Online Supplementary Materials for

Measuring Complexity in Financial Data

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1 Data description

Source of data: Center for Research in Security Prices (CRSP) database: http://www.crsp.org/; accessed through Wharton Research Data Services (WRDS)).

Description of the data: Daily stock price data from NASDAQ spanning over a period of 47 years, from 1972 to 2018. We denote the length of the entire data in terms of years by $\mathbb{T}$ ($\mathbb{T} = 47$). We have considered 4-year wide moving windows, viz., 1972-75, 1973-76 and so on till 2015-18 (windows are denoted by $k = 1,\ 2,\ ...,\ 44$).

Stock selection: In the analysis, For each window, we have calculated the market capitalization of all stocks at the end of the period and chosen top $N = 300$, with a restriction that the data for chosen stocks cannot have more than 5% missing values within a window (which we fill by zeros). This dataset covers pre-crisis, crisis and post-crisis periods (the crisis period was 2007-09). Due to missing data, the first window (1972-75) contains only 124 stocks.

Description of each window: The k-th window has a size of $N \times T_k$ where $N = 300$ (except the first one, where $N = 124$). T_k varies within 1002-1011 as there roughly 250 trading days per year and each window covers four consecutive calender years.

1.1 Constructing log return series

All the computational analysis have been conducted on the log-return data, obtained from each of the window data. For each window, we denote each price series by $S_i^k(t)$ where i denotes the stock, t denotes the time period within a window and k denotes the window. A four-year long window has a roughly 1000 days (each year has slightly more than 250 trading days) denoted by T_k. Log-return data is defined as

$$G_i^k(t) \ = \ log\ S_i^k(t+1) - log\ S_i^k(t). \tag{1}$$

Next we normalize the log return as follows,

$$g_i^k(t) = \frac{G_i^k(t) - < G_i^k(t) >}{\sigma_i^k} \tag{2}$$

where $< . >$ denotes the sample average and σ_i^k is the sample standard deviation of G_i.

2 Quantification of Linear and Nonlinear Relationships

In this section, the steps to compare the information content in linear and nonlinear relationships are explained. We have created the cross-correlation matrix to capture the linear relationship and mutual-information matrix to capture the non-linear relationship for each data-window.

2.1 Dominant Eigenvector of the Correlation-based Distance Matrix

1. For the k-th window, we have created the cross-correlation matrix of size 300×300 (except for the 1972-75 window, which is of size 124×124) from the normalised log-return matrix.

$$C_{ij}^k =< g_i^k(t) g_j^k(t) > \tag{3}$$

2. Then we calculate the distance matrix using the following transformation proposed by [12],

$$d_{ij}^k = \sqrt{2(1 - C_{ij}^k)}. \tag{4}$$

3. Then we conduct an eigendecomposition of the distance matrix D_{ij}^k where k corresponds to the k^{th} window. Eigendecomposition of a square matrix D of size $N \times N$ can be written as

$$D = V \Lambda V^{-1}. \tag{5}$$

   Here, V is a square $N \times N$ matrix whose i^{th} column is the i-th eigenvector v^i of M and Λ is the diagonal matrix whose diagonal elements are the corresponding eigenvalues, $\Lambda_{ii} = \lambda_i$.

2.2 Dominant eigenvector of the Mutual Information matrix

Let us first define Shannon entropy, joint entropy and mutual information.

Definition 1 *For the probability distribution $p(x)$ of a discrete variable X defined over a domain $[x_1, x_2 \ldots x_N]$, the* **Shannon entropy** *is given by [4],*

$$H(X) = -\sum_i p(x_i) \log_2 p(x_i). \tag{6}$$

Definition 2 *For two discrete variables X and Y with probability distributions $p(x)$ and $p(y)$, the* **joint entropy** *is given by [4],*

$$H(X, Y) = -\sum_i \sum_j p(x_i, y_j) \log_2 p(x_i, y_j) \tag{7}$$

where $p(x_i, y_j)$ denotes joint probability.

Definition 3 *For two variables X and Y having probability distributions $p(x)$ and $p(y)$,* **mutual information** *is defined as [4],*

$$I(X; Y) = \sum_{i,j} p(x_i, y_j) \log_2 \frac{p(x_i, y_j)}{p(y_j) p(x_i)}. \tag{8}$$

Now we are in a position to describe the steps.

