

Supplementary Material

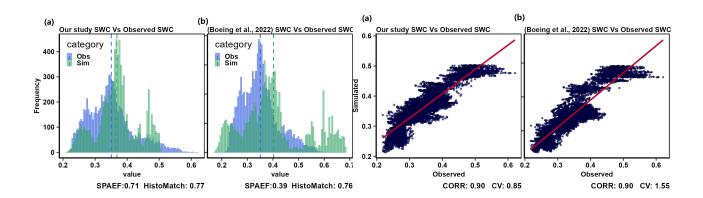


Figure S1. Comparison of observed and simulated soil water content using performance metrics: SPAEF (Spatial Efficiency), HistoMatch (Histogram Matching), CORR (Correlation), and CV (Coefficient of Variation). Panel (a) shows mHM results with Boeing et al. (2022) parameters vs. observed values. Panel (b) presents simulations using the mean of the top 10 calibrated parameter sets vs. observed CRNS data.

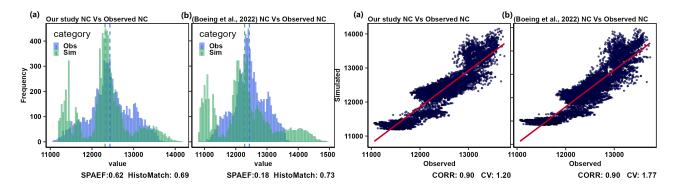


Figure S2. Comparison of observed and simulated neutron counts data using performance metrics: SPAEF (Spatial Efficiency), HistoMatch (Histogram Matching), CORR (Correlation), and CV (Coefficient of Variation). Panel (a) shows results with (Boeing et al., 2022) parameters in mHM vs. observed neutron counts. Panel (b) shows calibrated top 10 parameter sets with mean values simulating the neutron counts from mHM vs. observed CRNS data. The frequency distributions of observed and simulated values are displayed for each case.

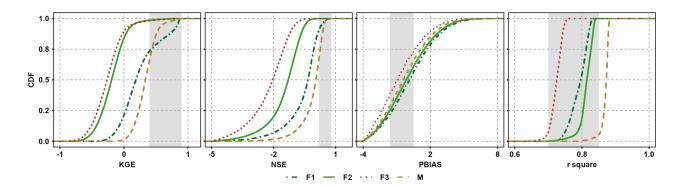


Figure S3. CDF of performance metrics (KGE, NSE, r-square, PBIAS) for neutron counts in the mHM simulation ensemble (100,000 realizations) across four land cover types: meadows (M), dense forests (F1), half-open forests (F2), and shunting areas (F3). The grey-shaded area indicates optimal performance thresholds.

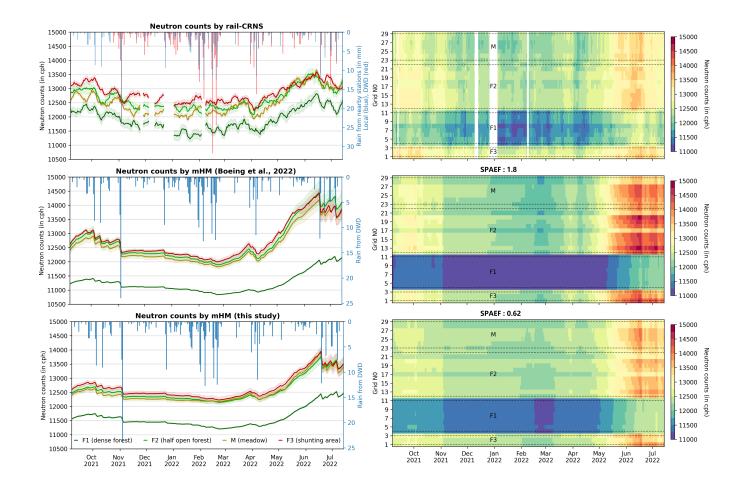


Figure S4. Comparison of neutron counts estimates from different methods across various land cover types. (a) neutron counts measurement using rail-based cosmic-ray neutron sensing (rail-CRNS). (b) Neutron counts were simulated using mHM from Boeing et al. (2022). (c) Neutron counts were simulated using mHM in this study. The left panels show a time series of neutron counts (cph) for different land cover types (F1 - dense forest, F2 - half-open forest, F3 - shunting area, M - meadow), along with precipitation data. The right panels display the spatial distribution of neutron counts over time from Oct 2021 to July 2022 for different grid locations.

