Supplementary Material

Table S1: Model equations solved in simulations (Katsman et al., 2013; Katsman, 2015)

Equation	**Formulation**	
Force Equilibrium Equation	$-\nabla \cdot \sigma = \overrightarrow{F_g}$	(S1)
Solute Conservation Equation	$\frac{\partial C_{CH_4}(aq)}{\partial t} + \nabla \cdot \left[- D \nabla C_{CH_4}(aq)) \right] = 0$	(S2)
Conservation of gaseous CH_4 inside the bubble	$\frac{\partial \left(C_{CH_4}(g) V_b \right)}{\partial t} = \int_{\alpha} \vec{n} \cdot (-D \phi \nabla C_{CH_4}(aq)) d\alpha$	(S3)
Pore-water concentration of dissolved CH_4 at the bubble surface (Henry's Law)	$C_{CH_4}(aq)(\alpha, t) = C_{CH_4}(g) \cdot k_H^{cc}$	(S4)
Bubble volume	$V_b = \int_{\alpha} \vec{n} \cdot \vec{u} d\alpha$	(S5)
Pressure of gaseous CH_4 in the bubble	$P_b = C_{CH_4}(g) RT$	(S6)
Mode I stress intensity factor (SIF) at the bubble front	$K_I = \frac{E}{4(1-\nu^2)} \sqrt{\frac{\pi}{2d}} 2 w_n^P$	(S7)
Distribution of crack increments along crack front (at fracturing)	$\Delta a = \Delta a_{max} \cdot \left(\frac{K_I}{K_{Imax}} \right)^2$	(S8)

Note: σ – Cauchy stress tensor; $\vec{F}_g$ - gravity load; $C_{CH_4}(aq)$ - local aqueous CH_4 concentration in pore waters; $C_{CH_4}(g)$ - gaseous CH_4 concentration in the bubble; V_b - bubble volume; ϕ - effective porosity of sediment; k_H^{cc} - dimensionless Henry's law constant defined as $k_H \cdot R \cdot T$, where k_H is Henry Law constant, R is gas constant, T is temperature,; $\vec{n}$ – normal vector at bubble surface, α; $\vec{u}$ - local elastic displacement at the bubble surface; P_b - uniform gas pressure in the bubble, t - time. The parameter D is tortuosity-corrected diffusion coefficient of methane in bulk sediment, related to molecular diffusion of methane (D_m) in free-solution, as $D = \frac{D_m}{\tau^2}$ (Berner, 1980), with dimensionless tortuosity factor $\tau = \sqrt{1 - 2 \ln \phi}$ (Boudreau, 1997). Also, ν is Poisson's ratio, E is Young's modulus and d is distance of points P (modeling setup, Figure 1) prescribed on the crack surface from crack front, where w_n^P (projection of the displacement $\vec{w}^p$, in direction to normal to the crack surface) is calculated to determine SIF, K_I. K_{Imax} is maximum value of SIF, K_I, along crack font, and Δa_{max} is maximum crack increment along crack front, prescribed to be smaller than dimension of plastic zone near crack tip (Citarella and Crici, 2010; Katsman et al., 2013).

Table S2: Geochemical and mechanical parameters of sediment used in simulations (input data)

Input Parameter	**Value**

Dissolved aqueous pore space CH_4 concentration (C_{CH_4}(aq); as at 1 m sediment depth, NRL site, Eckernförde Bay, Summer conditions, Martens et al., 1998)	0.1 kg m^{-3}
Molecular diffusion coefficient of CH_4 in free solution (D_m; Iversen and Jørgensen, 1993)	10^{-9} m^2 s^{-1}
Sediment bulk density (ρ; Silva and Brandes, 1998)	1240 kg m^{-3}
Effective porosity (ϕ; Sevee, 2010; Mitchell and Soga, 2005)	0.2
Young's modulus (E; Algar and Boudreau, 2010; Barry et al., 2012; Barry et al., 2013)	$5.5{\cdot}10^5$ Pa
Poission's ratio (v; L'Esperance et al., 2013; Dorgan et al., 2007)	0.45
Henry's constant (k_H; Sander, 1999)	$9.2 \cdot 10^{-5}$ mol m^{-3} Pa^{-1}
Temperature (T)	298.15 K

