## APPENDIX

### A. BNC-SMA Adapter Phase Shift

To calibrate a VNA with SMA cables but a BNC calibration kit, appropriate BNC-SMA adapters must be attached to the cables coming off port 1 and port 2 of the VNA. After calibrating the VNA with the BNC calibration kit, the BNC-SMA adapters are removed from the SMA cables, shortening the calibrated line length and adding a phase shift. The adapters are assumed to be ideal transmission line lengths, modeled by Eq. 1, where a non-zero length causes a phase shift ($\phi$). Since the ideal phase for a calibrated line is zero, the unwanted phase shift is compensated by subtracting it from all subsequent 2-port measurements. This effectively shifts the calibration plane back to the SMA connections. The phase shift is calculated by taking a 2-port measurement without the BNC-SMA adapters attached and leaving the cables unconnected (open). The phaseshift for port 1 ($\phi_1$) and port 2 ($\phi_2$) is half the measured phase of $S_{11}$ and $S_{22}$, respectively.

$$S_{Adpt} = \begin{bmatrix} 0 & e^{j\phi} \\ e^{j\phi} & 0 \end{bmatrix} \tag{1}$$

The S-parameters are converted to T-parameters, using Eq. 11. The adapters, on either end, are cascaded to determine the measured T-parameter ($T'$) that the VNA would see, Eq. 2.

$$\begin{bmatrix} T'_{11} & T'_{12} \\ T'_{21} & T'_{22} \end{bmatrix} = \begin{bmatrix} e^{j\phi_1} & 0 \\ 0 & e^{j\phi_1} \end{bmatrix} \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} e^{j\phi_2} & 0 \\ 0 & e^{j\phi_2} \end{bmatrix} \tag{2}$$

From this equation, the true T-parameters ($T$) are calculated by inverting the adapter matrices and moving them to the left side. The the true T-parameters are converted back to the true S-parameters using Equation 12. The resulting phases shifts, Eq. 3, can be applied to future S-parameter measurements ($S'$) to determine the true S-parameters. The process should be repeated for all subsequent VNA calibrations.

$$S_{11} = e^{-2j\phi_1} S'_{11} \tag{3a}$$

$$S_{12} = e^{-j(\phi_1+\phi_2)} S'_{12} \tag{3b}$$

$$S_{21} = e^{-j(\phi_1+\phi_2)} S'_{21} \tag{3c}$$

$$S_{22} = e^{-2j\phi_2} S'_{22} \tag{3d}$$

### B. Measuring a Cable Propagation Constant

If the stem attenuation coefficient or phase constant are unknown, then their values can be measured from a length ($l$) of the same type of cable. The 2-port S-parameters of the cable are measured with a calibrated VNA. The S-parameters of the transmission line are assumed to be defined by Eq. 4, so that $S_{21} = e^{-\gamma l}$.

$$S_{TL} = \begin{bmatrix} 0 & e^{-\gamma l} \\ e^{-\gamma l} & 0 \end{bmatrix} \tag{4}$$

The propagation constant ($\gamma$) is modeled using Eq. 5, with the circuit elements (resistance (R), inductance (L), capacitance (C), and shunt conductance (G)) per unit length.[1]

$$\gamma = \sqrt{(R + i\omega L)(G + i\omega C)} \tag{5}$$

Assuming that the cable is a "low-loss line" ($R \ll \omega L$ and $G \ll \omega C$), then the propagation constant is reduced to Eq. 6, using a Taylor Expansion.

$$\gamma = j\omega\sqrt{LC}\left[1 - \frac{j}{2}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)\right] \tag{6}$$

Assuming that the center conductor is well insulated by the cable dielectric, the shunt conductance is ignored ($R \gg G$). In addition, resistance is assumed to be dominated by the finite conductivity of the metal in the cable's conductors, making the resistance ($R \propto \sqrt{\omega}$). The phase constant ($\beta$) and attenuation coefficient ($\alpha$) are then defined by Eq. 7 and Eq. 8, where $R_o$, $L$, and $C$ are constants.

$$\alpha = \frac{R_o l \sqrt{\omega}}{2Z_o} \tag{7}$$

$$\beta = \omega\sqrt{LC} \tag{8}$$

The attenuation coefficient is fitted to the natural log of the measured magnitude, Eq. 9, and the phase constant is fitted to the measured phase angle, Eq. 10. For proper linear fitting of $\beta$ over a long cable, the phase should be "unwrapped," where the phase accumulates to higher values rather than being bounded by $-\pi$ and $\pi$, as demonstrated in Figure 1.

