Supplementary Material: Liquid computing on and off the edge of chaos with a striatal microcircuit

Carlos Toledo-Suárez[1,2,3], **Renato Duarte**[1,2,4,6] **and Abigail Morrison**[1,4,5*]

[1] *Bernstein Center Freiburg, Albert-Ludwig University of Freiburg, Freiburg im Breisgau, Germany*
[2] *Faculty of Biology, Albert-Ludwig University of Freiburg, Freiburg im Breisgau, Germany*
[3] *Department of Computational Biology, School of Computer Science and Communication, KTH Stockholm, Sweden*
[4] *Institute for Advanced Simulation (IAS-6) and Institute of Neuroscience and Medicine (INM-6), Jülich Research Centre and JARA Jülich, Germany*
[5] *Institute of Cognitive Neuroscience, Faculty of Psychology, Ruhr-University Bochum, Bochum, Germany*
[6] *Institute of Adaptive and Neural Computation, School of Informatics, University of Edinburgh, United Kingdom*

Correspondence*:
Abigail Morrison
Jülich Research Center and JARA,
Institute of Neuroscience and Medicine (INM-6) and Institute for Advanced Simulation (IAS-6),
Building 15.22, 52425 Jülich, Germany, a.morrison@fz-juelich.de

1 SUPPLEMENTARY TABLES

Table 1. Tabular description of network model.

A: Model Summary	
Populations	Input encoding cortical neurons (Cs), striatal medium spiny neurons (MSNs) and striatal fast spiking interneurons (FSIs)
Connectivity	Feed forward from Cs to MSNs and FSIs, feed forward from FSIs to MSNs, recurrent between MSNs
Neuron model	Leaky integrate-and-fire with exponential post-synaptic currents for Cs, leaky integrate-and-fire with exponential post-synaptic currents and multi-timescale adaptive threshold tuned for intrinsic bursting for MSNs and for fast spiking for FSIs
Input	Independent fixed-rate Poisson spike trains to Cs, gaussian current profile to Cs
Measurements	Spike activity

B: Populations		
Name	**Elements**	**Size**
Cs	iaf psc exp neuron	50, 25 per axis
MSNs	mat2 iaf psc exp neuron	500
FSIs	mat2 iaf psc exp neuron	50

C: Connectivity			
Name	**Source**	**Target**	**Pattern**
$FF_{\mathrm{C-MSN}}$	Cs	MSNs	Random convergent, $C_{\mathrm{Ex}} \rightarrow 1$, weight $w_c J_{\mathrm{CMSN}}$, delay d_{C}
$FF_{\mathrm{C-FSI}}$	Cs	FSIs	Random convergent, $C_{\mathrm{Ex}} \rightarrow 1$, weight $w_c J_{\mathrm{CFSI}}$, delay d_{C}
INH_{FSI}	FSIs	MSNs	Random divergent, $1 \rightarrow C_{\mathrm{FSI}}$, weight $w_s J_{\mathrm{FSI}}$, delay d_{FSI}
INH_{MSN}	MSNs	MSNs	Random divergent, $1 \rightarrow C_{\mathrm{MSN}}$, weight $w_s J_{\mathrm{MSN}}$, delay d_{MSN}

D: Neuron Models	
Name	iaf_psc_exp neuron
Type	Leaky integrate-and fire with exponential post synaptic currents
Subthreshold dynamics	if $(t > t^* + \tau_{\mathrm{ref}}) \quad \tau_{\mathrm{m}} \frac{dV}{dt} = -V + \frac{I_{\mathrm{syn}}(t)}{C_{\mathrm{m}}}$ else $V(t) = V_{\mathrm{reset}}$ $I_{\mathrm{syn}} = \sum_i \sum_{s \in S_i} w_i J_i I_{exp}(t - s - d)$ $I_{exp}(t) = e^{-t/\tau_{\mathrm{exp}}}$
Spiking	If $V(t-) < \Theta$ OR $V(t+) \geq \Theta$ 1. set $t^* = t$ 2. emit spike with time stamp t^* Membrane potential reset:$V(t) = V_{\mathrm{reset}}$
Threshold dynamics	Fixed threshold
Name	mat2_psc_exp neuron (**Kobayashi et al.**, 2009)
Type	Leaky integrate-and-fire with exponential post-synaptic currents and multi-timescale adaptive threshold
Subthreshold dynamics	Same as iaf_psc_exp neuron
Spiking	Same as iaf_psc_exp neuron, without membrane potential reset
Threshold dynamics	$\Theta(t) = \sum_k H(t - t_k) + \omega, \qquad H(t) = \sum_{j=1}^{2} \alpha_j \exp(-t/\tau_j)$,

E: Input		
Type	**Target**	**Description**
Poisson generator	Cs	Independent for each neuron, rate ν_{back}, weight J_{back}
Gaussian current profile	Cs	$I_{\mathrm{Gauss}} \exp(-(i - x)^2/(2\sigma^2))$, per axis, where: $i \in \{1, 2, \ldots, 25\}$ runs among Cs, $0 \leq x \leq 25$ tells the position on axis

F: Measurements
Low pass filtering of spikes from MSNs and FSIs

Table 2. Simulation parameters.