Text S1: Quantification of bubble expansion and contraction under oscillating pressure field

Oscillating pressure field of surface waves causes bubbles to perform periodic expansion and contraction, synchronized with wave loadings. Such motion for penny-shaped bubbles can be quantified by estimating the normal displacements at bubble's (crack) surface, w_n, which is a function of superposition of normal stresses at crack surface:

$$\Delta\sigma = P_b - \sigma_y \quad \text{(S9)}$$

where σ_y is an ambient compressive local stress from sediment to bubble surface, P_b is uniform inner bubble pressure. Under the uniaxial (vertical) strain boundary condition (as prescribed in present model), σ_y is a function of the vertical remote stress (σ_z), $\sigma_y = \frac{\nu}{1-\nu} \cdot \sigma_z$, where ν is Poisson's ratio (Katsman et al., 2013; Katsman 2015). Due to effect of gravity, σ_z rises linearly with depth and is given as, $\sigma_z = \sigma_z^0 + \rho_s g(h_s - z)$, where ρ_s is density of bulk sediment, g - gravity, h_s - height of modeled sediment cell above origin (located at center of bubble's initial geometry; Figure 1) and z is local vertical coordinate of the point under consideration. Therefore, $\Delta\sigma = \left(P_b - \sigma_y^{con}\right) + \sigma_y^{lin}$, were $\sigma_y^{con} = \frac{\nu}{1-\nu} \cdot (\sigma_z^0 + \rho_s g h_s)$ is the constant component of σ_y, and $\sigma_y^{lin} = \frac{\nu}{1-\nu} \cdot \rho_s \cdot g \cdot z$ quantifies the linear variation induced by gravity in sediment. Thus, for a 3D penny-shaped bubble, the total w_n, is obtained by superimposing displacements due to constant normal load, $P_b - \sigma_y^{con}$, and due to linearly varying load, σ_y^{lin}. Following Eq. 17 and 18 from Katsman (2015) (and also, Atroshenko, 2010), w_n can be specified as:

$$w_n(r,\theta) = 2c\frac{(1-\nu^2)}{E}\sqrt{1-r^2}\left[\frac{(P_b-\sigma_y^{con})}{E(k)} - \frac{k^2}{(1-2k^2)E(k)-k'^2K(k)} \cdot \frac{r\,a\,\cos\theta}{z}\,\sigma_y^{lin}\right] \quad \text{(S10)}$$

where E is Young's modulus; $K(k)$, $E(k)$ are complete elliptic integrals of first and second kinds, respectively; k' is ratio of semi minor axis (c) to semi major axis (a), $k = \sqrt{1 - k'^2}$; and r, θ denotes the non-dimensional radius and polar angle of points at the bubble surface, respectively (see, Katsman 2015). Evolution of normal displacements at bubble surface ($w_n(r, \theta)$), determines the accrued stress concentration around the crack front, which is quantified in our model as Mode I stress intensity factor (SIF), K_I (Eq. S7).

Table S3: Input conditions used in additional simulations focused on water depth verifications (performed under a very shallow water depth (0.5 m) associated with near shore aquatic sites).

Run no	Mean water column height, H_{eq} (m)	Wave amplitude, A (m)	Wave periods, T (s)	Wave amplitude to water column height ratio ($\bar{r} = A/H_{eq}$)	Bubble maturity time, t_m (s)	$\bar{t}$
S1	0.5	0.22	0	NA	123 sec	NA
S2	0.5	0.22	3	0.44	86 sec	30.08 %

Note: In this modelled theoretical scenario no sediment displacement by waves in the shallow water environment is assumed.

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