Measured by rail-CRNS (upscaled): Monthly Mean Gravimetric SWC Maps

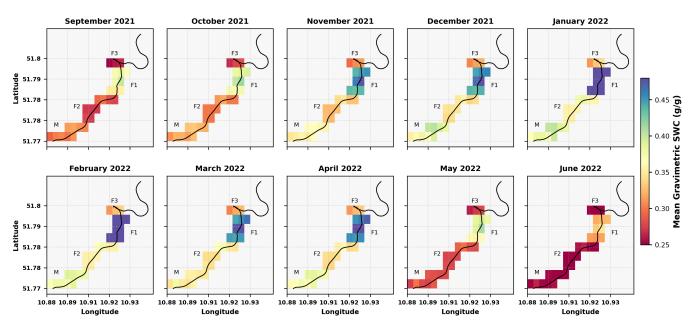


Figure S5. Monthly mean gravimetric SWC (g/g) from rail-CRNS (upscaled) using the inverse distance weighting (IDW) approach, overlaid with the 9 km rail track (black line), from September 2021 to June 2022.

mHM (this study): Monthly Mean Gravimetric SWC Maps

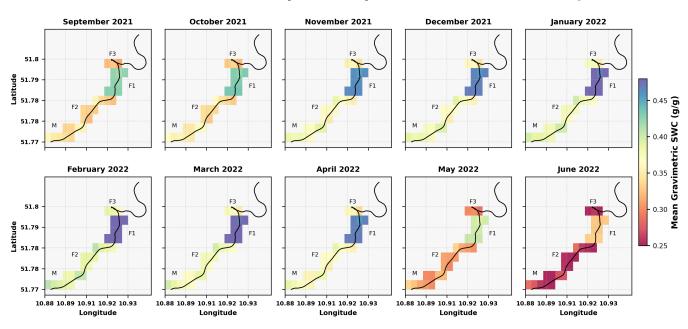


Figure S6. Monthly mean gravimetric SWC (g/g) simulated by mHM (calibrated parameters), overlaid with the 9 km rail track (black line), from September 2021 to June 2022.