A: Connectivity		
Name	**Value**	**Description**
C_{Ex}	6	Number of outgoing feed-forward connections received by every striatal neuron from Cs
C_{FSI}	variable	Number of feed-forward connections from a FSI to MSNs found on a circular area centered on the FSI with radius R_{FSI}, chosen with uniform probability P_{FSI}
C_{MSN}	variable	Number of recurrent connections from a MSN to other MSNs found on a circular area centered on it with radius R_{MSN}, chosen with Gaussian probability, such that the total probability a standard deviation away is fixed to P_{MSN}
J_{CMSN}	$100\ \mathrm{pA}$	Amplitude of excitatory connection from a cortical neuron to a MSN
J_{CFSI}	$1.7\times\ \mathrm{pA}$	Amplitude of excitatory connection from a cortical neuron to a FSI
J_{FSI}	variable	Amplitude of inhibitory connection from a FSI to a MSN, taken from a random uniform distribution on $[-480, -50]\ \mathrm{pA}$ (**Koos et al.**, 2004)
J_{MSN}	variable	Amplitude of inhibitory connection from a MSN to a MSN, taken from a random uniform distribution on $[-90, -10]\ \mathrm{pA}$ (**Koos et al.**, 2004)
d_C	$1\ \mathrm{ms}$	Synaptic transmission delay from Cs to MSNs and FSIs
d_{FSI}	$1\ \mathrm{ms}$	Synaptic transmission delay from FSIs to MSNs
d_{MSN}	$2\ \mathrm{ms}$	Synaptic transmission delay from MSNs to MSNs
R_{FSI}	$0.1\ \mathrm{mm}$	Radius for circular area around FSI to make a connection with MSN (**Planert et al.**, 2010)
R_{MSN}	$1\ \mathrm{mm}$	Radius for circular area around MSN to make a connection with another MSN
P_{FSI}	0.74	Uniform probability to make a connection between FSI and MSN (**Planert et al.**, 2010)
P_{MSN}	0.2	Probability within a Gaussian standard deviation away from MSN to make a connection with another MSN (**Planert et al.**, 2010)

B: Neuron Model		
Name	**Value**	**Description**
τ_{mC}	$10\ \mathrm{ms}$	C membrane time constant
C_{mC}	$250\ \mathrm{pF}$	C membrane capacitance
Θ_C	-55 mV	C Fixed firing threshold
V_{0C}	$-70\ \mathrm{mV}$	C resting potential
$V_{\mathrm{reset}C}$	V_{0C}	C reset potential
τ_{refC}	$2\ \mathrm{ms}$	C absolute refractory period
τ_{mM}	$5\ \mathrm{ms}$	MSN membrane time constant
C_{mM}	$200\ \mathrm{pF}$	MSN membrane capacitance (**Gertler et al.**, 2008)
V_{0M}	$-58\ \mathrm{mV}$	MSN resting potential
τ_{refM}	$1\ \mathrm{ms}$	MSN absolute refractory period
τ^{+}_{sM}	$0.2\ \mathrm{ms}$	MSN time constant of post-synaptic excitatory currents
τ^{-}_{sM}	$2\ \mathrm{ms}$	MSN time constant of post-synaptic inhibitory currents
α_{1M}	$7.5\ \mathrm{mV}$	Weight of MSN multi-timescale adaptive threshold first time constant
α_{2M}	$1.5\ \mathrm{mV}$	Weight of MSN multi-timescale adaptive threshold second time constant
ω_M	$19\ \mathrm{mV}$	MSN multi-timescale adaptive threshold resting value

B: Neuron Model		
Name	**Value**	**Description**
τ_{mF}	$5\ \mathrm{ms}$	FSI membrane time constant
C_{mF}	$500\ \mathrm{pF}$	FSI membrane capacitance
V_{0F}	$-68\ \mathrm{mV}$	FSI resting potential
τ_{refF}	$2\ \mathrm{ms}$	FSI absolute refractory period
τ^{+}_{sF}	$0.3\ \mathrm{ms}$	FSI time constant of post-synaptic excitatory currents
τ^{-}_{sF}	$2\ \mathrm{ms}$	FSI time constant of post-synaptic inhibitory currents
α_{1F}	$10\ \mathrm{mV}$	Weight of FSI multi-timescale adaptive threshold first time constant
α_{2F}	$0.2\ \mathrm{mV}$	Weight of FSI multi-timescale adaptive threshold second time constant
ω_{F}	$10\ \mathrm{mV}$	FSI multi-timescale adaptive threshold resting value

C: Input		
Name	**Value**	**Description**
ν_{back}	10^4 Hz	Background independent Poisson rate to Cs
J_{back}	$20\ \mathrm{pA}$	Amplitude of background independent Poisson process to Cs
I_{Gauss}	$1\ \mathrm{nA}$	Maximum amplitude for Gaussian current profile
σ	2	Standard deviation of Gaussian current profile

D: Supervised learning	
Parameter	**Value**
Time step	$0.1\ \mathrm{ms}$
Learning rate	0.1
Runtime per step	$300\ \mathrm{ms}$
Training samples per step	50 after $50\ \mathrm{ms}$
Time in current position to take sample for advocated action	$50\ \mathrm{ms}$
Low pass filter decay	$30\ \mathrm{ms}$

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