Supplementary Information: Single Qubit Multi-Party Transmission Using Universal Symmetric Quantum Cloning

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A Density Matrix and Bloch Sphere Conversion Formulas

In order to convert from x, y, z coordinates (of the head of the Bloch vector representing the quantum state) into a density matrix, the following matrix form can be used (i denotes the imaginary component):

$$\begin{bmatrix} 0.5 + 0.5z & 0.5x - 0.5yi \\ 0.5x + 0.5yi & 0.5 - 0.5z \end{bmatrix}$$

In order to convert from the following density matrix into Bloch sphere coordinates,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The x coordinate is $\mathrm{Re}(c + b)$, the y coordinate is $\mathrm{Im}\{c - b\}$ and the z coordinate is $\mathrm{Re}(d - a)$.

B Distribution of Error Rates for the Quantum Cloning Transmission

Most of the result figures in the main text all present mean error rates, however an important aspect of the scaling of this transmission protocol is the full distribution of error rates. Figure 1 shows what this distribution is for $M = 10$ (using the clone emulation method), as the number of shots used in the quantum state tomography increases.

C Example Quantum Cloning Circuit Diagram

Figure 2 shows an example quantum cloning circuit diagram for generating 3 approximate copies of a single qubit.

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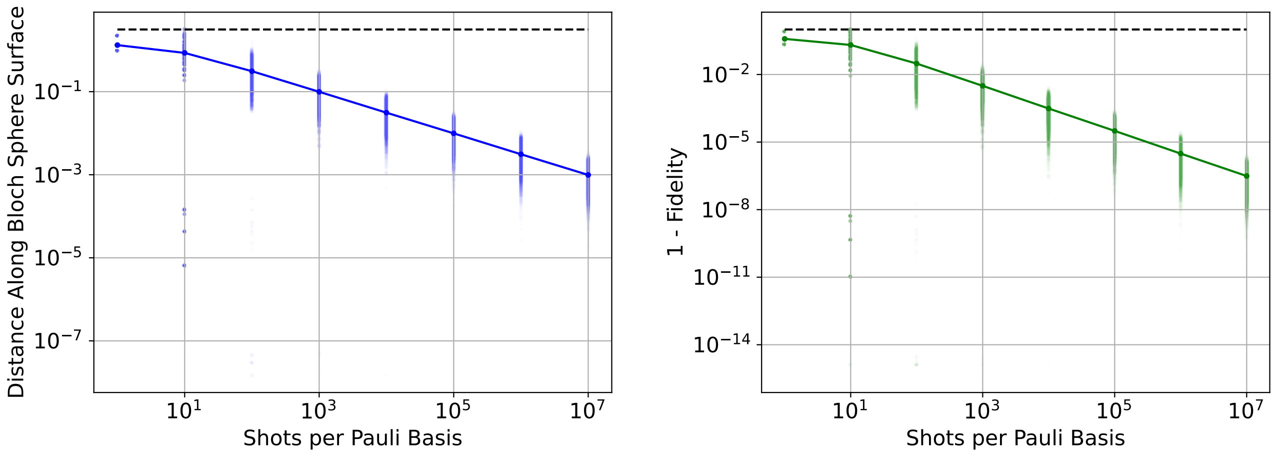

Figure 1: Full distribution of error rates (y-axis) for an $M = 10$ quantum cloning distribution as the number of shots per Pauli basis is increased (x-axis). For each number of shots on the x-axis, using the clone emulation method $M \cdot 1000$ separate simulations are executed. Therefore, for each point that is plotted on the x-axis, a total of $M \cdot 1000$ error datapoints are gathered, thereby plotting the full distribution of error rates that result from the quantum cloning emulation and subsequent geometric extrapolation technique. The solid line connects the average error rate of the distributions at each shot count per Pauli basis (x-axis). The maximum possible error rate is shown by the horizontal dashed black line. Log-log scale axis.

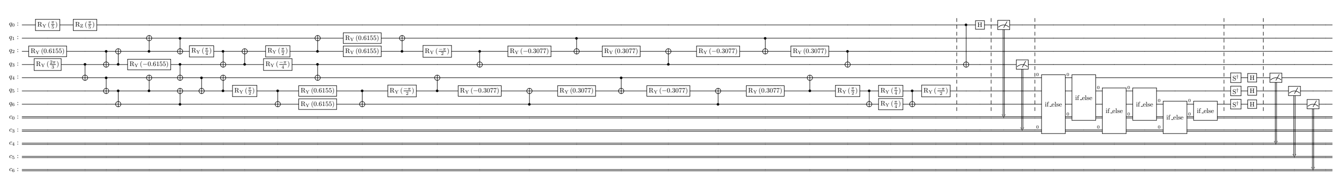

Figure 2: Circuit diagram from ref. [1]. An example $1 \rightarrow 3$ quantum cloning circuit. This is specifically a quantum telecloning circuit thus requiring classical conditional and feed-forward operations, which implements ideal universal symmetric quantum cloning. Qubits 4, 5, 6 are the approximate copies of qubit 0. Qubit 3 is the port qubit, a required part of the quantum telecloning operation [1–3]. Qubits 1 and 2 are ancilla qubits. The state that is prepared on qubit line 0 can be arbitrary - in this case it is two single qubit rotation gates to serve as a simple case. This particular circuit layout is targeting a linear line of qubits – other qubit topologies could be used to implement the same unitary. The circuit barriers (vertical dashed lines) denote the logical phases of the quantum telecloning protocol, beginning with the quantum telecloning state preparation, followed by a Bell state measurement mid-circuit, followed by conditional if-else statements, followed by a rotation into the Pauli-Y basis (this basis change would rotate through different Pauli bases in order to perform full quantum state tomography), and ending with the measurement of the 3 clone qubits.

References

[1] Elijah Pelofske et al. "Probing Quantum Telecloning on Superconducting Quantum Processors". In: *IEEE Transactions on Quantum Engineering* 5 (2024), 1–19. ISSN: 2689-1808. DOI: `10.1109/tqe.2024.3391654`. URL: `http://dx.doi.org/10.1109/TQE.2024.3391654`.

[2] Elijah Pelofske et al. "Quantum Telecloning on NISQ Computers". In: *2022 IEEE International Conference on Quantum Computing and Engineering (QCE)*. IEEE, 2022. DOI: `10.1109/qce53715.2022.00083`. URL: `https://doi.org/10.1109%2Fqce53715.2022.00083`.

[3] Elijah Pelofske, Andreas Bärtschi, and Stephan Eidenbenz. "Optimized Telecloning Circuits: Theory and Practice of Nine NISQ Clones". In: *2022 IEEE International Conference on Rebooting Computing (ICRC)*. IEEE, 2022. DOI: `10.1109/icrc57508.2022.00009`. URL: `https://doi.org/10.1109%2Ficrc57508.2022.00009`.