frontiers

Supplementary Material

1 PLANE WAVE ANALYSIS

The equilibrium equations of fluid and porous media are presented as below:

$$\rho\ddot{\mathbf{u}} + \rho_{\mathrm{f}}\ddot{\mathbf{w}} = (H - \mu + \alpha^2 M)\nabla(\nabla\cdot\mathbf{u}) + G\nabla^2\mathbf{u} + \alpha M\nabla(\nabla\cdot\mathbf{w}) \tag{S1}$$

$$\rho_{\mathrm{f}}\ddot{\mathbf{u}} + m\ddot{\mathbf{w}} + bF(t) * \dot{\mathbf{w}} = \alpha M\nabla(\nabla\cdot\mathbf{u}) + M\nabla(\nabla\cdot\mathbf{w}) \tag{S2}$$

Thus, in this section, only the plane wave analysis in the poroelastic medium is described in detail. According to the Helmholtz theorem, the displacement of solid phase and fluid phase could be defined as the below equations:

$$\mathbf{u} = \mathbf{u}_{\mathrm{p}} + \mathbf{u}_{\mathrm{s}} = \nabla\Omega_{\mathrm{s}} + \nabla\times\boldsymbol{\psi}_{\mathrm{s}}$$

$$\mathbf{w} = \mathbf{w}_{\mathrm{p}} + \mathbf{w}_{\mathrm{s}} = \nabla\Omega_{\mathrm{f}} + \nabla\times\boldsymbol{\psi}_{\mathrm{f}}$$

where Ω is a scalar field called "scalar potential", and ψ is a vector field, called a vector potential. Noted that $\mathbf{u}_{\mathrm{p}}$, $\mathbf{w}_{\mathrm{p}}$ are the gradient of the scalar field, and $\mathbf{u}_{\mathrm{s}}$, $\mathbf{w}_{\mathrm{s}}$ are the curl of the vector field. Thus the below expressions could be obtained:

$$\nabla\times\mathbf{u}_{\mathrm{p}} = \mathbf{0} \;\text{ and }\; \nabla\times\mathbf{w}_{\mathrm{p}} = \mathbf{0}$$

$$\nabla\cdot\mathbf{u}_{\mathrm{s}} = 0 \;\text{ and }\; \nabla\cdot\mathbf{w}_{\mathrm{s}} = 0$$

2 COMPRESSIONAL WAVES

Firstly, applying the $\mathbf{u}_{\mathrm{p}}$ and $\mathbf{w}_{\mathrm{p}}$ into the Eq. (S1) and Eq. (S2), we obtain

$$\rho\ddot{\mathbf{u}}_{\mathrm{p}} + \rho_{\mathrm{f}}\ddot{\mathbf{w}}_{\mathrm{p}} = (H + G)\nabla^2\mathbf{u_p} + \alpha M\nabla^2\mathbf{w_p} \tag{S3}$$

$$\rho_{\mathrm{f}}\ddot{\mathbf{u}}_{\mathrm{p}} + m\ddot{\mathbf{w}}_{\mathrm{p}} + bF(t) * \dot{\mathbf{w}}_{\mathrm{p}} = \alpha M\nabla^2\mathbf{u_p} + M\nabla^2\mathbf{w_p} \tag{S4}$$

In order to simplify, the propagation of plane waves in single x direction will be investigated and plane waves are assumed as below:

$$\mathbf{u}_{\mathrm{p}} = \mathbf{u}_{\mathrm{p0}}\exp[i\omega(t - v_c x)]$$

$$\mathbf{w}_{\mathrm{p}} = \mathbf{w}_{\mathrm{p0}}\exp[i\omega(t - v_c x)]$$

where ω is the angular frequency; and $\hat{k}$ is the wavenumber, possibly complex number, $v_c = \hat{k}/\omega$ is the complex velocity.

