

Supplement 2

Estimation for the OIA error in section 4.1

The OIA is defined as the sum of the SSH and SEP anomalies, as shown in Eq. 2 in the main text. This means that the error of the OIA anomalies includes errors of both the SSH and SEP anomalies. Due to the lack of number of the *in-situ* CTD observations in the analysis area as mentioned in section 1, we could not estimate directly the error of the SEP using observational CTD data. Thus, we estimated the OIA error using the regression relationship between the SSH and SEP anomalies shown in Figs. 7a and 8.

The standard deviation (SD) of the OIA is written as,

$$SD_{OIA}^2 = SD_{ssh}^2 + SD_{SEP}^2 + 2 \times SD_{SSH} \times SD_{SEP} \times r, \quad (S1)$$

where, SD_{OIA} is the standard deviation of the OIA; SD_{ssh} is the standard deviation of the SSH anomaly; SD_{SEP} is the standard deviation of the SEP anomaly; and r is the correlation coefficient between SSH and SEP anomalies.

When SD_{SEP} is estimated using SD_{ssh} and the regression coefficient between the SSH and SEP anomalies, SD_{SEP} is approximately 92% of the SD_{ssh} at both PG1 and PG2, since the regression coefficient between the SSH and SEP anomalies is -0.9167 (-0.9153) at PG1 (PG2) (see section 3.4). The correlation coefficient between SSH and SEP anomalies are -0.9983 and -0.9980 at PG1 and PG2, respectively (also see section 3.4). Using the regression and correlation coefficients between SSH and SEP anomalies, SD_{OIA}^2 at PG1 is estimated from Eq. (S1) as,

$$\begin{aligned} SD_{OIA}^2 &= SD_{ssh}^2 + (0.9167 \times SD_{SSH})^2 \\ &\quad + 2 \times SD_{SSH} \times (0.9167 \times SD_{SSH}) \times (-0.9983) \\ &= 0.0101 \times SD_{ssh}^2. \end{aligned} \quad (S2)$$

From Eq. (S2), SD_{OIA} is estimated to be,

$$SD_{OIA} = 0.1 \times SD_{ssh}. \quad (S3)$$

Therefore, SD_{OIA} is estimated to be approximately 10% of the SD_{ssh} at PG1. In the same way, SD_{OIA} at PG2 is also estimated to be approximately 10% of SD_{ssh} at PG2. Therefore, it is indicated that the error of the OIA is roughly estimated to be 10% of the error of the JCOPE2M SSH anomaly (i.e., SDdif, if SDdif perfectly represents the error of the JCOPE2M SSH anomalies; see section 4.1). The OIA error is estimated to be approximately $\pm 6.0 \times 10^{-4}$ (m) at PG1 and $\pm 1.7 \times 10^{-3}$ (m) at PG2, since SDdif is approximately 6.0×10^{-3} (m) at PG1 and 1.7×10^{-2} (m) at PG2 (see section 3.1).