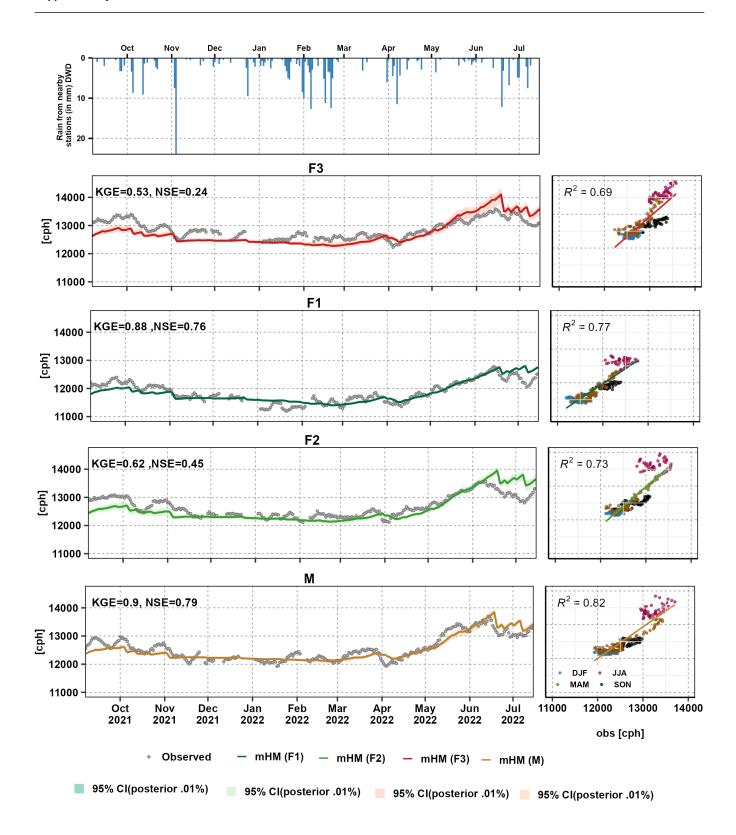


Figure S7. Neutron counts simulated by mHM at four different land cover types (M: meadow, F2: open forest, F1: dense forest, and F3: shunting area) from Sep 2021 to Jul 2022 between Blankenburg-Rübeland. Measured rail-based CRNS data are shown in gray dots.

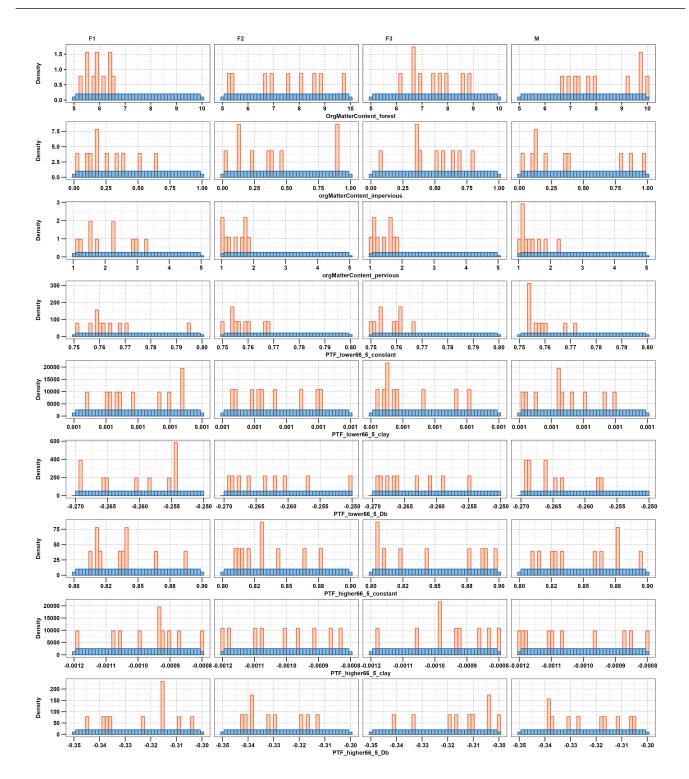


Figure S8. Probability Density Function (PDF) for nine parameters (soil moisture module) of the mHM taken from $N_{\rm Des,W}$ method. The prior PDF of the original sample, consisting of LHS 100 000 data points, is represented by the blue color. The behavioral PDF LHS 10, obtained after applying the objective function, is shown in orange for four sites.

