APPENDIX A: The parameterization of MDT

In this study, the MDT is parameterized by the LBF in a regular rectangular mesh grid, and the bilinear polynomials containing 4 parameters are taken as the basis function.

Considering the maximum expansion of satellite-only GGM, the estimated MDT will be parameterized into 1 ° rectangular grids. And in order to reduce the correlation between the grids, the MSS and GGM are extracted into 0.5 ° grids, respectively (Becker, 2012). In each rectangular grid, a local coordinate system is setup, where the four nodes have the coordinates of (0, 0), (1, 0), (1, 1) and (0, 1) indexed counterclockwise from the left bottom node. The coordinate of the point $\left(\bar{\theta}, \bar{\lambda}\right)$ in the rectangular grid will be transformed to this local system through an affine transformation:

$$\theta = \frac{\bar{\theta} - \theta_1}{\theta_3 - \theta_1}, \lambda = \frac{\bar{\lambda} - \lambda_1}{\lambda_3 - \lambda_1} \tag{1}$$

where $\left(\theta_1, \lambda_1\right), \left(\theta_2, \lambda_2\right), \left(\theta_3, \lambda_3\right)$ are the nodes starting from the left bottom counterclockwise. The MDT's parameterization using the bilinear polynomials in Cartesian coordinates is given by:

$$\boldsymbol{MDT}\left(\theta, \lambda\right) = \kappa_1 + \kappa_2 \lambda + \kappa_3 \theta + \kappa_4 \lambda \theta = \begin{bmatrix} 1 & \lambda & \theta & \lambda\theta \end{bmatrix} \cdot \begin{bmatrix} \kappa_1 \\ \kappa_2 \\ \kappa_3 \\ \kappa_4 \end{bmatrix}, \tag{2}$$

where $\left\{\kappa_i\right\}, i = 1, 2, 3, 4$ are the coefficients of the parameterization polynomials. The four nodes in the rectangular grid could be expressed as

$$\begin{bmatrix} MDT\left(0,0\right) \\ MDT\left(0,1\right) \\ MDT\left(1,1\right) \\ MDT\left(1,0\right) \end{bmatrix} == \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \kappa_1 \\ \kappa_2 \\ \kappa_3 \\ \kappa_4 \end{bmatrix} \tag{3}$$

Consequently, the $\left\{\kappa_i\right\}, i = 1, 2, 3, 4$ could be solved based on Eq.(3), and the MDT could be parameterized as

$$\boldsymbol{MDT}(\theta, \lambda) = \begin{bmatrix} 1 & \lambda & \theta & \lambda\theta \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} MDT\left(0,0\right) \\ MDT\left(0,1\right) \\ MDT\left(1,1\right) \\ MDT\left(1,0\right) \end{bmatrix} \tag{4}$$

APPENDIX B: The determination of $\mathbf{A}_{cs}$ and $\mathbf{A}_{mdt}$

In this study, the MSS is represented as the sum of geoid height ($\mathbf{N}$) and MDT ($\boldsymbol{\eta}$):

$$\begin{aligned}\mathbf{MSS}+\mathbf{v}&=\mathbf{N}+\boldsymbol{\eta}\\&=[\mathbf{A}_{cs}\quad \mathbf{A}_{mdt}]\begin{bmatrix}\mathbf{x}_{cs}\\ \mathbf{x}_{mdt}\end{bmatrix}\end{aligned} \tag{5}$$

In order to construct observation equation, the geoid height and the MDT are parameterized using the SHs and basis function with 4 parameters, respectively, which can be given as:

$$\begin{aligned}\mathbf{N}&=\frac{GM}{R\gamma(B)}\sum_{n=0}^{N}\sum_{m=0}^{n}\left(\frac{R}{r}\right)^{n+1}\overline{P}_{nm}(\cos\theta)\left(\overline{C}_{nm}\cos(m\lambda)+\overline{S}_{nm}\sin(m\lambda)\right)\\&=\frac{GM}{R\gamma(B)}\Bigg[\left(\frac{R}{r}\right)^{1}\overline{P}_{00}(\cos\theta)\cos(m\lambda)\quad\left(\frac{R}{r}\right)^{1}\overline{P}_{00}(\cos\theta)\sin(m\lambda)\quad\left(\frac{R}{r}\right)^{2}\overline{P}_{10}(\cos\theta)\cos(m\lambda)\quad\left(\frac{R}{r}\right)^{2}\overline{P}_{10}(\cos\theta)\sin(m\lambda)\quad\cdots\\&\qquad\cdots\quad\left(\frac{R}{r}\right)^{N+1}\overline{P}_{NN}(\cos\theta)\cos(m\lambda)\quad\left(\frac{R}{r}\right)^{N+1}\overline{P}_{NN}(\cos\theta)\sin(m\lambda)\Bigg]\cdot\begin{bmatrix}\overline{C}_{00}\\ \overline{S}_{00}\\ \overline{C}_{10}\\ \overline{S}_{10}\\ \vdots\\ \overline{C}_{NN}\\ \overline{S}_{NN}\end{bmatrix}\\&=\mathbf{A}_{cs}\cdot\mathbf{X}_{cs}\end{aligned} \tag{6}$$

