**SUPPLEMENTARY MATERIAL**

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**Supplementary Appendix SA. Kuusk’s Model for Hot Spot Effect**

A single scattering approximation of radiance, leaving the canopy through the upper boundary (*z*=0) can be estimated as ([Kuusk 1991](#_ENREF_3)), p. 141,

|  |  |
| --- | --- |
|  | (SA1) |

Here is the flux density of direct solar beam; stands for the BiDirectional Gap Propability (BDGP); denotes scattering anisotropy at depth *z* defined as

|  |  |
| --- | --- |
|  | (SA2) |

 signifies the extinction coefficient; , , and represent leaf area scattering phase function, geometry factor, leaf albedo (or single scattering albedo) and leaf area volume density, respectively. The following assumptions underlie a model for BDGP proposed in ([Kuusk 1991](#_ENREF_3)): (a) the geometry factor, leaf area scattering phase function and leaf area volume density do not depend on *z* and (b) the cross-correlation coefficient, which characterizes correlation between the directions of photon flight, can be approximated by an exponential function with a leaf-size dependent parameter, i.e., . Under these assumptions the BDGP becomes

|  |  |
| --- | --- |
|  | (SA3) |

Here and where represents the canopy Hot Spot coefficient defined as

|  |  |
| --- | --- |
|  | (SA4) |

with and

|  |  |
| --- | --- |
|  | (SA5) |

Note that we use instead of the Hot Spot factor, , introduced in ([Kuusk 1991](#_ENREF_3)). These variables are related as where is the optical distance between point at depth *z* and top of the canopy in the direction (cf. Eq. 9a in ([Kuusk 1991](#_ENREF_3))). Here we approximate by a constant, i.e., , where denotes a hot spot parameter and is an effective canopy depth. In such an approximation the effective depth depends on SZA.

The above model can be expressed as a solution of 1D radiative transfer equation. Indeed, where satisfies the following boundary value problem,

|  |  |
| --- | --- |
|  | (SA6) |

with no incoming radiation through the upper () and lower () boundaries, i.e., for downward directions () and for upward directions (). The canopy leaving radiances are solutions of **Eq. SA6** at the canopy top and bottom, namely,

|  |  |
| --- | --- |
|  | (SA7a) |
|  | (SA7b) |

Here , and . **Eqs. SA7** are used to approximate the directional escape and recollision probabilities.

**Supplementary Appendix SB. Escape and Recollision Probabilities**

Consider a vegetation canopy with not absorbing leaves, i.e., . Let and be the horizontal average radiance of photons scattered *m* times and associated mean irradiance on leaf sides. The directional escape and recollision probabilities for photons scattered *m* times are defined as and . Here represents the upper boundary, , in the case of upward directions and the surface beneath the canopy, , for downward directions. The probabilities are related as = ([Yang et al. 2017](#_ENREF_6); [Huang et al. 2008](#_ENREF_2)). The sequence convergences to the unique positive eigenvalue of the radiative transfer equation, corresponding to the unique positive (normalized to unity) eigenvector ([Huang et al. 2007](#_ENREF_1); [Vladimirov 1963](#_ENREF_5)). For the vegetation canopy illuminated by a mono-directional solar beam, . Here is the canopy interceptance.

The Directional Area Scattering Function (DASF) can be expanded in successive order of scattering, or in Neumann series ([Yang et al. 2017](#_ENREF_6); [Huang et al. 2007](#_ENREF_1)), i.e.,

|  |  |
| --- | --- |
|  | (SB1) |

Here , . Since , the geometric mean, , also convergences to . To quantify contributions of each term in our Neumann series, we introduce weights, , defined as where Obviously, the weights sum to unity.

The average recollision, , and directional escape, , probabilities are defined as ([Stenberg, Mõttus, and Rautiainen 2016](#_ENREF_4); [Yang et al. 2017](#_ENREF_6))

|  |  |
| --- | --- |
|  | (SB2) |

Note that can also be expressed as . Solving this equation for *X*, one gets and consequently . Thus, the contributions of summands in **Eqs. SB2** decrease as a geometric progression with the common ratio . In terms of these notations, **Eq. SB1** rearranges to **Eq. 1**.

**Supplementary Appendix SC. Single Scattering Approximation**

The directional escape probability for single scattered photons is for up- and for downward directions. Upon integrating **Eq. SA6** over the canopy space and unit sphere and normalizing by one obtains

|  |  |
| --- | --- |
|  | (SC1) |

Terms in the numerator on the left-hand side are fluxes of radiation leaving the canopy through the upper (term ) and lower () boundaries, namely

|  |  |
| --- | --- |
|  | (SC2) |

Here and denote up- and downward hemispheres of directions. We use this equation to estimate .