We could get

$$\left(H+G-v_c^2\rho\right)\mathbf{u}_{\mathrm{p}0}+\left(\alpha M-v_c^2\rho_{\mathrm{f}}\right)\mathbf{w}_{\mathrm{p}0}=\mathbf{0} \tag{S5}$$

$$\left(\alpha M-v_c^2\rho_{\mathrm{f}}\right)\mathbf{u}_{\mathrm{p}0}+\left(M-v_c^2 m+\frac{i}{\omega}b\tilde{F}(\omega)\right)\mathbf{w}_{\mathrm{p}0}=\mathbf{0} \tag{S6}$$

The dispersion relation is obtained by taking the determinant of the system.

$$\begin{aligned}&-\left(\rho_{\mathrm{f}}^2-m\rho+\frac{i}{\omega}b\tilde{F}(\omega)\rho\right)v_c^4\\&+\left[\frac{i}{\omega}(mi\omega+b\tilde{F}(\omega))(H+G)+M(2\alpha\rho_{\mathrm{f}}-\rho)\right]v_c^2+ME_m=0\end{aligned} \tag{S7}$$

where, $E_m=K+\frac{4}{3}G$ is P wave modulus of solid matrix.

Multiplying this equation by ω and taking the limit $\omega\to 0$, we could get

$$i(mi\omega+b\tilde{F}(\omega))v_c^2(\rho v_c^2-(H+G))=0 \tag{S8}$$

get the results,

$$\begin{aligned}&{v^0}_{\mathrm{pI}}=\sqrt{(H+G)/\rho}\\&{v^0}_{\mathrm{pII}}=0\end{aligned}$$

when $\omega\to\infty$, is equivalent to considering $\mu_{\mathrm{f}}=0$,

$$\left(m\rho-\rho_{\mathrm{f}}^2\right)v_c^4-\left(mH-M(2\alpha\rho_{\mathrm{f}}-\rho)\right)v_c^2+ME_m=0 \tag{S9}$$

It can be verified that the equation has two real number solutions greater than zero. This indicates that the attenuation of both P waves is close to zero.

3 SHEAR WAVE

Substituting the $\mathbf{u}_{\mathrm{s}}$ and $\mathbf{w}_{\mathrm{s}}$ into the Eq. (S1) and Eq. (S2), we obtain

$$\rho\ddot{\mathbf{u}}_{\mathrm{s}}+\rho_{\mathrm{f}}\ddot{\mathbf{w}}_{\mathrm{s}}=G\nabla^2\mathbf{u_s} \tag{S10}$$

$$\rho_{\mathrm{f}}\ddot{\mathbf{u}}_{\mathrm{s}}+m\ddot{\mathbf{w}}_{\mathrm{s}}+bF(t)*\dot{\mathbf{w}}_{\mathrm{s}}=0 \tag{S11}$$

As the investigation of the compressional wave, the shear wave traveling in x direction and polarized in the z direction is assumed as below:

$$\mathbf{u}_{\mathrm{s}} = \mathbf{u}_{\mathrm{s}0} \exp[i\omega(t - v_c x)]$$
$$\mathbf{w}_{\mathrm{s}} = \mathbf{w}_{\mathrm{s}0} \exp[i\omega(t - v_c x)]$$

We could get

$$\left(G - v_c^2 \rho\right) \mathbf{u}_{\mathrm{s}0} - \rho_{\mathrm{f}} v_c^2 \mathbf{w}_{\mathrm{s}0} = 0 \tag{S12}$$

$$-(\rho_{\mathrm{f}} + m)\mathbf{u}_{\mathrm{s}0} + \frac{i}{\omega} b\tilde{F}(\omega)\mathbf{w}_{\mathrm{s}0} = 0 \tag{S13}$$

The dispersion relation is obtained by taking the determinant of the system.

$$v_c = \sqrt{\frac{G}{\rho - i\omega\rho_{\mathrm{f}}^2 \tilde{Y}^{-1}}} \tag{S14}$$

when $\omega \to 0$,

$$v_c^0 = \sqrt{\frac{G}{\rho}} \tag{S15}$$

when $\omega \to \infty$,

$$v_c^\infty = \sqrt{\frac{G}{\rho - \rho_{\mathrm{f}} \phi \tau^{-1}}} \tag{S16}$$

We could find $v_c^\infty > v_c^0$