1. For the k-th window, we construct mutual information matrix M^k from the log-return matrix, where element M_{ij}^k is defined as
$$M_{ij}^k = I(S_i^k; S_j^k). \tag{9}$$
where, S_i^k and S_j^k are log-returns of i^{th} and j^{th} stocks in k^{th} window and $I()$ is mutual information defined above.

2. Then we conduct an eigendecomposition of the mutual information matrix M_{ij}^k, where k corresponds to the k^{th} window. Here, eigendecomposition of the square matrix M of size $N \times N$ can be written as
$$M = V\Lambda V^{-1}. \tag{10}$$
Here, V is a square $N \times N$ matrix whose i^{th} column is the i-th eigenvector v^i of the mutual information matrix M and Λ is the diagonal matrix whose diagonal elements are the corresponding eigenvalues, $\Lambda_{ii} = \lambda_i$.

2.3 Comparison Between Linear and Nonlinear Relationships

Definition 4 *Linear regression is a linear model that captures the relationship between a response variable y and a set of explanatory variables x:*
$$y_i = \alpha + \beta x_i + \boldsymbol{\varepsilon}_i, \quad \textit{for} \;\; i = 1, ..., N, \tag{11}$$
where α represents the intercept, β represents the slope and $\boldsymbol{\varepsilon}$ represent the error term (also called noise).

We employ a regression framework to check for a relationship between the distance matrix and the mutual information matrix for each time window k. Note that each of them contain N^2 number of elements where $N = 300$. Rather than comparing all the elements, we simply consider the dominant eigenvectors of the respective matrices since they explain a large part of the total variability of the empirical matrices. This choice is also motivated by the fact that the dominant eigenvector can be treated as the '*market mode*' of these matrices [6].

1. We regress the dominant eigenvector of mutual information matrix on the dominant eigenvector of distance matrix. The regression specification is
$$v_i^{mi,k} = \alpha + \beta v_i^{D,k} + \boldsymbol{\varepsilon}_i, \quad \text{for} \;\; i = 1, ..., N, \tag{12}$$
In Fig. 1 of the main text, we have plotted the evolution of R^2 value (extracted from all windows).

2. Note that for calculating mutual information, one needs to discretize the data. In order to ensure the robustness of the results, the analysis has been done by converting each series into an ordinal categorical series with different bin classes denoted by b. We have done the analysis for $b = 8$, 12 and 16. As Fig. 1 in the main text shows, the results are nearly identical.

3. In tables 1, 2 and 3, we describe the summary of the regression results (intercept, slope, standard error of the slope coefficient, R^2 and p-value of the slope).

3 Complexity through systemic risk

We construct *Granger causal network* (*GCN* hereafter) for each window of data (we have excluded the first window as the network size was not comparable with the rest). The network is constructed as follows.

1. The j-th asset's return is said to *Granger cause* i-th asset's return, if β_{ij} in the following regression is significant:
$$r_{it} = \alpha + \beta_{ii} r_{i,t-1} + \beta_{ij} r_{j,t-1} + \epsilon_{it}. \tag{13}$$

2. For each pair $\{i, j\}$ of stocks, if the $j-$th stock *Granger causes* the i-th stock, then there exists an edge from j to i. We represent the edge by 1 and if there is no causation, we represent it by 0.

3. We have evaluated the existence of causal relationship at the standard 5% level of significance. All estimation exercises have been carried out using *lmtest* package in *R*.

4. Thus we get an $N \times N$ matrix which is binary in nature. We call this the *Granger causal network* or *GCN* (denoted by G) which has edges connecting pairs of nodes $\{i, j\}$ described by $\{g_{ij}\}$.

5. Once the *GCN* is created, we find PageRank [7] of *GCN*. The PageRank vector of a network with adjacency matrix G is given by a vector v such that the j-th element satisfies
$$v_j = \frac{1-\alpha}{N} + \alpha \sum_{k \in G(j)} \frac{g_{jk} v_k}{d_k} \tag{14}$$
where α is a tuning parameter (a standard value is 0.15), d_k is the number of outbound links on j and $G(j)$ is the neighborhood of node j.

   This value represents a measure of risk. A high PageRank would indicate higher propensity of lagged movement with respect to other stocks. Thus this measure quantifies higher risk of spillover within the network.

