SI Appendix

Supplementary Information "Impact of Variable Speed on Collective Movement of Animal Groups"

Pascal P. Klamser[1,2], Luis Gómez Nava[1,3], Tim Landgraf[3,4], Jolle W. Jolles,[5], David Bierbach,[3,6,7], Pawel Romanczuk[1,2,3,*]

1 Institute for Theoretical Biology, Department of Biology, Humboldt Universität zu Berlin, Berlin, Germany.
2 Bernstein Center for Computational Neuroscience, 10115 Berlin, Germany.
3 Cluster of Excellence, Science of Intelligence, Technische Universität Berlin, Berlin, Germany,
4 Department of Mathematics and Computer Science, Freie Universität Berlin, Berlin, Germany.
5 Center for Ecological Research and Forestry Applications (CREAF), Campus de Bellaterra (UAB), Barcelona, Spain.
6 Department of Biology and Ecology of Fishes, Leibniz-Institute of Freshwater Ecology and Inland Fisheries, Berlin, Germany.
7 Faculty of Life Sciences, Albrecht Daniel Thaer-Institute of Agricultural and Horticultural Sciences, Humboldt Universität zu Berlin, Berlin, Germany.

I Details on experimental setups

The experimental data presented in this article was published in Jolles et al. [30] (guppies and RoboFish), Bierbach et al. [38] (single mollies) and Doran et al. [40] (groups of mollies). In the following we summarize the setups of each study.

I.1 Individual guppy and RoboFish trajectories

For this experiments, only adult female individuals (Trinidadian guppies) were used, which have a standard body length of (31.7 ± 0.8 mm). The single individual trajectories were obtained by putting single fish in a 88cm $\times$ 88cm white glass tank. The behavior was recorded over an observation period of 10 minutes using an acquisition frame rate of 30 FPS. The videos were then processed using the software BioTracker [41] to obtain the tracking of each fish.

For the RoboFish trials, a three-dimensional-printed fish replica was used. This replica was connected to a two-wheeled robot below the tank (see Fig. S1 of the SI in [30]). The robot was controlled by a closed-loop system whereby the movements of the fish were identified and fed back to the robot control. The robot was able to adjust its position and direction of motion to mimic natural responses. The robot's behavior was based on the zonal model explained in [21]. For more details on the acquisition of the experimental data, see [30].

I.2 Mollies trajectories: single individual experiments

The experiments were performed using clonal Amazon mollies using a open field circular tank (48.5cm of diameter) made of white plastic. Single fish were introduced and observed for periods of 5 minutes. Its behavior and motion were recorded and its positions were acquired with the tracking software Ethovision Version 10.1 (Noldus Information Technologies Inc.). For more details on the data acquisition, see [38].

I.3 Mollies trajectories: group experiments

For these experiments, 32 fish were used. They were assembled in groups of 4 individuals, leading to 8 groups in total. The fish were adult sized-matched mollies of a standard body lenght of (6.14 ± 0.76 cm). The experiments were

	RoboFish		Guppy		Molly		Mollies(N=4)		Model	
	F	α	F	α	F	α	F	α	F	α
$\frac{d\varphi}{dt} = F/v$	0.73		8.96		0.46		0.24		58.35	
$\frac{d\varphi}{dt} = F/(v+\alpha)$	3.81	0.31	22.21	1.11	3.51	0.31	3	0.39	129.91	2.51

Table S4: **Statistical model parameters.** Fitting parameters for the fits displayed in Fig. 2.

			standard	Fig. 2	Fig. 3	Fig. 4	Figs. 5, 6
single	preferred speed	v_0	1		[0.1, 4]		2
	speed relaxation	β	0.2		[0.25, 10.76]	[1, 64]	[0.025, 10.76]
	turn friction	α	1	0.1			
	angular noise	D_φ	1				
	velocity noise	D_v	0.4				
collective	group size	N	400	1			
	alignment strength	μ_{alg}	2	-		[0, 5.9]	
	distance strength	μ_d	2	-			
	distance slope	m_d	2	-			
	preferred distance	r_d	1	-			
simulation	simulation step	dt	0.02			0.005	0.002
	output step	dt_{out}	1				
	sample runs	N_{sam}	3	1			10
	equilibration time	t_{equi}	100	20			
	simulation time	t_{simu}	300	30000		100	
	boundary cond.	BC	open				