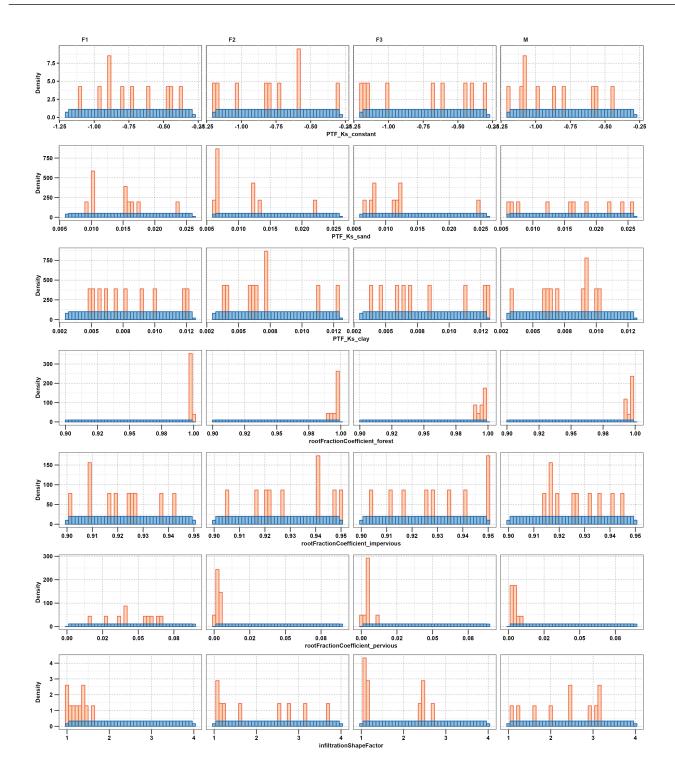


Figure S9. Probability Density Function (PDF) for seven parameters (soil moisture module) of the mHM taken from $N_{\rm Des,W}$ method. The prior PDF of the original sample, consisting of LHS 100 000 data points, is represented by the blue color. The behavioral PDF LHS 10, obtained after applying the objective function, is shown in orange for four sites.

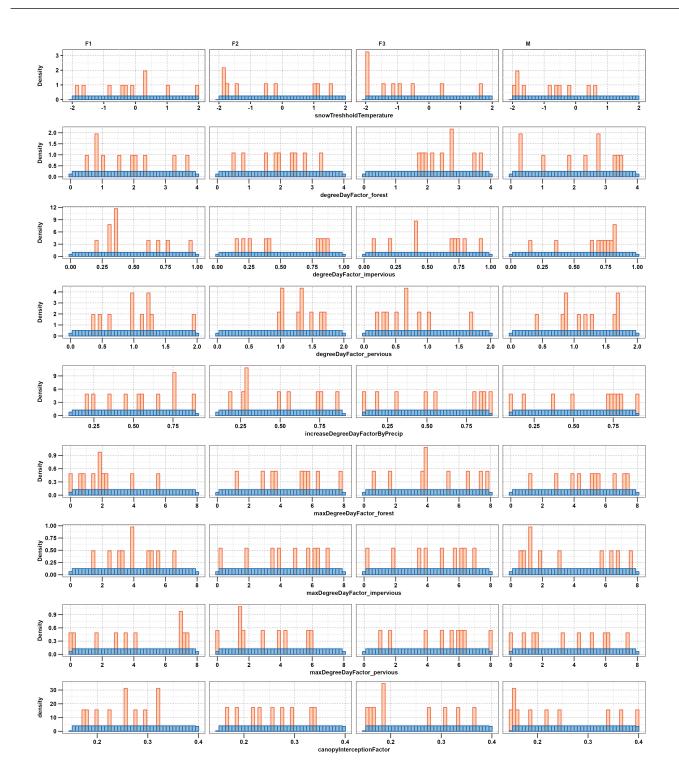


Figure S10. Probability Density Function (PDF) for nine parameters (snow module) of the mHM taken from $N_{\rm Des,W}$ method. The prior PDF of the original sample, consisting of LHS 100 000 data points, is represented by the blue color. The behavioral PDF LHS 10, obtained after applying the objective function, is shown in orange for four sites.

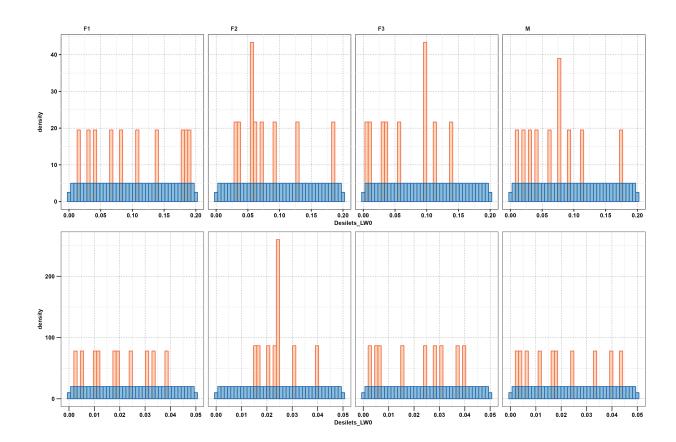


Figure S11. Probability Density Function (PDF) for two parameters (neutron module) of the mHM taken from $N_{\rm Des,W}$ method. The prior PDF of the original sample, consisting of LHS 100 000 data points, is represented by the blue color. The behavioral PDF LHS 10, obtained after applying the objective function, is shown in orange for four sites.

