Supplementary Material

1 SUPPLEMENTARY DATA

1.1 Number of networks

Given V unique species in an assembly, how many interaction networks can they form? We assume that interactions are causal, therefore they are both directed, and there are thus two links possible between species a and b, i.e., $a \rightarrow b$ and $a \leftarrow b$, with the direction of an arrow indicating the causal direction of the interaction. Furthermore, self-loops are permitted, e.g. a's self-regulation, or true cannibalism.

The maximum number of species pairs, designated $|E|_{\texttt{max}}$ (E edges between V nodes on a graph), is

$$|E|_{\max} = \frac{|V| \times (|V| - 1)}{2} \tag{S1}$$

Check this for $|V| = 5$, where $|E|_{\max} = 10$.

Permitting edges to be bi-directional and self-looped as explained above, the maximum number of interactions in the network is $2|E|_{\max} + |V|$, because each undirected link can become a pair of directed interactions pointing in either direction between species a and b, and self-loops are permitted. The complexity of networks possible given $|V|$ ranges from a minimum number of interactions of zero in an acausal network, e.g. because one could hypothetically have a community consisting solely of non-interacting autotrophs, to simple chains, to a fully connected network in which each species interacts causally and directionally with every other. Futhermore, for every possible web of interspecific interactions, one can associate arrangements of intraspecific loops ranging in number from zero to $|V|$. Taken together, the number of network topologies given $|V|$ species is

$$|W| = 2 \left[\sum_{e=0}^{|E|_{\max}+|V|} \binom{|E|_{\max} + |V|}{e} \right] \left[\sum_{v=1}^{|V|} \binom{|V|}{v} \right] \tag{S2}$$

where e is the number of edges or interactions, and v is the number of vertices or species. The number of possible networks therefore grows at least twice as fast as undirected graphs of the same size, and generally more than that if we relax the constraints of reciprocal connections and self-loops.