Table S5: Parameters used in simulations. The different columns after the "standard" column list parameters which differ from standard parameters for the simulations represented by the respective figures. "open" boundary condition (BC) means that the agents can move in an unlimited space. The equilibration time t_{equi} was prior to the simulation time t_{simu}, i.e. the total simulation time is $t_{total} = t_{simu} + t_{equi}$.

performed in a 60cm $\times$ 30cm arena. The groups were left unperturbed for periods of 5 minutes, in which their behavior was recorded using an acquisition frame rate of 30 FPS. The positions of the individuals were obtained from the videos using the tracking software Ethovision XT12 (Noldus Information Technology, Inc.). For more details on the data acquisition, see [40].

II Derivation of turning dependence on speed

Here we show in detail how to derive the dynamics of speed and turning angle from the change in velocity:

$$\frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = \frac{1}{m}\mathbf{F}_i \tag{S1}$$

with $\mathbf{v}_i$ as the velocity vector of the object, m its mass and $\mathbf{F}_i$ as the force acting on it. The velocity vector can be expressed via the speed v_i and the heading angle φ_i to $\mathbf{v}_i = v_i[\cos\varphi_i, \sin\varphi_i]^T = v_i\,\hat{\mathbf{e}}_{v,i}$. We can reformulate the velocity dynamics by the speed and heading angle dynamics to

$$\frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(v_i\,\hat{\mathbf{e}}_{v,i}) = \frac{\mathrm{d}v_i}{\mathrm{d}t}\hat{\mathbf{e}}_{v,i} + \frac{\mathrm{d}\varphi_i}{\mathrm{d}t}\frac{\mathrm{d}\hat{\mathbf{e}}_{v,i}}{\mathrm{d}\varphi_i}v_i \tag{S2}$$

$$= \frac{\mathrm{d}v_i}{\mathrm{d}t}\hat{\mathbf{e}}_{v,i} + \frac{\mathrm{d}\varphi_i}{\mathrm{d}t}\begin{bmatrix}-\sin\varphi_i\\ \cos\varphi_i\end{bmatrix}v_i \tag{S3}$$

$$= \frac{\mathrm{d}v_i}{\mathrm{d}t}\hat{\mathbf{e}}_{v,i} + \frac{\mathrm{d}\varphi_i}{\mathrm{d}t}\,\hat{\mathbf{e}}_{\varphi,i}\,v_i\ . \tag{S4}$$

where $\hat{\mathbf{e}}_{v,i} = \big[\cos\varphi_i(t), \sin\varphi_i(t)\big]^T$ and $\hat{\mathbf{e}}_{\varphi,i} = \big[-\sin\varphi_i(t), \cos\varphi_i(t)\big]^T$ are unitary vectors in the particle's heading and turning directions respectively. Thus, the change in speed and heading angle is

$$\frac{\mathrm{d}v_i}{\mathrm{d}t} = \frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} \cdot \hat{\mathbf{e}}_{v,i} = \frac{\mathbf{F}_i}{m} \cdot \hat{\mathbf{e}}_{v,i} \tag{S5}$$

$$\frac{\mathrm{d}\varphi_i}{\mathrm{d}t} = \frac{1}{v_i}\frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} \cdot \hat{\mathbf{e}}_{\varphi,i} = \frac{\mathbf{F}_i}{v_i\, m} \cdot \hat{\mathbf{e}}_{\varphi,i} \tag{S6}$$

where the upper index T indicates transposed vectors.