$$\begin{aligned}\boldsymbol{\eta}&=\begin{bmatrix}1-\lambda-\theta+\lambda\theta & \lambda-\lambda\theta & \theta & \theta-\lambda\theta\end{bmatrix}\cdot\begin{bmatrix}MDT(0,0)\\ MDT(0,1)\\ MDT(1,1)\\ MDT(1,0)\end{bmatrix}\\&=\boldsymbol{A}_{mdt}\cdot\boldsymbol{X}_{mdt}\end{aligned} \tag{7}$$

Accordingly, the MSS is represented by the geoid height and the MDT in a matrix format.

APPENDIX C: The estimation of $\mathbf{x}_{mdt}$ based on the Schur complement theory

In this study, the authors tried to use the LS method for MDT estimation, and we only focus on the estimation of $\mathbf{x}_{mdt}$. The design matrix can be written as:

$$\begin{bmatrix} \mathbf{A}_{cs1}^T\mathbf{K}_{MSS}^{-1}\mathbf{A}_{cs1}+\mathbf{K}_{cs1}^{-1} & \mathbf{A}_{cs1}^T\mathbf{K}_{MSS}^{-1}\mathbf{A}_{cs2} & \mathbf{A}_{cs1}^T\mathbf{K}_{MSS}^{-1}\mathbf{A}_{mdt} \\ \mathbf{A}_{cs2}^T\mathbf{K}_{MSS}^{-1}\mathbf{A}_{cs1} & \mathbf{A}_{cs2}^T\mathbf{K}_{MSS}^{-1}\mathbf{A}_{cs2}+\mathbf{K}_{cs2}^{-1} & \mathbf{A}_{cs2}^T\mathbf{K}_{MSS}^{-1}\mathbf{A}_{mdt} \\ \mathbf{A}_{mdt}^T\mathbf{K}_{MSS}^{-1}\mathbf{A}_{cs1} & \mathbf{A}_{mdt}^T\mathbf{K}_{MSS}^{-1}\mathbf{A}_{cs2} & \mathbf{A}_{mdt}^T\mathbf{K}_{MSS}^{-1}\mathbf{A}_{mdt}+\nabla\mathbf{A}_x^T\nabla\mathbf{A}_x+\nabla\mathbf{A}_y^T\nabla\mathbf{A}_y \end{bmatrix}\cdot\begin{bmatrix}\mathbf{x}_{cs1}\\ \mathbf{x}_{cs2}\\ \mathbf{x}_{mdt}\end{bmatrix} = \begin{bmatrix}\mathbf{A}_{cs1}^T\mathbf{K}_{MSS}^{-1}\cdot\mathbf{MSS}+\mathbf{K}_{cs1}^{-1}\cdot\mathbf{GGM}_{cs1}\\ \mathbf{A}_{cs2}^T\mathbf{K}_{MSS}^{-1}\cdot\mathbf{MSS}\\ \mathbf{A}_{mdt}^T\mathbf{K}_{MSS}^{-1}\cdot\mathbf{MSS}\end{bmatrix} \tag{8}$$

in short:

$$\begin{bmatrix}\mathbf{N}_{cs} & \mathbf{N}_{cs,mdt}\\ \mathbf{N}_{mdt,cs} & \mathbf{N}_{mdt}\end{bmatrix}\begin{bmatrix}\mathbf{x}_{cs}\\ \mathbf{x}_{mdt}\end{bmatrix}=\begin{bmatrix}\mathbf{n}_{cs}\\ \mathbf{n}_{mdt}\end{bmatrix} \tag{9}$$

Based on our test, the inversion of $\mathbf{N}_{cs}$ is stable (full rank and condition number is ~1e8), so according to the Schur complement theory (Haynsworth, 1968), $\mathbf{x}_{mdt}$ can be solved directly since there is no ill condition problems for $\mathbf{N}_{cs}$:

$$\mathbf{x}_{mdt}=\left(\mathbf{N}_{mdt}-\mathbf{N}_{mdt,cs}\mathbf{N}_{cs}^{-1}\mathbf{N}_{cs,mdt}\right)^{-1}\cdot\left(\mathbf{n}_{mdt}-\mathbf{N}_{mdt,cs}\mathbf{N}_{cs}^{-1}\mathbf{n}_{cs}\right) \tag{10}$$