We study the evolution of assets in *GCN*. A high dispersion in the PageRank would imply high inequality in influence across the stocks. In Fig. 2 in the main text, we present the time series of the standard deviation and differential entropy of the PageRank of *GCN* over all time windows. Differential entropy is defined as
$$\mathscr{E} = -\int p(x) \log\left(\frac{p(x)}{m(x)}\right) dx \tag{15}$$
where $m(x)$ represents information invariance and measurement scale (see for example [5]).

Table 1: Results for regression with dominant eigenvectors of the mutual information matrix (number of bins= 8) and the distance matrix.

Year	Intercept	Slope	Standard Error	R square	p-value
1972-75	1.149	-11.828	1.433	0.358	2.117e-13
1973-76	0.02	0.44	1.977	0.0	0.824
1974-77	0.065	-0.365	2.301	0.0	0.874
1975-78	0.286	-4.171	2.591	0.009	0.108
1976-79	0.066	-0.391	2.513	0.0	0.876
1977-80	0.622	-9.913	1.474	0.132	9.040e-11
1978-81	0.796	-12.972	1.787	0.15	3.386e-12
1979-82	1.001	-16.603	2.227	0.157	9.831e-13
1980-83	1.585	-26.848	2.467	0.284	1.916e-23
1981-84	1.543	-25.988	2.277	0.304	2.828e-25
1982-85	1.532	-25.68	1.592	0.466	1.673e-42
1983-86	1.769	-29.799	2.233	0.374	3.665e-32
1984-87	0.959	-15.701	0.633	0.674	1.654e-74
1985-88	0.943	-15.419	0.608	0.683	2.169e-76
1986-99	0.847	-13.762	0.601	0.638	1.234e-67
1987-90	0.621	-9.826	0.489	0.575	2.541e-57
1988-91	0.753	-12.126	0.97	0.344	3.990e-29
1989-92	0.786	-12.689	1.122	0.3	6.406e-25
1990-93	0.797	-12.902	1.141	0.3	6.629e-25
1991-94	0.892	-14.557	1.509	0.238	2.401e-19
1992-95	0.646	-10.287	1.881	0.091	9.606e-08
1993-96	0.77	-12.428	1.817	0.136	4.461e-11
1994-97	0.937	-15.313	1.466	0.268	5.719e-22
1995-98	1.077	-17.729	0.956	0.536	1.297e-51
1996-09	1.08	-17.783	0.888	0.574	4.200e-57
1997-00	0.938	-15.312	0.57	0.707	1.619e-81
1998-01	0.882	-14.369	0.408	0.806	2.841e-108
1999-02	0.798	-12.928	0.409	0.771	2.952e-97
2000-03	0.671	-10.741	0.292	0.819	1.016e-112
2001-04	0.621	-9.904	0.227	0.865	1.239e-131
2002-05	0.638	-10.212	0.243	0.856	2.260e-127
2003-06	0.747	-12.103	0.365	0.787	5.983e-102
2004-07	0.742	-11.971	0.439	0.714	5.408e-83
2005-08	0.343	-5.002	0.216	0.644	9.517e-69
2006-09	0.319	-4.602	0.18	0.686	6.712e-77
2007-10	0.281	-3.939	0.157	0.68	1.193e-75
2008-11	0.244	-3.296	0.142	0.643	1.098e-68
2009-12	0.284	-4.018	0.149	0.709	8.222e-82
2010-13	0.334	-4.895	0.149	0.784	2.858e-101
2011-14	0.334	-4.901	0.196	0.677	4.294e-75
2012-15	0.446	-6.852	0.266	0.69	7.491e-78
2013-16	0.43	-6.569	0.263	0.677	3.334e-75
2014-17	0.41	-6.211	0.304	0.584	9.844e-59
2015-18	0.397	-5.988	0.258	0.645	6.480e-69

Table 2: Results for regression with dominant eigenvectors of the mutual information matrix (number of bins= 12) and the distance matrix.

