\frac{D_{\parallel}t}{a^2}}\sum_{k=0}^{\infty}c_{kp}\left(aq\right)^{2\left(p+k\right)}\sum_{j=0}^{p+k}\binom{p+k}{j}\left(-1\right)^j\frac{\Gamma\left(j+\frac{1}{2}\right)-\Gamma\left(j+\frac{1}{2},q^2D_{\parallel}t\right)}{\left(q^2D_{\parallel}t\right)^{j+1/2}}\Bigg],\end{aligned} \tag{55}$$

Substituting Eq. (49) into Eq. (55) and considering Lori's q-space correction, we obtain the final spherical mean signal model in Eq. (17).

Note that for $q=0$ the previous equation exhibits a singularity due to the division by q. To address this issue, the following asymptotic limit for $q\to 0$ is employed to avoid numerical issues: $\frac{\Gamma\left(j+\frac{1}{2}\right)-\Gamma\left(j+\frac{1}{2},q^2D_{\|}t\right)}{\left(q^2D_{\|}t\right)^{j+1/2}}=\frac{\Gamma\left(j+\frac{1}{2}\right)}{\Gamma\left(j+\frac{3}{2}\right)}=\frac{1}{j+\frac{1}{2}}$.

Appendix E: Derivation of the distribution of cylinder radius

To solve Eq. (25), let us focus on the integral

$$I=\int_{a\cdot g}^{a/g}a_i^{\mu-2}e^{-\kappa a_i}da_i=\frac{1}{\kappa^{\mu-1}}\int_{\kappa\cdot a\cdot g}^{\kappa\cdot a/g}v^{\mu-2}e^{-v}dv\,, \tag{56}$$

where we used the substitution $v=\kappa a_i$, thus, $da_i=dv/\kappa$.

We recognize this integral can be written in terms of the upper incomplete Gamma function,

$$\Gamma(s,x)=\int_x^{\infty}t^{s-1}e^{-t}dt\,. \tag{57}$$

However, since our limits of integration are from $\kappa\cdot a\cdot g$ to $\kappa\cdot a/g$, we need to use the general form:

$$\int_a^b t^{s-1}e^{-t}dt=\Gamma(s,a)-\Gamma(s,b)\,. \tag{58}$$

In our case, we get

$$I=\frac{1}{\kappa^{\mu-1}}\left[\Gamma(\mu-1,a\cdot g\cdot\kappa)-\Gamma\left(\mu-1,\frac{a\cdot\kappa}{g}\right)\right]. \tag{59}$$

On the other hand, the normalization constant η is estimated from $\int_0^{\infty}P(a)da=1$, thus

$$\eta\frac{\kappa}{\Gamma(\mu)}\frac{g}{(1-g)}\left[\int_0^{\infty}\Gamma(\mu-1,a\cdot g\cdot\kappa)da-\int_0^{\infty}\Gamma\left(\mu-1,\frac{a\cdot\kappa}{g}\right)da\right]=1, \tag{60}$$

where we inserted the result in Eq. (59) in Eq. (25).

Making the substitution $x = a \cdot g \cdot \kappa$ and thus $dx = g \cdot \kappa da$ in the first integral and $x = a \cdot \kappa / g$, $dx = (\kappa / g) da$ in the second one, we obtain

$$\eta \frac{\kappa}{\Gamma(\mu)} \frac{g}{(1-g)} \left(\frac{1}{g \cdot \kappa} - \frac{g}{\kappa} \right) \int_0^\infty \Gamma(\mu - 1, x) \, dx = 1. \qquad (61)$$

The integral in the last equation is equal to $\Gamma(\mu)$ [119], and thus,

$$\eta = \frac{(1-g)}{(1-g^2)}. \qquad (62)$$

Substituting Eqs. (62) and (59) into Eq. (25), we obtain the final solution.

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