Supplementary Material

TISSUE DEGRADATION MODEL

The tissue degradation model used in this work has been developed by Daniel Balzani and coworkers (Balzani et al., 2012; Anttila et al., 2019). In a previous study, we have used this model to study the interaction between tissue degradation and blood flow (Wang et al., 2021). Here we give a brief outline of this method.

The total strain energy function of the tissue degradation model is chosen to be (Balzani et al., 2012)

$$\Psi^{\mathrm{tot}} = \Psi^{\mathrm{iso}} + \Psi^{\mathrm{vol}} + \sum_{i=1}^{2} \Psi_i^{\mathrm{ti}}. \tag{S1}$$

Here, $\Psi^{\mathrm{iso}} = c_1(I_1/I_3^{1/3} - 3)$ describing the ground matrix material (also known as elastin) as incompressible Neo-Hookean; $\Psi^{\mathrm{vol}} = \epsilon_1(I_3^{\epsilon_2} + 1/I_3^{\epsilon_2} - 2)$ (Hartmann and Neff, 2003) serving as volumetric penalty function to account for near-incompressibility. $I_1 = \mathrm{tr}\boldsymbol{C}$ and $I_3 = \mathrm{det}\boldsymbol{C}$ denote the first and the third invariants of the right Cauchy-Green tensor $\boldsymbol{C} = \boldsymbol{F}^{\mathrm{T}}\boldsymbol{F}$, while $\boldsymbol{F}$ is the deformation gradient tensor. The effective elasticity tensor is computed via $\mathbb{C} = 4\partial^2_{\boldsymbol{CC}}\Psi$ (Balzani et al., 2012).

The effect of damage is accounted for across two phenomenological fiber families $i = 1, 2$ as

$$\Psi_i^{\mathrm{ti}} = \alpha_1 \langle (1 - D_i)[\kappa I_1 + (1 - 1.5\kappa) K_3^i] - 2 \rangle^{\alpha_2}, \tag{S2}$$

describing the material behavior of collagen fibers. D_i are scalar damage functions for each fiber family i used to capture remnant strains (i.e., strain at zero stress level after unloading) within the fibers and the stress-softening effect. The Macaulay brackets, $\langle (\cdot) \rangle = [(\cdot) + |(\cdot)|]/2$, filter out positive values. $K_3^i = \mathrm{tr}[\mathrm{cof}\boldsymbol{C}(\boldsymbol{1} - \boldsymbol{M_i})]$ is the fundamental polyconvex function (Schröder and Neff, 2003) with the definition of the structural tensor $\boldsymbol{M_i} = \boldsymbol{A_i} \otimes \boldsymbol{A_i}$, given in terms of each fiber direction vector $\boldsymbol{A}_i$. The cofactor is defined as $\mathrm{cof}\boldsymbol{C} = \mathrm{det}\boldsymbol{C}\boldsymbol{C}^{-1}$. Here, $c_1,\ \epsilon_1,\ \epsilon_2,\ \alpha_1$, α_2 and κ are material parameters.

The damage function D_i for a fiber family i is defined as

$$D_i(\beta_i) = D_{\mathrm{s},\,i}\left[1 - \exp\left(\frac{\ln(1 - r_{\mathrm{s}})}{\beta_{\mathrm{s}}}\beta_i\right)\right], \tag{S3}$$

where the maximally reachable damage value for fixated load levels is denoted by $D_{\mathrm{s},\,i} \in [0, 1)$, the fraction of the maximum damage is $r_{\mathrm{s}} = 0.99$, and $\beta_{\mathrm{s}} > 0$ is the value of the internal variable β_i corresponding to damage saturation. The internal variable is defined as $\beta_i = \langle \tilde{\beta}_i - \tilde{\beta}_i^{\mathrm{ini}} \rangle$, with $\tilde{\beta}_i = \int_0^t \langle \dot{\Psi}_i^{\mathrm{ti},\,0}(s) \rangle \mathrm{d}s$ allowing for continuous damage evolution for loading and re-loading paths and $\tilde{\beta}_i^{\mathrm{ini}}$ denoting the damage initiation threshold. t indicates the time at the current state, and $\dot{\Psi}_i^{\mathrm{ti},\,0}$ is the first time derivative of the fictitiously undamaged (effective) strain energy density $\Psi_i^{\mathrm{ti},0}$ in fiber direction i. The maximally reachable damage value for fixated load levels $D_{\mathrm{s},\,i}$ is expressed as

$$D_{\mathrm{s},\,i}(\gamma) = D_\infty\left[1 - \exp\left(\frac{\ln(1 - r_\infty)}{\gamma_\infty}\gamma_i\right)\right], \tag{S4}$$

with the predefined converging limit for the overall damage value $D_\infty \in [0,1)$ and $\gamma_\infty > 0$ representing the value of the internal variable γ_i reached at the limit fraction $r_\infty = 0.99$.

In order to ensure that $D_{\mathrm{s},\,i}$ remains unchanged during a cyclic process under fixed maximum load levels, the second internal variable

$$\gamma_i = \max_{s\in[0,t]} \langle \Psi_i^{\mathrm{ti},\,0}(s) - \Psi_{\mathrm{ini},\,i}^{\mathrm{ti},\,0} \rangle \tag{S5}$$

is defined as the maximum value of the effective energy reached up to the current state at t. $\Psi_{\mathrm{ini},\,i}^{\mathrm{ti},\,0}$ denotes the effective strain energy density at an initial damage state obtained at the limit of the physiological domain. The damage saturation criterion is expressed as

$$\phi_i := \langle \Psi_i^{\mathrm{ti},\,0}(s) - \Psi_{\mathrm{ini},\,i}^{\mathrm{ti},\,0} \rangle - \gamma_i \leq 0. \tag{S6}$$

In this model, different degradation intensities can be simulated by adapting the damage parameter γ_∞. A small degree of damage can be simulated by increasing the value of γ_∞ (Wang et al., 2021). Compared with $\gamma_\infty = 11$ kPa, we use $\gamma_\infty = 18$ kPa to simulate a smaller degradation (which may represent the accumulation (overall effect) of damage for a shorter time).

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