Year	Intercept	Slope	Standard Error	R square	p-value
1972-75	0.852	-8.509	1.317	0.255	2.225e-09
1973-76	0.372	-5.53	1.281	0.059	2.164e-05
1974-77	0.436	-6.642	1.406	0.07	3.577e-06
1975-78	0.655	-10.444	1.705	0.112	2.861e-09
1976-79	0.48	-7.434	1.732	0.058	2.402e-05
1977-80	0.668	-10.655	1.132	0.229	1.330e-18
1978-81	0.845	-13.741	1.316	0.268	5.731e-22
1979-82	1.019	-16.78	1.492	0.298	1.072e-24
1980-83	1.509	-25.35	1.746	0.414	1.706e-36
1981-84	1.502	-25.165	1.571	0.463	4.207e-42
1982-85	1.381	-23.037	1.507	0.44	2.391e-39
1983-86	1.624	-27.274	2.093	0.363	5.110e-31
1984-87	0.834	-13.525	0.607	0.625	1.938e-65
1985-88	0.814	-13.169	0.582	0.632	1.181e-66
1986-89	0.733	-11.764	0.56	0.597	9.573e-61
1987-90	0.511	-7.909	0.484	0.473	2.432e-43
1988-91	0.587	-9.226	0.955	0.239	2.174e-19
1989-92	0.621	-9.82	1.095	0.213	3.331e-17
1990-93	0.619	-9.795	1.101	0.21	5.741e-17
1991-94	0.711	-11.405	1.458	0.17	8.897e-14
1992-95	0.498	-7.709	1.784	0.059	2.110e-05
1993-96	0.617	-9.763	1.715	0.098	3.004e-08
1994-97	0.787	-12.699	1.358	0.227	2.145e-18
1995-98	0.912	-14.864	0.877	0.491	1.444e-45
1996-99	0.9	-14.645	0.797	0.531	5.750e-51
1997-00	0.803	-12.968	0.497	0.695	6.355e-79
1998-01	0.768	-12.374	0.347	0.81	2.008e-109
1999-02	0.706	-11.297	0.331	0.796	8.072e-105
2000-03	0.602	-9.517	0.217	0.866	2.953e-132
2001-04	0.55	-8.639	0.178	0.887	3.155e-143
2002-05	0.572	-9.021	0.185	0.889	4.991e-144
2003-06	0.666	-10.656	0.29	0.819	8.580e-113
2004-07	0.678	-10.847	0.376	0.736	3.082e-88
2005-08	0.31	-4.42	0.181	0.666	6.073e-73
2006-09	0.288	-4.038	0.151	0.707	1.843e-81
2007-10	0.256	-3.494	0.13	0.707	2.480e-81
2008-11	0.226	-2.977	0.119	0.677	4.476e-75
2009-12	0.263	-3.632	0.13	0.723	5.910e-85
2010-13	0.303	-4.328	0.123	0.805	8.682e-108
2011-14	0.303	-4.328	0.166	0.694	1.049e-78
2012-15	0.414	-6.264	0.23	0.713	1.048e-82
2013-16	0.39	-5.838	0.221	0.701	4.274e-80
2014-17	0.369	-5.478	0.251	0.615	8.922e-64
2015-18	0.353	-5.194	0.213	0.666	6.365e-73

Table 3: Results for regression with dominant eigenvectors of the mutual information matrix (number of bins= 16) and the distance matrix.

