APPENDIX

In this section, a prescription for the IGM bias parameter will be derived based on the results of the Press-Schechter theory. Let, $\rho_{\rm c}$ be the critical density of the universe, $\Omega_{\rm M}$ be the matter fraction, v be the volume of each grid cell, δ be the overdensity of a grid cell, $f_{\rm coll}$ be the collapse fraction of the grid cell, m be the mass content of the grid cell. Let us assume that all the haloes with mass greater than $M_{\rm min} = 10^8 M_\odot$ are massive and have collapsed to form galaxies. And all the haloes with mass less than $M_{\rm min}$ do not form galaxies and hence have baryons that make up the entire IGM. Let $m_>$ be the total mass in haloes which are more massive than $M_{\rm min}$ and $m_<$ be total mass in haloes which are less massive than $M_{\rm min}$. Let $\delta_{\rm cmh}$ be the overdensity measured for the collapsed massive haloes and $\delta_{\rm lmh}$ be the overdensity measured for low mass haloes. $f_{\rm coll}$ can be obtained exactly in the Press-Schechter framework and is given by

$$f_{\rm coll} = {\rm erfc}\left\{\frac{\delta_c(z)-\delta_m}{\sqrt{2\left[\sigma^2_{\rm min}-\sigma^2(m)\right]}}\right\} \quad (1)$$

Here, δ_m is mean linear overdensity of the grid-cell, m is the total mass in the grid-cell, $\sigma^2(m)$ is the variance of density fluctuations on the scale m, $\sigma^2_{\rm min} = \sigma^2(M_{\rm min})$, $M_{\rm min}$ is minimum mass of the ionizing sources and $\delta_c(z)$ is the critical density for collapse at redshift z. We have,

$$\begin{aligned} m &= \Omega_{\rm M}\rho_{\rm c} v(1+\delta) \\ m_> &= f_{\rm coll} m \\ m_< &= m - m_> \end{aligned}$$

Let, $\bar{m}_>$ and $\bar{m}_<$ be average values of $m_>$ and $m_<$ respectively. We also have,

$$\begin{aligned} \bar{m}_> &= \bar{f}_{\rm coll}\Omega_{\rm M}\rho_{\rm c} v & (2) \\ \bar{m}_< &= (1-\bar{f}_{\rm coll})\Omega_{\rm M}\rho_{\rm c} v & (3) \end{aligned}$$

Where average value of collapse fraction $\bar{f}_{\rm coll}$ is given by

$$\bar{f}_{\rm coll} = {\rm erfc}\left\{\frac{\delta_c(z)}{\sqrt{2\sigma^2_{\rm min}}}\right\} \quad (4)$$

Now we have,

$$\begin{aligned} \delta_{\rm cmh} &= \frac{m_> - \bar{m}_>}{\bar{m}_>} & (5) \\ &= \frac{f_{\rm coll}(1+\delta) - \bar{f}_{\rm coll}}{\bar{f}_{\rm coll}} & (6) \\ \delta_{\rm lmh} &= \frac{m_< - \bar{m}_<}{\bar{m}_<} & (7) \\ &= \frac{(1-f_{\rm coll})(1+\delta) - (1-\bar{f}_{\rm coll})}{(1-\bar{f}_{\rm coll})} & (8) \end{aligned}$$

Together we have,

$$\alpha \equiv \frac{\delta_{\rm lmh}}{\delta_{\rm cmh}} = \frac{(1+\delta)(1-f_{\rm coll}) - (1-\bar{f}_{\rm coll})}{(1+\delta)f_{\rm coll} - \bar{f}_{\rm coll}} \times \frac{\bar{f}_{\rm coll}}{1-\bar{f}_{\rm coll}} \quad (9)$$

Both, $\delta_{\rm cmh}$ and $\delta_{\rm lmh}$, can be obtained as function of true overdensity δ and hence the IGM bias parameter α can be obtained as a function of overdensity of collapsed massive haloes $\delta_{\rm cmh}$.