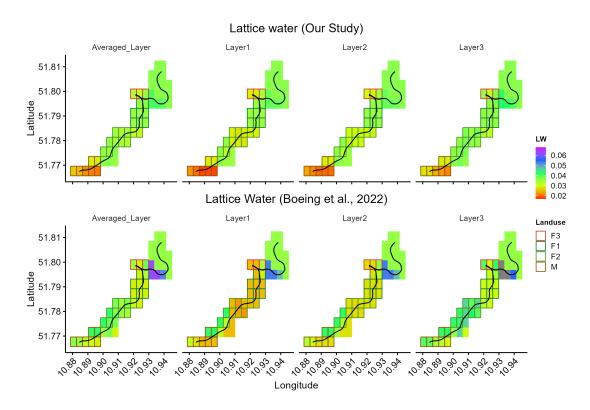


Figure S12. Spatial variability of lattice water across three soil layers (0–5 cm, 5–25 cm, and 25–60 cm) and the average profile (0–60 cm).

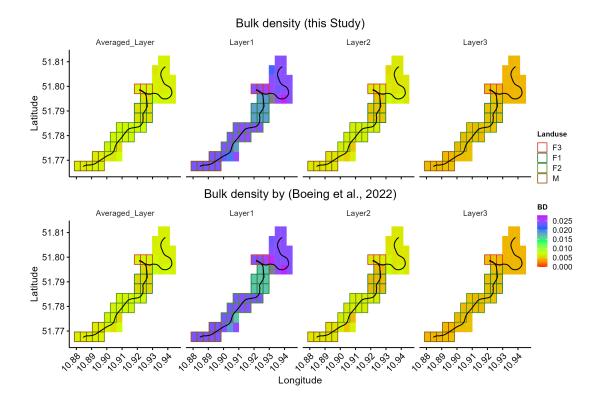


Figure S13. Spatial variability of bulk density across three soil layers (0–5 cm, 5–25 cm, and 25–60 cm) and the average profile (0–60 cm).

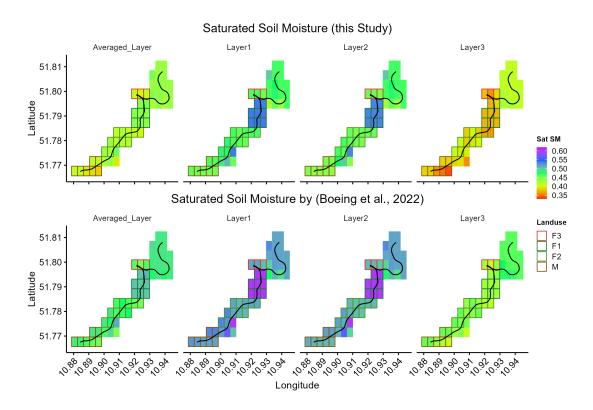


Figure S14. Spatial variability of the effective porosity (θ_s) of the three soil layers (0–5 cm, 5–25 cm, and 25–60 cm) and an average depth of (0–60 cm) at \sim 200 m resolution.

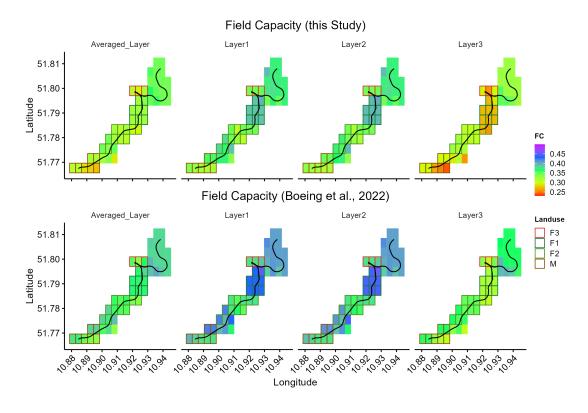


Figure S15. Spatial variability of field capacity (FC) across three soil layers (0-5 cm, 5-25 cm, and 25-60 cm) and the average profile (0-60 cm).