III Neighborhood polarization and speed reduction

In Order to understand the sudden change in individual speed at group-sizes larger than $N = 3$, we show that the dependence of the neighborhood polarization Φ_{N_i} changes qualitatively at this threshold. For group sizes

The alignment force acting on a focal agent i from Eq. 8 is

$$\mathbf{F}_{i,a}(t) = \frac{\mu_{alg}}{|\mathbb{N}_i|} \sum_{j \in \mathbb{N}_i} \mathbf{v}_{ji}(t) = \frac{\mu_{alg}}{|\mathbb{N}_i|} \sum_{j \in \mathbb{N}_i} \left(\mathbf{v}_j(t) - \mathbf{v}_i(t)\right) . \tag{S7}$$

If we now assume that each agent swims with the same speed $v_i(t) = v(t),\ \forall i$, it becomes clear that the alignment force directly depends on the neighborhood polarization $\mathbf{\Phi}_{\mathbb{N}_i}(t) = \frac{1}{|\mathbb{N}_i|}\sum_{j\in\mathbb{N}_i}\frac{\mathbf{v}_j}{|\mathbf{v}_j|} = \frac{1}{|\mathbb{N}_i|}\sum_{j\in\mathbb{N}_i}\mathbf{u}_j$:

$$\mathbf{F}_{i,a}(t) = \frac{\mu_{alg}}{|\mathbb{N}_i|} \sum_{j \in \mathbb{N}_i} \left(\mathbf{v}_j(t) - \mathbf{v}_i(t)\right) \tag{S8}$$

$$= \frac{\mu_{alg} v(t)}{|\mathbb{N}_i|} \sum_{j \in \mathbb{N}_i} \left(\mathbf{u}_j(t) - \mathbf{u}_i(t)\right) \tag{S9}$$

$$= \mu_{alg} v(t) \left(\mathbf{\Phi}_{\mathbb{N}_i}(t) - \mathbf{u}_i(t)\right) . \tag{S10}$$

The alignment force acts on the speed of the focal agent i:

$$\frac{\mathrm{d}v_i(t)}{\mathrm{d}t} = \beta\big(v_0 - v_i(t)\big) + F_{i,v}(t) + \sqrt{2D_v}\ \xi_v(t) \tag{S11}$$

$$\propto \mathbf{F}_{i,a}(t) \cdot \mathbf{u}_i(t) \propto \left(\mathbf{\Phi}_{\mathbb{N}_i}(t) - \mathbf{u}_i(t)\right) \cdot \mathbf{u}_i(t) \tag{S12}$$

$$\propto \mathbf{\Phi}_{\mathbb{N}_i}(t) \cdot \mathbf{u}_i(t) - 1 \tag{S13}$$

$$\propto \Phi_{\mathbb{N}_i} \cos\angle_{\mathbf{\Phi}_{\mathbb{N}_i},\mathbf{u}_i} - 1 . \tag{S14}$$

Thus, the stronger the heading direction deviates from the mean neighborhood heading direction $\hat{\mathbf{\Phi}}_{\mathbb{N}_i} = \mathbf{\Phi}_{\mathbb{N}_i}/\Phi_{\mathbb{N}_i}$, the stronger is the speed decreased. Of course, the neighborhood polarization $\Phi_{\mathbb{N}_i}$ affects the speed, the less polarized the neighborhood the stronger the speed decrease.

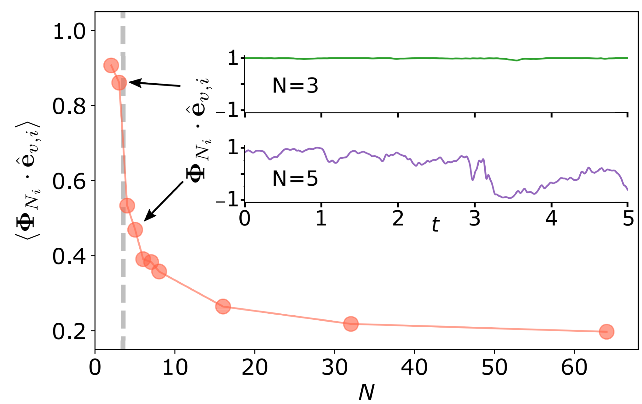

Figure S1: **Directional synchronization with neighbors.** Average vector product of an agents i heading direction $\hat{\mathbf{e}}_{v,i}$ and the polarization vector of its neighborhood $\mathbf{\Phi}_{\mathbb{N}_i}$. If each agent is perfectly synchronized in its direction with its neighborhood, the vector product is 1. The time series of the vector product reveals a distinct difference between groups of $N \leq 3$ and $N > 3$.