APPENDIX C

The coefficients in the second-order solution (17)-(18) are as follows ($k = k_c$ everywhere below)

$$\begin{aligned} a_{22} = \{&-5Gk^4 \,\mathrm{Ma}_c^{\,2} + 144Gk^4 \,\mathrm{Ma}_c - 126\,\mathrm{Ma}_c^{\,2}\, k^4 + 60G(\beta - \varkappa_l)\,\mathrm{Ma}_c\, k^2 \\ &+24i\omega_0 Gk^2 \,\mathrm{Ma}_c - 144i\omega_0 \,\mathrm{Ma}_c\, k^2 - 12i\omega_0 \,\mathrm{Ma}_c^{\,2}\, k^2 - 72i\omega_0(\beta - \varkappa_l)\,\mathrm{Ma}_c \\ &+[16G(Gk^2 - 3k^2 - 3(\beta - \varkappa_l) + 6i\omega_0) + (G+72)\,\mathrm{Ma}_c\, k^2]\varkappa_q\} \,/ \\ &\{\mathrm{Ma}_c[4Ck^6 \,\mathrm{Ma}_c + Gk^4 \,\mathrm{Ma}_c - 96i\omega_0 Ck^4 - 32i\omega_0 Gk^2 - 192Ck^6 - Gk^4 \\ &+32i\omega_0 \,\mathrm{Ma}_c\, k^2 + 72k^4 \,\mathrm{Ma}_c - 96i\omega_0 k^2 - 48C(\beta - \varkappa_l)k^4 - 24i\omega_0(\beta - \varkappa_l)]\}, \end{aligned} \tag{C1}$$

$$\begin{aligned} b_{22} = -\{&72(3\,\mathrm{Ma}_c - 4)\,\mathrm{Ma}_c\, k^4[2G(\beta - \varkappa_l) - 3\,\mathrm{Ma}_c\, k^2] + [-24(G^2 + 7GCk^2 + 54Ck^2) \\ &+(588 - 9\,\mathrm{Ma}_c)G]k^6 \,\mathrm{Ma}_c^{\,2} + 8(2G + 3Ck^2)Gk^4 \,\mathrm{Ma}_c + 576(2G + 3Ck^2)k^4 \,\mathrm{Ma}_c \\ &+192(2G + 14Ck^2 - 3)Gk^6 \,\mathrm{Ma}_c + 216[8(2Ck^2 + G) - (2Ck^2 + 19)\,\mathrm{Ma}_c]i\omega_0 k^4 \,\mathrm{Ma}_c \\ &+[128(G^2 + 5GCk^2 - 2C^2k^4)Gk^4 - 384Gk^2(2G + 3Ck^2)(k^2 + \beta - \varkappa_l) \\ &+12i\omega_0(G+72)k^2 \,\mathrm{Ma}_c + 192i\omega_0(5G + 12Ck^2 - 3)Gk^2 - 576i\omega_0 G(\beta - \varkappa_l)]\varkappa_q\} \\ &/\{18\,\mathrm{Ma}_c^{\,2}\, k^2[4Ck^6 \,\mathrm{Ma}_c + Gk^4 \,\mathrm{Ma}_c - 96i\omega_0 Ck^4 - 32i\omega_0 Gk^2 - 192Ck^6 - Gk^4 \\ &+32i\omega_0 \,\mathrm{Ma}_c\, k^2 + 72\,\mathrm{Ma}_c\, k^4 - 96i\omega_0 k^2 - 48C(\beta - \varkappa_l)k^4 - 24i\omega_0(\beta - \varkappa_l)]\}, \end{aligned} \tag{C2}$$

$$b_{20} = \frac{-9\,\mathrm{Ma}_c^{\,2}\, k^6 + \varkappa_q(8G^2k^4 + 72\omega_0^{\,2})}{9\,\mathrm{Ma}_c^{\,2}\, k^4(\beta - \varkappa_l)}\,. \tag{C3}$$

The coefficients of the amplitude equation (19) can be presented as

$$\gamma = \frac{k^2 Ma_2}{6}\left[1 - i\,\frac{k^2(G+72)}{48\omega_0}\right], \tag{C4}$$

$$\tilde{K} = \frac{k^2}{6}(-G + 3Ck^2 + 6) + \frac{ik^4}{288\omega_0}\left[9Ck^2(3G-16) - 5G^2 - 15G - 432\right], \tag{C4}$$

$$K_0 = \frac{K_{01}}{2i\omega_0}\left(i\omega_0 + \frac{Ma_0}{6}k^2 + k^2 + \beta\right) - \frac{K_{02}}{4i\omega_0}Ma_0 k^2\,, \tag{C5}$$

$$K_{01} = Gk^2 + \frac{1}{2}Ma_0 k^2(2\alpha - \alpha^*) + a_{22}(G + 6Ck^2 - Ma_0\alpha^*)k^2 + 2Ma_0 k^2 b_{22}\,, \tag{C6}$$

$$
\begin{aligned}
K_{02} &= \frac{3}{4}Gk^2 - Gk^2\alpha^* + Ma_0k^2\alpha(\alpha+1) - \frac{1}{2}k^2Ma_0\alpha^* - 2Ma_0k^2\alpha\alpha^* \\
&\quad + a_{22}k^2\left(\frac{1}{2}G + 3Ck^2 - \frac{2}{3}\alpha^*G - 2\alpha^*Ck^2 + 3 - \alpha^* - \frac{1}{2}Ma_0\alpha^*\right) \\
&\quad + b_{22}k^2\left(Ma_0 - 2Ma_0\alpha^* - \frac{2}{3}G + 2\right) + i\omega_0(a_{22} - \alpha^*a_{22} + 2b_{22}) - 2\varkappa_q(\alpha^*b_{22} + \alpha b_{20})
\end{aligned}
\tag{C7}
$$

$$
\lambda_2 = \frac{2}{3}Ck^2 - \frac{i}{288\omega_0}\left[144\omega_1^2 + Ck^4(71Ck^2 + G - 27) - k^2(6G^2 + 162G + 1296)\right]. \tag{C8}
$$