Appendix

The mass matrix **M**, stiffness matrix **K**, damping matrix **C**, and force vector **F** in Eq. (24) are expressed as

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_{\mathrm{NN}} & \mathbf{M}_{\mathrm{NI}} & \mathbf{0} \\ \mathbf{M}_{\mathrm{IN}} & \mathbf{M}_{\mathrm{II}}^{\mathrm{N}} + \mathbf{M}_{\mathrm{II}}^{\mathrm{E}} & \mathbf{M}_{\mathrm{IE}} \\ \mathbf{0} & \mathbf{M}_{\mathrm{EI}} & \mathbf{M}_{\mathrm{EE}} \end{bmatrix} \tag{34}$$

with

$$\mathbf{M}_{\mathrm{NN}} = \mathbf{diag}(m_1, m_2, m_3, m_4, m_5, m_6)$$

$$\mathbf{M}_{\mathrm{IN}} = \mathbf{M}_{\mathrm{NI}}^{\mathrm{T}} = \begin{bmatrix} 0 & 0 & 0 & m_{47} & m_{47} & m_{47} \\ m_{18} & m_{28} & m_{38} & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{M}_{\mathrm{II}}^{\mathrm{N}} = (3m_0 + M_{nac} + M_{hub}) \cdot \mathbf{I}$$

where $\mathbf{diag}(\cdot)$ indicates a diagonal matrix; **I** indicates identity matrix; $m_1 = m_2 = m_3 = \int_0^R \overline{m}\phi_{1e}^2 dr$, $m_4 = m_5 = m_6 = \int_0^R \overline{m}\phi_{1f}^2 dr$, $m_{47} = m_{57} = m_{67} = \int_0^R \overline{m}\phi_{1f} dr$, $m_{j8} = \int_0^R \overline{m}\phi_{1e} dr \cos\psi_j$, ($j =$ 1,2,3).

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{\mathrm{NN}} & \mathbf{K}_{\mathrm{NI}} & \mathbf{0} \\ \mathbf{K}_{\mathrm{IN}} & \mathbf{K}_{\mathrm{II}}^{\mathrm{N}} + \mathbf{K}_{\mathrm{II}}^{\mathrm{E}} & \mathbf{K}_{\mathrm{IE}} \\ \mathbf{0} & \mathbf{K}_{\mathrm{EI}} & \mathbf{K}_{\mathrm{EE}} \end{bmatrix} \tag{35}$$

with

$$\mathbf{K}_{\mathrm{NN}} = \mathbf{diag}\left(k_{b1,eg}, k_{b2,eg}, k_{b3,eg}, k_{b1,fp}, k_{b2,fp}, k_{b3,fp}\right)$$

$$\mathbf{K}_{\mathrm{IN}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -\Omega^2 m_{18} & -\Omega^2 m_{28} & -\Omega^2 m_{38} & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{K}_{\mathrm{NI}} = \mathbf{0}$$

$$\mathbf{K}_{\mathrm{II}}^{\mathrm{N}} = \mathbf{0}$$

where $k_{bj,eg} = k_{eg} + k_{ge,eg} - k_{gr,eg}\cos\psi_j - \Omega^2 \int_0^R \overline{m}\phi_{1e}^2 dr$, $k_{bj,fp} = k_{fp} + k_{ge,fp} - k_{gr,fp}\cos\psi_j$, ($j = 1,2,3$).

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_{\mathrm{NN}} & \mathbf{C}_{\mathrm{NI}} & \mathbf{0} \\ \mathbf{C}_{\mathrm{IN}} & \mathbf{C}_{\mathrm{II}}^{\mathrm{N}} + \mathbf{C}_{\mathrm{II}}^{\mathrm{E}} & \mathbf{C}_{\mathrm{IE}} \\ \mathbf{0} & \mathbf{C}_{\mathrm{EI}} & \mathbf{C}_{\mathrm{EE}} \end{bmatrix} \tag{36}$$

with

$$\mathbf{C}_{\mathrm{NN}} = \mathbf{diag}\left(c_{b1,eg}, c_{b2,eg}, c_{b3,eg}, c_{b1,fp}, c_{b2,fp}, c_{b3,fp}\right)$$

$$\mathbf{C}_{\mathrm{IN}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -2\Omega\overline{m}_1 & -2\Omega\overline{m}_2 & -2\Omega\overline{m}_3 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{C}_{\mathrm{NI}} = \mathbf{0}$$

$$\mathbf{C}_{\mathrm{II}}^{\mathrm{N}} = \mathbf{diag}\left(c_{aero,x}, c_{aero,y}\right)$$

where $\overline{m}_j = \int_0^R \overline{m}\phi_{1e} dr \sin\psi_j$, ($j = 1,2,3$).

