frontiers

Supplementary Material

1 The correction of nodal acceleration for the rainfall boundary condition

When the prescribed seepage velocity is known, i.e., rainfall intensity (mm/hr), the nodal accelerations of each phase ($\boldsymbol{a}_s^{n,k}$, $\boldsymbol{a}_l^{n,k}$) are adjusted in order to comply with the rainfall data after the computational cycle in the second step (II). For the correction of nodal acceleration, the temporarily nodal accelerations ($\widetilde{\boldsymbol{a}}_s^{n,k}$, $\widetilde{\boldsymbol{a}}_l^{n,k}$) are obtain assuming that $\boldsymbol{F}_l^{ext}$ and $\boldsymbol{F}_s^{ext}$ equals to zero in the computational cycle of second step (II). Then, $\widetilde{\boldsymbol{a}}_s^{n,k}$ and $\widetilde{\boldsymbol{a}}_l^{n,k}$ are used to calculated the temporarily nodal velocities ($\widetilde{\boldsymbol{v}}_s^{n,k+1}$, $\widetilde{\boldsymbol{v}}_l^{n,k+1}$) using equation (12). Because rainfall boundary condition is imposed at soil surface, as visualized in Figure 3., the outer normal unit vector at node ($\boldsymbol{n}^{n,k}$) is required and can be determined by

$$\boldsymbol{n}^{n,k} = \frac{\sum_{i=1}^{N^p}\left[\left(1-n^{i,k}\right)\rho_s + n^i S_l^{i,k}\rho_l\right]\nabla N^{i,k} V^{i,k}}{\left|\sum_{i=1}^{N^p}\left[\left(1-n^{i,k}\right)\rho_s + n^i S_l^{i,k}\rho_l\right]\nabla N^{i,k} V^{i,k}\right|}. \qquad \text{(A)}$$

If the temporarily seepage velocity is smaller than the prescribed seepage velocity, which reads by

$$\left[n^{n,k} S_l^{n,k}\left(\widetilde{\boldsymbol{v}}_l^{n,k+1} - \widetilde{\boldsymbol{v}}_s^{n,k+1}\right) - \boldsymbol{w}^{n,k}\right]\cdot \boldsymbol{n}^{n,k} \le 0, \qquad \text{(B)}$$

$\widetilde{\boldsymbol{a}}_s^{n,k}$ and $\widetilde{\boldsymbol{a}}_l^{n,k}$ have to correct by the compensation of velocities along the normal direction ($\Delta\boldsymbol{v}_l^{n,k+1}$, $\Delta\boldsymbol{v}_s^{n,k+1}$). In (B), $n^{n,k}$ and $S_l^{n,k}$ are the nodal porosity and saturation, and these can be mapping from MP using (9). The compensation of velocities along the normal direction is the difference between the temporarily seepage velocity and the prescribed seepage velocity, which reads as

$$\left\{n^{n,k} S_l^{n,k}\left[\left(\widetilde{\boldsymbol{v}}_l^{n,k+1} + \Delta\boldsymbol{v}_l^{n,k+1}\boldsymbol{n}^{n,k}\right) - \left(\widetilde{\boldsymbol{v}}_s^{n,k+1} + \Delta\boldsymbol{v}_s^{n,k+1}\boldsymbol{n}^{n,k}\right)\right] - \boldsymbol{w}^{n,k}\right\}\cdot \boldsymbol{n}^{n,k} = 0, \qquad \text{(C)}$$

and it has to satisfy with the conservation of momentum of the mixture as follow

$$\left(\sum_{i=1}^{N^p} n^i S_l^{i,k}\rho_l N^{i,k}\right)\Delta\boldsymbol{v}_l^{n,k+1} + \left(\sum_{i=1}^{N^p}\left(1-n^{i,k}\right)\rho_s N^{i,k}\right)\Delta\boldsymbol{v}_s^{n,k+1} = 0. \qquad \text{(D)}$$

Hence, $\Delta\boldsymbol{v}_l^{n,k+1}$ and $\Delta\boldsymbol{v}_s^{n,k+1}$ can be obtained using (C) and (D). With help of these, the nodal accelerations of each phases can be corrected as follow:

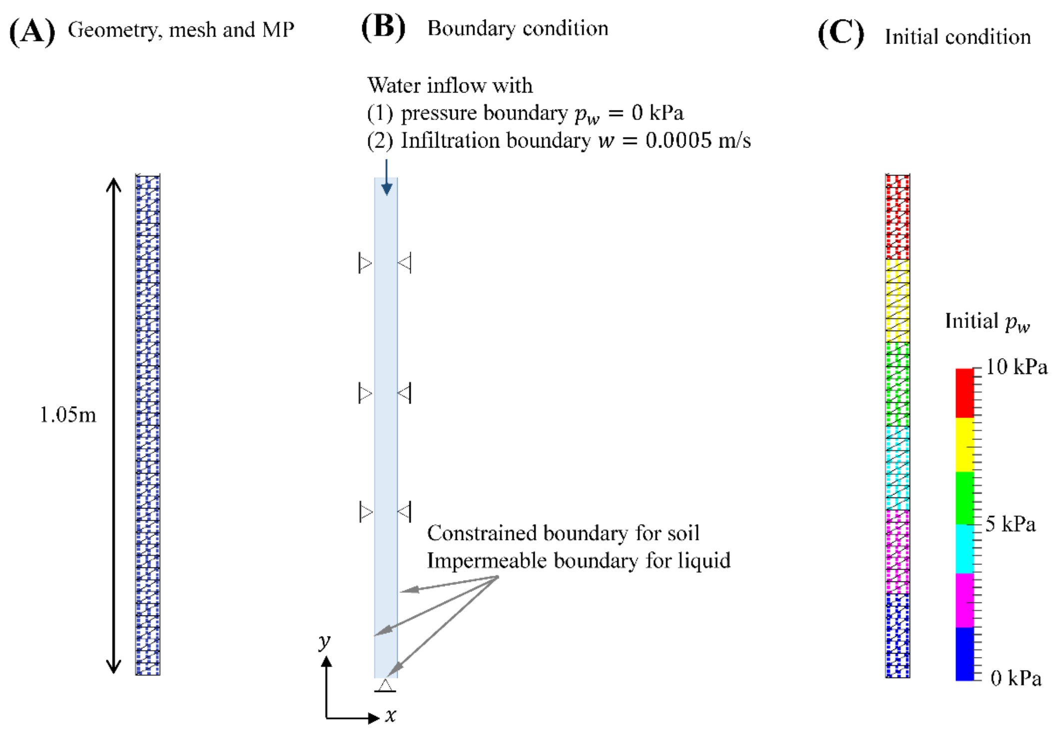
$$\boldsymbol{a}_l^{n,k} = \frac{\left(\widetilde{\boldsymbol{v}}_l^{n,k+1} + \Delta\boldsymbol{v}_l^{n,k+1}\boldsymbol{n}^{n,k}\right) - \boldsymbol{v}_l^{n,k}}{\Delta t}, \boldsymbol{a}_s^{n,k} = \frac{\left(\widetilde{\boldsymbol{v}}_s^{n,k+1} + \Delta\boldsymbol{v}_s^{n,k+1}\boldsymbol{n}^{n,k}\right) - \boldsymbol{v}_s^{n,k}}{\Delta t}. \qquad \text{(E)}$$

2 Validation of the rainfall boundary condition

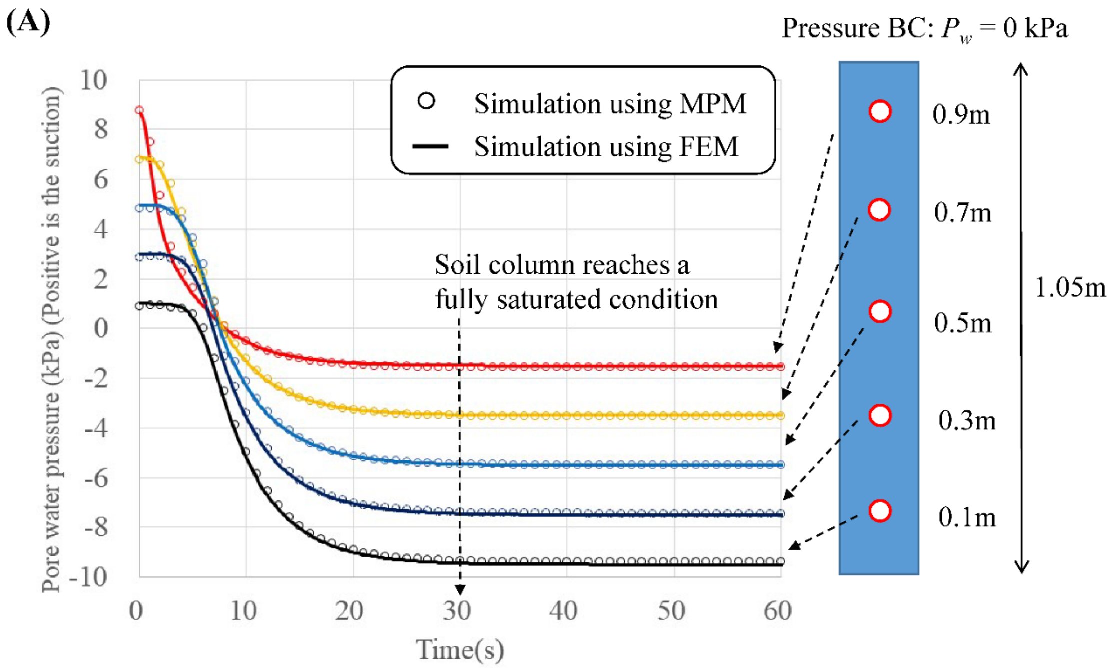
In this section, 1-D infiltration problems in a van Genuchten soil model are performed to validate the MPM code. Since the analytical solution is difficult to derive, the MPM results are compared against the FE commercial PLAXIS 2D code (Galavi, 2010).

