# Supplementary Appendix. $^3P_0$ pair-creation model


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The $^3P_0$ pair-creation model is an effective model to compute $A \rightarrow BC$ open-flavor strong decays [1–4]. Here, a hadron decay takes place in its rest frame and proceeds via the creation of an additional $q\bar{q}$ pair. The quark-antiquark pair is created with the quantum numbers of the vacuum, i.e. $J^{PC} = 0^{++}$ (see **Figure 1**), and the decay width can be expressed as [1–8]

$$\Gamma_{A\rightarrow BC} = \Phi_{A\rightarrow BC}(k_0) \sum_{\ell} \left| \langle BCk_0\, \ell J | \, T^\dagger \, |A\rangle \right|^2 \ . \quad \text{(S1)}$$

The final state is characterized by the relative orbital angular momentum $\ell$ between $B$ and $C$ and a total angular momentum $\mathbf{J} = \mathbf{J}_B + \mathbf{J}_C + \boldsymbol{\ell}$. One usually assumes harmonic oscillator wave functions for the parent, $A$, and daughter, $B$ and $C$, hadrons, depending on a h.o. parameter, $\alpha_{\rm ho}$. In the meson case, one has

$$\begin{aligned} \Phi_{n_j l_j m_j}(\mathbf{q}_j) \ &= \ \mathcal{N}_{n_j l_j}(\alpha_j) L_{n_j}^{l_j+1/2}(\alpha_j^{-2} q_j^2) \ e^{-\frac{1}{2} q_j^2/\alpha_j^2} \\ & \quad \mathcal{Y}_{l_j m_j}(\mathbf{q}_j) \ , \end{aligned} \quad \text{(S2)}$$

where the index $j = A, B$ or $C$ distinguishes among parent and daughter mesons, $\alpha_j = \alpha_{\rm ho}$, $n_j$ is the radial quantum number, $L_{n_j}^{l_j+1/2}(\alpha_j^{-2} q_j^2)$ a Laguerre polynomial and $\mathcal{Y}_{l_j m_j}(\mathbf{q}_j)$ a solid spherical harmonic [9];

$$\mathcal{N}_{n_j l_j}(\alpha_j) = \sqrt{\frac{2 n_j!}{\Gamma(n_j + l_j + 3/2)}} \ \alpha_j^{-l_j - \frac{3}{2}} \quad \text{(S3)}$$

is the normalization factor of the h.o. wave function of **Eq. S2**. The coefficient $\Phi_{A\rightarrow BC}(k_0)$ in **Eq. S1** is the phase-space factor (PSF) for the decay. Several prescriptions for $\Phi_{A\rightarrow BC}(k_0)$ are possible [10], including the non-relativistic one,

$$\Phi_{A\rightarrow BC}(k_0) = 2\pi k_0 \frac{M_B M_C}{M_A} \ , \quad \text{(S4)}$$

depending on the relative momentum $k_0$ between $B$ and $C$ and on the masses of the parent, $M_A$, and daughter hadrons, $M_B$ and $M_C$. The second option is the standard relativistic form,

$$\Phi_{A\rightarrow BC}(k_0) = 2\pi k_0 \frac{E_B(k_0) E_C(k_0)}{M_A} \ , \quad \text{(S5)}$$

where $E_B = \sqrt{M_B^2 + k_0^2}$ and $E_C = \sqrt{M_C^2 + k_0^2}$ are the energies of the daughter hadrons. The third possibility is to use an effective PSF [11, 12],

$$\Phi_{A\rightarrow BC}(k_0) = 2\pi k_0 \frac{\tilde{M}_B \tilde{M}_C}{M_A} \ , \quad \text{(S6)}$$

where $\tilde{M}_B$ and $\tilde{M}_C$ are the effective masses of the daughter hadrons, evaluated by means of a spin-independent interaction. Our choice here is to use the PSF of **Eq. S5**. However, it is worth to note that in the case of heavy baryons and mesons, whose internal dynamics is almost non-relativistic and the hyperfine interactions are relatively small, the three types of phase-space factors are expected to provide very similar results.

