

# Supplementary Material

## **1 POPULATION DYNAMICS**



**Figure S1.** Raster plot and firing rate of one isolated neural population with  $I = I_0 = 11 \ \mu A/cm^2$  (A). Linear dependency of the oscillating population frequency on the detuning  $\Delta I = I - I_0$  (B).

### 2 V-MOTIF: SYNCHRONIZATION AND PHASE-LOCKING



**Figure S2. V-motif**: Phase locking between population 1 and 2 (A-C), population 1 and 3 (D-F), and population 2 and 3 (G-I) as a function of the delay  $\tau$  and the frequency mismatch  $\Delta I$  in population 1 for different values of the synaptic strength g. The phase locking is measured with the index  $\mathcal{D}_{ij}$  which implies a better locking as its value is closer to zero (see Methods in the main text).



**Figure S3.** V-motif: Phase difference between population 1 and 2  $\theta_{12}$  (A-C), population 1 and 3  $\theta_{13}$  (D-F), and population 2 and 3  $\theta_{23}$  (G-I) as a function of the delay  $\tau$  and the frequency mismatch  $\Delta I$  in population 1 for different values of the synaptic strength g.



**Figure S4.** V-motif: Phase locking between population 1 and 2 (A-C), population 1 and 3 (D-F), and population 2 and 3 (G-I) as a function of the delay  $\tau$  and the frequency mismatch  $\Delta I$  in population 2 for different values of the synaptic strength g.



**Figure S5.** V-motif: Phase difference between population 1 and 2  $\theta_{12}$  (A-C), population 1 and 3  $\theta_{13}$  (D-F), and population 2 and 3  $\theta_{23}$  (G-I) as a function of the delay  $\tau$  and the frequency mismatch  $\Delta I$  in population 2 for different values of the synaptic strength g.

#### **3 V-MOTIF: INFORMATION TRANSMISSION**



**Figure S6.** V-motif: Zero-lag cross covariance (ZLC) of the firing rates of the second (A-C) and third (D-F) population with the slow modulation injected as function of the delay  $\tau$  and the frequency mismatch  $\Delta I$  in population 1 for different values of the synaptic strength g.



**Figure S7.** V-motif: Difference  $\Delta MI_{ij}$  between the firing rates of population 1 and 2 (A-C) and population 1 and 3 (D-F) when a slow modulation is injected as a function of the delay  $\tau$  and the frequency mismatch  $\Delta I$  in population 1 for different values of the synaptic strength g.



**Figure S8.** V-motif: Integral of the absolute value of the nPRC of the population 2 (A-C) and population 3 (D-F) when a fast signal is injectd as a function of the delay  $\tau$  and the frequency mismatch  $\Delta I$  in population 1 for different values of the synaptic strength g.



**Figure S9. V-motif**: Zero-lag cross covariance (ZLC) of the firing rates of the first (A-C) and third (D-F) population with the slow modulation injected as function of the delay  $\tau$  and the frequency mismatch  $\Delta I$  in population 2 for different values of the synaptic strength g.



**Figure S10.** V-motif: Difference  $\Delta MI_{ij}$  between the firing rates of population 2 and 1 (A-C) and population 2 and 3 (D-F) when a slow modulation is injected as a function of the delay  $\tau$  and the frequency mismatch  $\Delta I$  in population 2 for different values of the synaptic strength g.



**Figure S11. V-motif**: Integral of the absolute value fo the nPRC of the population 1 (A-C) and population 3 (D-F) when a fast modulation is injected as a function of the delay  $\tau$  and the frequency mismatch  $\Delta I$  in population 2 for different values of the synaptic strength g.



**Figure S12.** V-motif: Histograms of ZLC<sub>i</sub> and  $\Delta$ MI<sub>1i</sub> (*i*=2,3) as a function of the phase-locking index D, for  $g = g_0$ . From the central plot, one can see how both measurements are non-linearly correlated when the phase-locking index is proximal to zero (perfect locking). Furthermore, high values of both measurements coincide with almost zero values of D, implying that a better communication is achieved when the sender and receptor exhibit a constant phase difference relation.