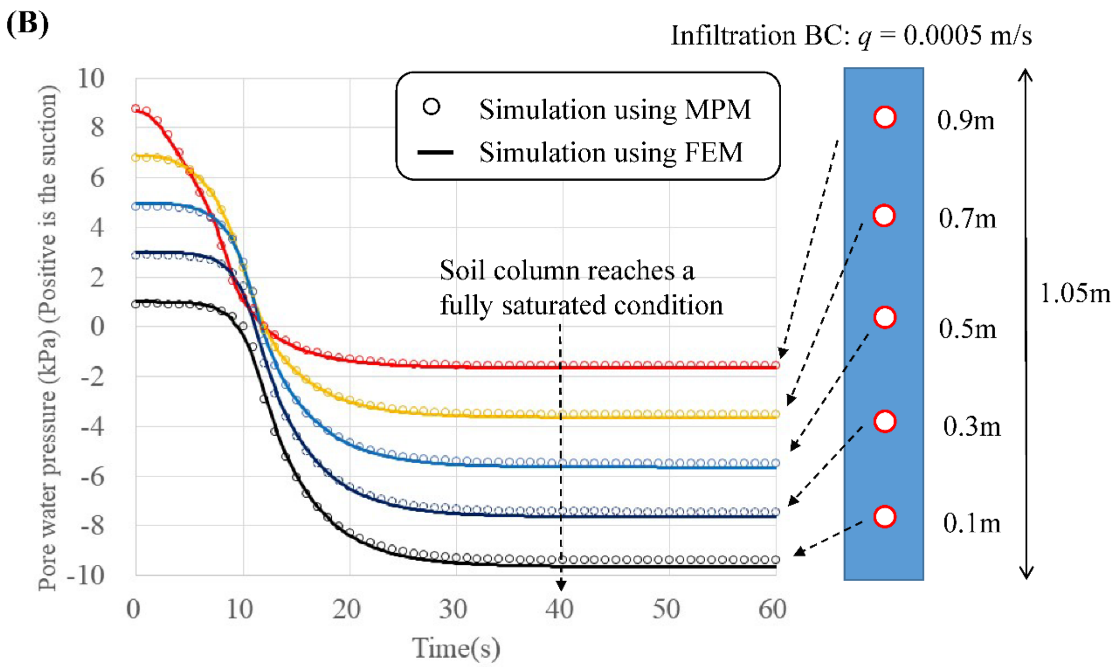
The height of soil in the column is 1.05 m; the computational domain is discretized using triangular elements, and 6 material points are initialized in each element, as shown in supplementary Figure 1-A. Due to the symmetry of the problem, the soil movement and water flow can only occur along the vertical direction (y axis), as shown in supplementary Figure 1-B. Two conditions are specified on the top surface of the soil column: (1) $p_w = 0$ kPa, i.e. "ponding" condition, and (2) infiltration velocity w=0.0005 m/s. The soil parameters are listed in Table A, which correspond to a silty sand. The hydraulic conductivity is set at $2\text{x}10^{-4}$ m/s. The densities of solid grain and water are given as 2700 kg/m^3 and 1000 kg/m^3, respectively. The porosity is set at 0.33. The Poisson's ratio is given as 0.

The results are illustrated in supplementary Figure 2, where the hollow circle represents the simulation using MPM, and the solid line indicates the simulation using FEM. The MPM results are in a good agreement with the FEM results for both "ponding" and the infiltration boundary condition.



Supplementary Figure 1. 1D consolidation of unsaturated flow in soil: (A) Geometry, mesh, and material point; (B) Assignment of boundary conditions; (C) Assignment of initial condition.





Supplementary Figure 2. Comparison of MPM and FEM: (A) Pressure boundary condition; (B) Infiltration boundary condition.

Table A. Material parameters for 1D infiltration

Parameters	Unit	Value
Young's modulus (E)	kPa	5000
Poisson's ratio (ν)	-	0
Density of solid grain (ρ_s)	kg/m^3	1000
Density of water (ρ_l)	kg/m^3	2700
Porosity (n)	-	0.33
Bulk modulus of water (K_l)	kPa	200000
Dynamic viscosity of liquid (μ_l)	kPa·s	0.000001
Intrinsic permeability (k_l)	m^2	0.00000000002
Air-entry pressure (p_0)	kPa	22.295
Empirical parameter (λ)	-	0.671
Residual degree of saturation (S_{min})	-	0.23
Maximum degree of saturation (S_{max})	-	1