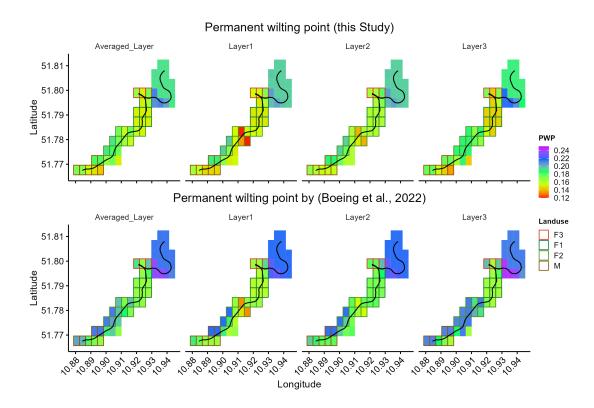


Figure S16. Spatial variability of permanent wilting point (pwp) across three soil layers (0–5 cm, 5–25 cm, and 25–60 cm) and the average profile (0–60 cm).

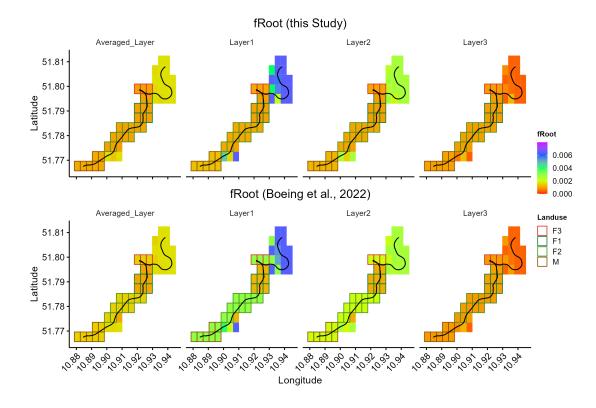


Figure S17. Spatial variability of fRoot across three soil layers (0–5 cm, 5–25 cm, and 25–60 cm) and the average profile (0–60 cm).

Table S1. Description of the mHM model parameter ranges used in this study.

Parameter	Description	Prior Range						
	Interception							
canintfact	Canopy interception factor	0.15-0.4						
Snow								
tsnow	Threshold temperature for snow/rain (°C)	-2.0–2.0						
degreeDayFactor _forest	Degree-day factor for forest	0.0001-4.0						
degreeDayFactor _impervious	Degree-day factor for impervious surface	0.00-1.0						
degreeDayFactor _pervious	Degree-day factor for pervious surface	0.0-2.0						
increaseDegreeDay FactorByPrecip	Degree-day factor per unit of precipitation (mm day $^{-1}$ $^{\circ}$ C $^{-1}$)	0.1–0.9						
maxDegreeDayFactor _forest	Maximum degree-day factor for forest	0.0-8.0						
maxDegreeDayFactor _impervious	Maximum degree-day factor for impervious surface	0.0-8.0						
maxDegreeDayFactor _pervious	Maximum degree-day factor for pervious surface	0.0–8.0						
	Soil moisture							
orgmatforest	Organic matter content for forest	5.0-10.0						
orgmatimper	Organic matter content for impervious zone	0.0 - 1.0						
orgmatperv	Organic matter content for pervious zone	1.0-5.0						
ptflowconst	PTF saturated water content: constant	0.75 - 0.8						
ptflowclay	PTF saturated water content: coefficient for clay content	0.0008-0.0012						
ptflowdb	PTF saturated water content: coefficient for bulk density	-0.27 – -0.25						
pfthighconst	PTF high saturated water content: constant	0.8-0.9						
ptfhighclay	Coefficient for clay content in high PTF	-0.35-0.30						
pfthighdb	Coefficient for bulk density in high PTF	-0.0012-0.00080						
ptfksconst	Constant in PTF for hydraulic conductivity of soils	-1.200–0.285						
ptfkssand	PTF hydraulic conductivity: coefficient for sand content	0.006-0.026						
pftksclay	PTF hydraulic conductivity: coefficient for clay content	0.003-0.013						
ptfksslp	Coefficient for slope in PTF	60.96 - 60.96						
rotfcoffore	Root fraction for forest areas	0.9–0.999						
rotfcofimperv	Root fraction for impervious areas	0.9-0.095						
rotfcofpervi	Threshold for pervious zone root fraction	0.001-0.090						
infshapef	Infiltration shape factor	1.0 – 4.0						
	Neutron counts	1.0 1.0						
N _{0(Des)}	Neutron intensity over dry soil under same conditions (cph)	600–1500						
Desliets _{lw0}	Lattice water content gg^{-1}	0.0-0.2						
Desliets _{lw1}	Lattice water content gg^{-1}	0.0-0.05						
D CONTOUNINI	Lattice water content gg	0.0 0.05						