**Figure S13.** V-motif: R-squared regression of the natural logarithmic of  $ZLC_i$  and  $\Delta MI_{1i}$  (i = 2,3) for values of the phase-locking index  $\mathcal{D}$  lower than 0.25. To avoid indeterminations, a bias term has been added to  $ZLC_i$  values before computing the logarithmic. This confirms the relation observerd in S12 between these two measurement over the condition of  $\mathcal{D}$  being proximal to zero.



**Figure S14. V-motif**: R-squared regression of the natural logarithmic of  $ZLC_i$  and  $\Delta MI_{2i}$  (i = 1,3) for values of the phase-locking index D lower than 0.25. To avoid indeterminations, a bias term has been added to  $ZLC_i$  values before computing the logarithmic.



#### **4 CIRCULAR MOTIF: SYNCHRONIZATION AND PHASE-LOCKING**

**Figure S15. Circular motif**: Phase locking between population 1 and 2 (A-D), population 1 and 3 (E-H), and population 2 and 3 (I-K) as a function of the delay  $\tau$  and the frequency mismatch  $\Delta I$  in population 1 for different values of the synaptic strength g'.



**Figure S16.** Circular motif: Phase difference between population 1 and 2  $\theta_{12}$  (A-D), population 1 and 3  $\theta_{13}$  (E-H), and population 2 and 3  $\theta_{23}$  (I-K) as a function of the delay  $\tau$  and the frequency mismatch  $\Delta I$  in population 1 for different values of the synaptic strength g'.



**Figure S17. Circular motif**: Phase locking between population 1 and 2 (A-D), population 1 and 3 (E-H), and population 2 and 3 (I-K) as a function of the delay  $\tau$  and the frequency mismatch  $\Delta I$  in population 2 for different values of the synaptic strength g.

![](_page_12_Figure_1.jpeg)

**Figure S18.** Circular motif: Phase difference between population 1 and 2  $\theta_{12}$  (A-D), population 1 and 3  $\theta_{13}$  (E-H), and population 2 and 3  $\theta_{23}$  (I-K) as a function of the delay  $\tau$  and the frequency mismatch  $\Delta I$  in population 2 for different values of the synaptic strength g'.

![](_page_13_Figure_1.jpeg)

![](_page_13_Figure_2.jpeg)

**Figure S19. Circular motif**: Zero-lag cross covariance (ZLC) of the firing rates of the first (A-D) and third (E-H) population with the slow modulation injected as function of the delay  $\tau$  and the frequency mismatch  $\Delta I$  in population 1 for different values of the synaptic strength g'.

![](_page_14_Figure_1.jpeg)

**Figure S20. Circular motif**: Difference  $\Delta MI_{ij}$  between the firing rates of population 1 and 2 (A-D) and population 1 and 3 (E-H) when a slow modulation is injected as a function of the delay  $\tau$  and the frequency mismatch  $\Delta I$  in population 1 for different values of the synaptic strength g.

![](_page_15_Figure_1.jpeg)

**Figure S21. Circular motif**: Integral of the absolute value of the nPRC of the population 2 (A-D) and population 3 (E-H) when a fast signal is injected as a function of the delay  $\tau$  and the frequency mismatch  $\Delta I$  in population 1 for different values of the synaptic strength g.

![](_page_16_Figure_1.jpeg)

**Figure S22.** Circular motif: Zero-lag cross covariance (ZLC) of the firing rates of the first (A-D) and third (E-H) population with the slow modultion injected as function of the delay  $\tau$  and the frequency mismatch  $\Delta I$  in population 2 for different values of the synaptic strength g'.

![](_page_17_Figure_1.jpeg)

**Figure S23. Circular motif**: Difference  $\Delta MI_{ij}$  between the firing rates of population 2 and 1 (A-D) and population 2 and 3 (E-H) when a slow modulation is injected as a function of the delay  $\tau$  and the frequency mismatch  $\Delta I$  in population 2 for different values of the synaptic strength g.

![](_page_18_Figure_1.jpeg)

**Figure S24.** Circular motif: Integral of the absolute value fo the nPRC of the population 1 (A-D) and population 3 (E-H) when a fast signal is injected as a function of the delay  $\tau$  and the frequency mismatch  $\Delta I$  in population 2 for different values of the synaptic strength g.