Supplementary Material

Enhancing the Performance and Bitrates in Brain–Computer Interface System with Phase-to-Amplitude Cross-Frequency Coupling: Evidences from traditional c-VEP, fast c-VEP and SSVEP Designs

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1. **Comodulograms from the whole dataset of c-VEP**

**S1 - S3 illustrates the** trial-Averaged **PACiPLV** patterns from the **c-VEP** responses for each target image and for both **attended vs non-attended images for the subjects 2 – 4 (disabled). S4 – S6 demonstrates the** trial-Averaged **PACiPLV** patterns from the **c-VEP** responses for each target image and for both **attended vs non-attended images for the subjects 7 – 9 (able bodied).**

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**S1. – subject 2 (disabled).**

**Demonstrating the level of CFC in c-VEP responses for each flashing image.**

Trial-Averaged **PACiPLV** patterns from the **c-VEP** responses for each target image and for both **attended vs non-attended images.**

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**S2. – subject 3 (disabled).**

**Demonstrating the level of CFC in c-VEP responses for each flashing image.**

Trial-Averaged **PACiPLV** patterns from the **c-VEP** responses for each target image and for both **attended vs non-attended images.**

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**S3. – subject 4 (disabled).**

**Demonstrating the level of CFC in c-VEP responses for each flashing image.**

Trial-Averaged **PACiPLV** patterns from the **c-VEP** responses for each target image and for both **attended vs non-attended images.**



**S4. – subject 7 (able bodied).**

**Demonstrating the level of CFC in c-VEP responses for each flashing image.**

Trial-Averaged **PACiPLV** patterns from the **c-VEP** responses for each target image and for both **attended vs non-attended images.**

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**S5. – subject 8 (able bodied).**

**Demonstrating the level of CFC in c-VEP responses for each flashing image.**

Trial-Averaged **PACiPLV** patterns from the **c-VEP** responses for each target image and for both **attended vs non-attended images.**

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**S6. – subject 9 (able bodied).**

**Demonstrating the level of CFC in c-VEP responses for each flashing image.**

Trial-Averaged **PACiPLV** patterns from the **c-VEP** responses for each target image and for both **attended vs non-attended images.**

1. **A Dissimilarity Measure for Dynamical Trajectories Based on the Wald-Wolfowitz (WW) Test**

The two-sample, non-parametric WW test was adopted in the present work to assess the degree of similarity between two nodal network metric time series based on global efficiency (nNMTSGE) derived from dynamic FCGs. The procedure entailed, first, transforming every pair of NMTSGE time series *x(t), t* = 1.2,…T into dynamic trajectories represented by multidimensional vectors *Xt* = [*x*(*t*), *x*(*t* + 1),…, *x*(*t* + *de*)] and *Yt* = [*y*(*t*), *y*(*t* + 1),…, *y*(*t* + *de*)] (*X* and *Y* correspond to two split-half segments from a single participant or from two participants). These vectors were formed by selecting the appropriate set of *de*, which is the embedding dimension parameter that controls the dimensionality of the vectors and *dt* is the time-delay. By adopting the Ragwitz criterion, we optimized the embedding dimension *de* and the embedding delay *dt* ([Ragwitz and Kantz, 2002](https://www.frontiersin.org/articles/10.3389/fninf.2017.00028/full#B87)), resulting in values ranging from 3 to 6 in both the complete and split-half temporal segments of NMTSGE series. The two point-samples {X*t*}*t*= 1:*m* and {Y*t*}*t*= 1:*n* were then formed and the wdist = w({X*t*},{Y*t*}) was computed.

Next, the minimal spanning tree (MST) graph of the overall sample was constructed (i.e., disregarding the sample identity of each point). In these graph points represent nodes with N − 1 edges (N = n + m) (i.e., paths within each pair of nodes). The second step of the procedure entails computing the R statistic which is the total number of consecutive sequences with identical sample identities (i.e., “runs”). Based on the number of edge pairs of MST sharing a common node and the degrees of the nodes, the mean and variance of R can be calculated ([Laskaris and Ioannides, 2001](https://www.frontiersin.org/articles/10.3389/fninf.2017.00028/full#B68)). This property of R permits computation of the initial form of the normally-distributed, WW Dissimilarity Index (w) as follows:

w=R−E[R]Var[R]√    (5)w=R-E[R]Var[R]    (1)

The measure used in classification schemes in the present work was derived from w using the Heaviside step function H(x) as follows: wdist = |w|.H(−w). The higher the value of wdist, the more dissimilar the two point-sets are considered to be.

**References**

Laskaris, N. A., and Ioannides, A. A. (2001). Exploratory data analysis of evoked response single trials based on minimal spanning tree. *Clin. Neurophysiol.* 112, 698–712. doi: 10.1016/S1388-2457(00)00560-5

Ragwitz, M., and Kantz, H. (2002). Markov models from data by simple nonlinear time series predictors in delay embedding spaces. *Phys. Rev. E* 65:056201. doi: 10.1103/physreve.65.056201