Table S2. Description of the best parameter values in mHM for each site and methods: NSE. To better assist the user we keep the model nomenclature name the same as that is there in the model code mHM.

Objective Function	NSE							
Parameters	<i>F1</i>	F2	<i>F3</i>	M				
Interception								
canintfact	0.2238	0.182	0.2093	0.2027				
Snow								
tsnow	-1.3158	-1.546	0.974	0.485				
degreeDayFactor_forest	1.29	3.364	0.485	2.748				
degreeDayFactor_impervious	0.949	0.1114	0.777	0.778				
degreeDayFactor_pervious	1.243	0.0053	1.828	0.626				
increaseDegreeDayFactorByPrecip	0.245	0.5112	0.723	0.566				
maxDegreeDayFactor_forest	0.32	6.686	1.502	7.35				
maxDegreeDayFactor_impervious	4.44	7.3873	7.958	6.036				
maxDegreeDayFactor_pervious	5.12	4.0472	0.836	0.752				
So	Soil moisture							
orgmatforest	9.70	5.636	5.553	6.393				
orgmatimper	0.619	0.630	0.965	0.081				
orgmatperv	3.438	1.7095	1.055	4.585				
ptflowconst	0.77	0.7807	0.775	0.753				
ptflowclay	0.00081	0.0008	0.00095	0.0011				
ptflowdb	-0.26	-0.265	-0.264	-0.255				
pfthighconst	0.857	0.898	0.867	0.850				
ptfhighclay	-0.0009	-0.0008	-0.001	-0.0008				
pfthighdb	-0.327	-0.316	-0.323	-0.32				
ptfksconst	-1.042	-0.575	-0.482	-0.83				
ptfkssand	0.0194	0.0167	0.015	0.011				
pftksclay	0.0128	0.0122	0.0106	0.0092				
ptfksslp	60.96	60.96	60.96	60.96				
rotfcoffore	0.9930	0.9972	0.998	0.9952				
rotfcofimperv	0.9455	0.9059	0.909	0.9254				
rotfcofpervi	0.060	0.0886	0.044	0.0053				
infshapef	1.619	1.232	2.068	1.048				
Neutron counts								
N _{0(Des)}	24310.5	24968.27	25094.81	24097.38				
Desliets _{lw0}	0.144	0.149	0.0997	0.139				
Desliets _{lw1}	0.0178	0.008	0.0212	0.016				

Table S3. Soil parameters for selected stations with measured values and simulated values of mHM.

Variables	Stations				
, ar impley	Grosses Bruch (Grassland)	Kall (Grassland)	Rollesbr1 (Grassland)	Hohes Holz (Forest)	Wildenrath (Forest)
Measurement					
Porosity	0.534	0.625	0.596	0.561	0.565
$BD (g/cm^3)$	1.176	0.959	1.032	1.136	1.126
LW(g/g)	0.048	0.037	0.032	0.038	0.002
Boeing et al. (2022)	1				
Porosity	0.322	0.499	0.477	0.5268	0.407
BD (g/cm ³)	1.428	1.267	1.364	1.1564	1.1322
LW(g/g)	0.0067	0.037	0.0407	0.0281	0.0044
M Parameters					
Porosity	0.376	0.46	0.437	-	-
$BD (g/cm^3)$	1.4297	1.26	1.36	-	-
LW(g/g)	0.038	0.054	0.057	-	-
F1 Parameters					
Porosity	-	-	-	0.47	0.458
$BD (g/cm^3)$	-	_	-	1.2	1.18
LW (g/g)	-	-	-	0.034	0.023