Year	Intercept	Slope	Standard Error	R square	p-value
1972-75	0.708	-6.909	1.291	0.19	4.142e-07
1973-76	0.442	-6.714	1.078	0.115	1.621e-09
1974-77	0.498	-7.69	1.176	0.126	2.671e-10
1975-78	0.709	-11.36	1.485	0.164	2.828e-13
1976-79	0.574	-9.039	1.5	0.109	4.917e-09
1977-70	0.653	-10.39	1.001	0.265	9.726e-22
1978-81	0.828	-13.426	1.142	0.317	1.743e-26
1979-82	0.97	-15.9	1.195	0.373	5.175e-32
1980-83	1.362	-22.736	1.344	0.49	1.870e-45
1981-84	1.432	-23.931	1.391	0.498	1.446e-46
1982-85	1.284	-21.352	1.493	0.407	1.136e-35
1983-86	1.532	-25.658	2.025	0.35	1.027e-29
1984-87	0.76	-12.228	0.622	0.564	1.037e-55
1985-88	0.736	-11.819	0.598	0.567	3.597e-56
1986-89	0.663	-10.538	0.586	0.521	1.665e-49
1987-90	0.436	-6.605	0.505	0.365	3.294e-31
1988-91	0.501	-7.745	0.974	0.175	3.843e-14
1989-92	0.514	-7.975	1.11	0.148	5.486e-12
1990-93	0.534	-8.315	1.098	0.161	4.716e-13
1991-94	0.612	-9.685	1.438	0.132	8.500e-11
1992-95	0.425	-6.445	1.746	0.044	0.000265
1993-96	0.538	-8.395	1.674	0.078	9.062e-07
1994-97	0.663	-10.558	1.356	0.169	1.151e-13
1995-98	0.816	-13.186	0.883	0.428	5.154e-38
1996-09	0.804	-12.982	0.792	0.474	1.762e-43
1997-00	0.719	-11.492	0.494	0.645	5.882e-69
1998-01	0.681	-10.854	0.334	0.78	6.181e-100
1999-02	0.612	-9.652	0.316	0.758	1.030e-93
2000-03	0.54	-8.425	0.199	0.857	4.205e-128
2001-04	0.503	-7.795	0.159	0.889	2.047e-144
2002-05	0.521	-8.116	0.168	0.887	3.472e-143
2003-06	0.602	-9.518	0.268	0.809	4.335e-109
2004-07	0.604	-9.53	0.34	0.725	1.561e-85
2005-08	0.284	-3.959	0.165	0.659	1.159e-71
2006-09	0.264	-3.618	0.138	0.698	1.446e-79
2007-10	0.239	-3.186	0.12	0.701	3.506e-80
2008-11	0.211	-2.71	0.109	0.673	2.163e-74
2009-12	0.244	-3.288	0.117	0.725	1.392e-85
2010-13	0.276	-3.852	0.111	0.803	4.051e-107
2011-14	0.281	-3.939	0.153	0.689	1.316e-77
2012-15	0.37	-5.489	0.206	0.704	8.534e-81
2013-16	0.353	-5.179	0.203	0.687	3.974e-77
2014-17	0.336	-4.893	0.233	0.597	1.084e-60
2015-18	0.327	-4.727	0.201	0.65	6.220e-70

4 Algorithmic Information Theory-based Measures

In this section, the steps to compute the algorithmic complexity of the stocks-returns using MDS scaling, have been explained.

1. For each data-window, we have created the cross-correlation matrix of size 300×300 (except for the $1972 - 75$ window, which is of size 124×124) using,
$$C_{ij}^k =< g_i^k(t) g_j^k(t) > . \tag{16}$$

2. Then we have calculated the dissimilarity matrix of size 300×300 (except for the $1972 - 75$ window, which is of size 124×124) using the correlation matrix, using the following equation,
$$d_{ij}^{sk} = 1 - C_{ij}^k \tag{17}$$
where d_{ij}^{sk} is the element in dissimilarity matrix D^S for k^{th} window. Note that this is not the same as the distance matrix given by Eqn. 4. Note that all elements of the D^{sk} matrix for all k now have to be within 0 and 2.

3. Then we projected the values on binary 2D plane using clustering technique (multi-dimensional scaling). For that, we calculate the 2D co-ordinates of the projected values (using Euclidean distance), fixing the angle of rotation for all of the windows. This step (fixing the rotation) is important since otherwise the code randomly rotates the projection on the 2 dimensional plane and that distorts the computation of the complexity measure.

4. We map these points on 300×300 grid (124×124 for the first window). Assigning the 1 to the cell if there are corresponding data-points within that cell, and 0 to the rest of the cells.

5. Then we calculated the algorithmic complexity of this binary grid, using BDM module (a *python* package) for 2 symbol, 2D array. Available at https://pypi.org/project/pybdm/.

6. Calculate the same for all the windows and plot the corresponding evolution of the complexity measure over the entire time period in Fig. 3 in the main paper.

5 Interactive Dynamics: Complexity Through Heterogeneity

In this part, the estimate heterogeneity in the interaction strength across assets. This heterogeneity is argued to capture complexity in the system.

1. First, we consider vector autoregression model with lag 1:
$$X_t = \hat{\Gamma} X_{t-1} + \epsilon_t \tag{18}$$
where X_t represents log return series, Γ represents the interaction matrix and ϵ_t represents eror term. Note that in this set up X_t is a vector of size $N \times 1$ where $N = 300$ and Γ is a matrix of size 300×300.

2. We estimate this VAR model on the k-th window (using the *stats.linregress* module of *python* available at https://www.statsmodels.org/dev/vector_ar.html) for all k.