The transition operator of the $^3P_0$ model is given by [1–4, 13, 14]:

$$\begin{aligned} T^\dagger \ &= \ -3\gamma_0 \int d\mathbf{p}_3\, d\mathbf{p}_4\, \delta(\mathbf{p}_3 + \mathbf{p}_4)\, C_{34}\, F_{34}\, V(\mathbf{p}_3 - \mathbf{p}_4) \\ &\times \ \left[\chi_{34} \times \mathcal{Y}_1(\mathbf{p}_3 - \mathbf{p}_4)\right]_0^{(0)} \, b_3^\dagger(\mathbf{p}_3)\, d_4^\dagger(\mathbf{p}_4) \ . \end{aligned} \quad \text{(S7)}$$

Here, $\gamma_0$ is the pair-creation strength, whose value is fitted to the reproduction of the experimental strong decay widths [15]; $b_3^\dagger(\mathbf{p}_3)$ and $d_4^\dagger(\mathbf{p}_4)$ are the creation operators for a quark and an antiquark with momenta $\mathbf{p}_3$ and $\mathbf{p}_4$, respectively; see **Figure 1**. The created $q\bar{q}$ pair is characterized by a color-singlet wave function, $C_{34}$, a flavor-singlet wave function, $F_{34}$, a spin-triplet wave function $\chi_{34}$, and a solid spherical harmonic $\mathcal{Y}_1(\mathbf{p}_3 - \mathbf{p}_4)$, because the quark and antiquark are in a relative $P$-wave.

$V(\mathbf{p}_3 - \mathbf{p}_4)$ in **Eq. S7** is the pair-creation vertex (PCV). In the original formulation of the $^3P_0$ model [1–3], one has $V(\mathbf{p}_3 - \mathbf{p}_4) = 1$. Possible refinements of the PCV were discussed in Refs. 13, 14. In the present calculations, we consider the PCV or quark form factor (QFF) [7, 14, 16, 17]

$$V(\mathbf{p}_3 - \mathbf{p}_4) = e^{-(\mathbf{p}_3 - \mathbf{p}_4)^2/(6\alpha_{\rm d}^2)} \ , \quad \text{(S8)}$$

where $\alpha_{\rm d}$ is the QFF parameter. The introduction of this particular PCV is motivated by the request that the $q\bar{q}$ pair of created quarks has an effective size.

In addition to that, another modification of the original $^3P_0$ model is worth to be taken into account. It consists in the substitution of the constant pair creation strength $\gamma_0$ of **Eq. S7** with an effective, flavor-dependent, pair-creation strength,

$$\gamma_0^{\rm eff} = \frac{m_{\rm u,d}}{m_{\rm i}} \, \gamma_0 \ . \quad \text{(S9)}$$

Its purpose is to suppress unphysical heavy quark pair-creation [7, 8, 14, 18]. The mechanism consists in multiplying $\gamma_0$ by a reduction factor $m_{\rm u,d}/m_{\rm i}$, with $i = u, d, s$


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or $c$; see **Table 1**. For example, the creation of $s\bar{s}$ pairs is suppressed with respect to $u\bar{u}$ ($d\bar{d}$) by a factor of $(m_{\rm u,d}/m_{\rm s})^2 = 0.36$, while that of $c\bar{c}$ by one of $(m_{\rm u,d}/m_{\rm c})^2 = 0.05$. In the case of $m_{\rm i} = m_{\rm u}$ or $m_{\rm d}$, $\gamma_0^{\rm eff} = \gamma_0$ and no suppression occurs. It is worth to note that this particular choice for the pair-creation strength breaks the $SU(3)$ $[SU(4)]$ flavor symmetry and its effect cannot be re-absorbed in a redefinition of the model parameters or in a different choice of the $^3P_0$ model vertex factor [14].

Finally, the $\langle BCk_0\, \ell J | \, T^\dagger \, |A\rangle$ $^3P_0$ amplitudes of **Eq. S1** can be calculated analytically by means of the formalism discussed in Ref. 4. For the open-flavor strong decays of baryons, see also Ref. 14.


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