APPENDIX A

Below we recall the derivation of the system (1)-(2) that governs the longwave Marangoni convection in a thin film heated from below.[6]

The thermocapillary convection in liquid layer is governed by the following dimensionless equations (Navier-Stokes, energy and continuity) and boundary conditions

$$\frac{1}{\mathrm{Pr}}\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \nabla^2\mathbf{v} - \mathrm{Ga}\mathbf{k}\,, \tag{A1}$$

$$\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T = \nabla^2 T\,,\ \nabla\cdot\mathbf{v} = 0\,, \tag{A2}$$

$$z = 0 : \mathbf{v} = 0,\ \frac{\partial T}{\partial z} = -1\,, \tag{A3}$$

$$z = h(x,y,t):\ \frac{\partial h}{\partial t} = w - \mathbf{u}\cdot\nabla h\,,\ \frac{\partial T}{\partial n} = -\mathrm{Bi}\,T, \tag{A4}$$

$$\sigma_{n\tau} = -\mathrm{Ma}\frac{\partial}{\partial \tau}\left. T\right|_{z=h}\,,\ \sigma_{nn} = p - \mathrm{Ca}K. \tag{A5}$$

where $\mathbf{v} = (\mathbf{u}, w)$ is the velocity field, T is the temperature, p is the pressure in the liquid, $\mathbf{k}$ is the upward unit vector, $\boldsymbol{\sigma}$ is the viscous stress tensor, $K = \nabla\cdot\mathbf{n}$ is the curvature of the free surface, $\mathbf{n}$ and $\boldsymbol{\tau}$ are the normal and tangential unit vectors at the interface; $\mathrm{Pr} = \nu / \chi$ is the Prandtl number; the rest of the dimensionless parameters are presented in Sec.2.

The conductive state that corresponds to the hydrostatic pressure and linear temperature distribution in a motionless liquid with a planar interface is given by the following expressions:

$$\bar{\mathbf{v}} = 0\,,\ \bar{p} = \mathrm{Ga}\,(1 - z)\,,\ \bar{T} = 1 - z + 1/\mathrm{Bi}\,,\ \bar{h} = 1\,. \tag{A6}$$

To study the stability of this solution with respect to long-wave perturbations and the evolution of the large-scale perturbations, we need to rescale the units

$$\tilde{x} = \varepsilon x\,,\ \tilde{y} = \varepsilon y\,,\ \tilde{z} = z\,,\ \tilde{t} = \varepsilon^2 t\,,\ \tilde{\mathbf{u}} = \frac{\mathbf{u}}{\varepsilon}\,,\ \tilde{w} = \frac{w}{\varepsilon^2}\,. \tag{A7}$$

Then we seek solution of the governing equations as the perturbation series in powers of small parameter ε^2

$$\tilde{\mathbf{u}} = \tilde{\mathbf{u}}_0 + \varepsilon^2\tilde{\mathbf{u}}_1 + \ldots,\ \tilde{w} = \tilde{w}_0 + \varepsilon^2\tilde{w}_1 + \ldots,\ p = p_0 + \varepsilon^2 p_1 + \ldots,\ T = -z + \frac{1}{\mathrm{Bi}} + T_0 + \varepsilon^2 T_1 + \ldots \tag{A8}$$

Following Shklyaev et al. [6] we assume large surface tension, low heat transfer from the free surface and normal gravity, $\mathrm{Ca} = \varepsilon^{-2} C$, $\mathrm{Bi} = \varepsilon^2 \beta$, $\mathrm{Ga} = O(1)$. Substituting this scaling with (A7) and (A8) into equations (A1)-(A5) we obtain at the leading order in ε the governing system; its solution is

$$p_0 = P(\tilde{x}, \tilde{y}, \tilde{t}) - \mathrm{Ga}\,\tilde{z}\,,\; T_0 = \Theta(\tilde{x}, \tilde{y}, \tilde{t})\,,\; \tilde{\mathbf{u}}_0 = \frac{\tilde{z}}{2}(\tilde{z} - 2h)\nabla P - \mathrm{Ma}\,\tilde{z}\nabla f\,,$$

$$\tilde{w}_0 = \frac{\tilde{z}^2}{2}\nabla \cdot \left[\left(h - \frac{\tilde{z}}{3}\right)\nabla P + \mathrm{Ma}\,\nabla f\right] \tag{A9}$$

If $\tilde{\mathbf{u}}_0$ is substituted into the mass conservation condition

$$\frac{\partial h}{\partial \tilde{t}} + \nabla \cdot \int_0^h \tilde{\mathbf{u}}\, d\tilde{z} = 0\,, \tag{A10}$$

one obtains the evolution equation for the interface (1).

The amplitude equation (2) is obtained from the first order of the expansion where we need only the equations that describe heat transfer in the liquid:

$$\frac{\partial^2 T_1}{\partial \tilde{z}^2} = \frac{\partial \Theta}{\partial \tilde{t}} - \nabla^2\Theta + \tilde{\mathbf{u}}_0 \cdot \nabla\Theta - \tilde{w}_0\,,\; \tilde{z} = 0: \frac{\partial T}{\partial \tilde{z}} = 0\,,\; \tilde{z} = h: \frac{\partial T_1}{\partial \tilde{z}} = \nabla\Theta \cdot \nabla h - \frac{1}{2}(\nabla h)^2 - \beta f\,.$$

Integration of the differential equation using boundary conditions results in the second amplitude equation (2). Within amplitude equations (1) and (2) one can investigate the behavior of small disturbances to the conductive state with motionless fluid and flat interface, that is corresponded to $h = 1$, $\Theta = 1$.

Note, that the Galileo number and capillary number are defined in such a way that the Prandtl number is scaled out from equations (1), (2). Thus, the large-scale Marangoni instability in this system does not depend on the Prandtl number. Recall, that similar disappearance of the Prandtl number from the linear theory of the buoyancy convection takes place when the Rayleight number is used instead of the Grashof number.

In [7,8] the feedback control was applied to the system under consideration. The physical mechanism of control is based on the suppression of the surface-tension-induced flow. It is assumed that system is equipped with the detectors and the actuators that can change the local heat flux on the solid substrate. Combining control techniques from [17] and [18], the authors reveal that one can effectively govern the instability by means of the following law

$$z = 0: \frac{\partial T}{\partial z} = -1 - K_f(f) f\,,\; f = T(z = h) - \bar{T}(z = 1)\,, \tag{A11}$$

where K_f is the non-dimensional scalar control gain, which should be rescaled as $K_f = \varepsilon^2 \varkappa$. Thus, additional term appears in Eq. (2) under feedback control as $\beta \to \beta - \varkappa(f)$.