3. After estimating the interaction matrix Γ, the standard deviation of the estimated parameters in the Γ matrix is calculated. We call this parameter σ and it represents the heterogeneity interactions across stocks. For extracting the elements of the $\hat{\Gamma}$ matrix, we had to employ *vars* package in R (*stats.linregress* package does not allow extraction of the interaction matrix).

4. We plot the evolution of the standard deviation parameter over the available data-length. Fig. 4 presents the evolution of the degree of heterogeneity in the interaction strengths of stocks.

5. Summary of the results of the VAR estimations are given in table 4. Num. of days (T) represents the number of days within each window for which we estimate the model. By construction, it is slightly above 1000 days for each stock (corresponding to 4 years' longitudinal data in each window). We also present the loglikelihood measure along with three information criteria (Akaike, Bayesian and Hannan-Quinn).

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Table 4: Summary of the result for VAR estimations ($N = 300$).

Year	Num. of days (T)	Log likelihood	AIC	BIC	HQIC
1972-85	1008	249051.06	-815.29	-739.7	-786.57
1973-76	1009	847905.24	-2353.06	-1913.04	-2185.89
1974-77	1009	872067.49	-2400.95	-1960.93	-2233.79
1975-78	1008	884220.04	-2426.6	-1986.23	-2259.3
1976-79	1008	888766.17	-2435.62	-1995.25	-2268.32
1977-80	1008	868455.86	-2395.32	-1954.96	-2228.02
1978-81	1009	865648.75	-2388.23	-1948.21	-2221.06
1979-82	1010	861156.86	-2377.81	-1938.14	-2210.79
1980-83	1010	831958.02	-2319.99	-1880.32	-2152.97
1981-84	1010	834343.79	-2324.72	-1885.05	-2157.69
1982-85	1010	820873.39	-2298.04	-1858.37	-2131.02
1983-86	1010	793303.97	-2243.45	-1803.78	-2076.42
1984-87	1010	780575.98	-2218.25	-1778.57	-2051.22
1985-88	1010	777923.27	-2212.99	-1773.32	-2045.97
1986-89	1009	790212.12	-2238.7	-1798.68	-2071.54
1987-90	1009	794435.93	-2247.07	-1807.05	-2079.91
1988-91	1009	781902.56	-2222.23	-1782.21	-2055.07
1989-92	1010	759536.33	-2176.58	-1736.91	-2009.56
1990-93	1011	744323.44	-2145.18	-1705.85	-1978.29
1991-94	1010	751615.68	-2160.9	-1721.23	-1993.87
1992-95	1009	742966.5	-2145.05	-1705.03	-1977.89
1993-96	1009	737347.94	-2133.92	-1693.9	-1966.75
1994-97	1009	745722.4	-2150.52	-1710.5	-1983.35
1995-98	1009	717267.15	-2094.11	-1654.09	-1926.95
1996-99	1009	670600.04	-2001.61	-1561.59	-1834.45
1997-00	1007	639061.65	-1941.26	-1500.54	-1773.81
1998-01	1002	647602.81	-1963.74	-1521.28	-1795.59
1999-02	1003	690594.27	-2048.36	-1606.25	-1880.35
2000-03	1002	718190.71	-2104.64	-1662.17	-1936.49
2001-04	1002	778859.22	-2225.73	-1783.27	-2057.58
2002-05	1006	818752.31	-2299.58	-1858.51	-2131.99
2003-06	1005	846233.71	-2355.71	-1914.3	-2187.98
2004-07	1004	857945.65	-2380.54	-1938.77	-2212.67
2005-08	1005	849217.43	-2361.65	-1920.23	-2193.92
2006-09	1005	838428.3	-2340.18	-1898.76	-2172.45
2007-10	1006	856462.82	-2374.55	-1933.49	-2206.96
2008-11	1007	877718.35	-2415.25	-1974.54	-2247.81
2009-12	1005	919092.36	-2500.7	-2059.29	-2332.97
2010-13	1005	948364.11	-2558.95	-2117.54	-2391.23
2011-14	1005	962470.09	-2587.02	-2145.61	-2419.3
2012-15	1005	985138.41	-2632.14	-2190.72	-2464.41
2013-16	1006	994160.95	-2648.3	-2207.24	-2480.72
2014-17	1005	1000336.59	-2662.38	-2220.97	-2494.65
2015-18	1004	1012073.34	-2687.56	-2245.8	-2519.7

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