$$ln(|S_{21}|) = -l\alpha \tag{9}$$

$$atan\left(\frac{Im(S_{21})}{Re(S_{21})}\right) = -l\beta \tag{10}$$

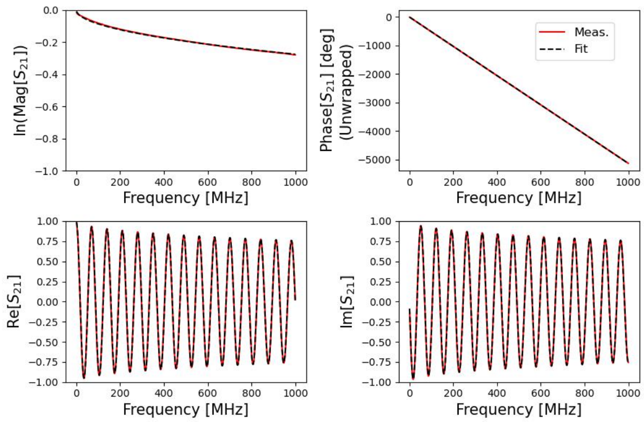


FIG. 1. The measured and curve-fitted $S_{21}$ of a 6 ft long RG-316 cable. The top two plots are Eq. 9 and Eq. 10. The bottom two plots are the measured S-parameters and Eq. 4 using the curve-fitted constants.

### C. Parameter Conversions

Eq. 11 converts the S-parameters to T-parameters.[2,3]

$$T_{11} = -\frac{S_{11}S_{22} - S_{12}S_{21}}{S_{21}} \tag{11a}$$

$$T_{12} = \frac{S_{11}}{S_{21}} \tag{11b}$$

$$T_{21} = -\frac{S_{22}}{S_{21}} \tag{11c}$$

$$T_{22} = \frac{1}{S_{21}} \tag{11d}$$

Eq. 12 converts T-parameters back to S-parameters.

$$S_{11} = \frac{T_{12}}{T_{22}} \tag{12a}$$

$$S_{12} = \frac{T_{11}T_{22} - T_{12}T_{21}}{T_{22}} \tag{12b}$$

$$S_{21} = \frac{T_{11}T_{22} - T_{12}T_{21}}{T_{22}} \tag{12c}$$

$$S_{22} = -\frac{T_{21}}{T_{22}} \tag{12d}$$

Eq. 13 calculates the self-impedances at each antenna ($Z_{11}$ and $Z_{22}$) and the mutual-impedance ($Z_{21}$ and $Z_{12}$) between the antennas, from the S-parameters. The characteristic impedance ($Z_o$) is the reference impedance, characteristic of the transmission line or VNA. In this work, the characteristic impedance is 50 $\Omega$, except for the differential mode along the stems and dipoles. Then the characteristic impedance is 100 $\Omega$.[3,4]

$$Z_{11} = Z_o\frac{(1+S_{11})(1-S_{22}) + S_{12}S_{21}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}} \tag{13a}$$

$$Z_{12} = Z_o\frac{2S_{12}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}} \tag{13b}$$

$$Z_{21} = Z_o\frac{2S_{21}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}} \tag{13c}$$

$$Z_{22} = Z_o\frac{(1-S_{11})(1+S_{22}) + S_{12}S_{21}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}} \tag{13d}$$

Eq. 14 converts the Z-parameters back to the S-parameters.

$$S_{11} = \frac{(Z_{11}-Z_o)(Z_{22}+Z_o) - Z_{12}Z_{21}}{(Z_{11}+Z_o)(Z_{22}+Z_o) - Z_{12}Z_{21}} \tag{14a}$$

$$S_{12} = \frac{2Z_oZ_{12}}{(Z_{11}+Z_o)(Z_{22}+Z_o) - Z_{12}Z_{21}} \tag{14b}$$

$$S_{21} = \frac{2Z_oZ_{21}}{(Z_{11}+Z_o)(Z_{22}+Z_o) - Z_{12}Z_{21}} \tag{14c}$$

$$Z_{22} = \frac{(Z_{11}+Z_o)(Z_{22}-Z_o) - Z_{12}Z_{21}}{(Z_{11}+Z_o)(Z_{22}+Z_o) - Z_{12}Z_{21}} \tag{14d}$$

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