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{\mathrm{N}} \\ \mathbf{F}_{\mathrm{I}}^{\mathrm{N}} + \mathbf{F}_{\mathrm{I}}^{\mathrm{E}} \\ \mathbf{F}_{\mathrm{E}} \end{bmatrix} \tag{37}$$

with

$$\mathbf{F}_{\mathrm{N}} = [Q_{wind,1} \quad Q_{wind,2} \quad Q_{wind,3} \quad Q_{wind,4} \quad Q_{wind,5} \quad Q_{wind,6}]^{\mathrm{T}}$$

$$\mathbf{F}_{\mathrm{I}}^{\mathrm{N}} = [Q_{wind,7} \quad Q_{wind,8}]^{\mathrm{T}}$$

$$\mathbf{F}_{\mathrm{I}}^{\mathrm{E}} = \mathbf{0}$$

$$\mathbf{F}_{\mathrm{E}} = \mathbf{F}_{wave}$$

where $Q_{wind,j} = \int_0^R p_{Tj}(r,t)\phi_{1e} dr$, $Q_{wind,j+3} = \int_0^R p_{Nj}(r,t)\phi_{1f} dr$, $(j = 1,2,3)$;

$Q_{wind,7} = \sum_{j=1}^{3} \int_0^R p_{Nj}(r,t) dr$, $Q_{wind,8} = \sum_{j=1}^{3} \int_0^R p_{Tj}(r,t) dr \cos\psi_j$; $\mathbf{F}_{wave}$ is the global force vector assembled from $\mathbf{F}_{wave,i}$ shown in Eq. (20).

In the above Eqs. (34)-(36), $\mathbf{M}_{\mathrm{II}}^{\mathrm{E}}$, $\mathbf{M}_{\mathrm{IE}}$, $\mathbf{M}_{\mathrm{IE}}$, $\mathbf{K}_{\mathrm{II}}^{\mathrm{E}}$, $\mathbf{K}_{\mathrm{IE}}$, $\mathbf{K}_{\mathrm{IE}}$, $\mathbf{C}_{\mathrm{II}}^{\mathrm{E}}$, $\mathbf{C}_{\mathrm{IE}}$, and $\mathbf{C}_{\mathrm{IE}}$ are all defined by partitioning the mass $\mathbf{M}_{tow}$, stiffness $\mathbf{K}_{tow}$, and damping $\mathbf{C}_{tow}$ matrices of the tower FE model obtained in Sec. “FE model for tower including foundation”. More specifically, they are

$$\begin{bmatrix} \mathbf{M}_{\mathrm{II}}^{\mathrm{E}} & \mathbf{M}_{\mathrm{IE}} \\ \mathbf{M}_{\mathrm{EI}} & \mathbf{M}_{\mathrm{EE}} \end{bmatrix} = \mathbf{M}_{tow}$$
$$\begin{bmatrix} \mathbf{K}_{\mathrm{II}}^{\mathrm{E}} & \mathbf{K}_{\mathrm{IE}} \\ \mathbf{K}_{\mathrm{EI}} & \mathbf{K}_{\mathrm{EE}} \end{bmatrix} = \mathbf{K}_{tow} \tag{38}$$
$$\begin{bmatrix} \mathbf{C}_{\mathrm{II}}^{\mathrm{E}} & \mathbf{C}_{\mathrm{IE}} \\ \mathbf{C}_{\mathrm{EI}} & \mathbf{C}_{\mathrm{EE}} \end{bmatrix} = \mathbf{C}_{tow}$$

where the subscripts “I”, “N” and “E” indicate the DOFs are related to the interfacing system, numerical component, and experimental component of the proposed RTHS framework, respectively; the superscripts “E” and “N” indicate the term originates from the experimental component or from the numerical component.