Supplementary Material for: Experimental investigation of quantum uncertainty relations with classical shadows

1 DERIVATIONS OF QUANTUM UNCERTAINTY RELATIONS IN TERMS OF RELATIVE ENTROPY OF COHERENCE

Considering a two-particle system, where particle $\mathcal{A}$ is the particle of interest entangled with a particle $\mathcal{B}$ acting as quantum memory, Berta et al. (2010) proposed to use the conditional Von Neumann entropy $H(A|\mathcal{B})(H(B|\mathcal{B}))$ to characterize the uncertainty of measurement outcomes of observable $A(B)$ obtained in the presence of information in $\mathcal{B}$. The entropic uncertainty relations proposed by Berta et al. (2010) is

$$H(A|\mathcal{B}) + H(B|\mathcal{B}) \geq -\log_2 c + H(\mathcal{A}|\mathcal{B}), \tag{S1}$$

where $H(\mathcal{A}|\mathcal{B})$ is the conditional Von Neumann entropy between $\mathcal{A}$ and $\mathcal{B}$. By considering particle $\mathcal{B}$ as a purification of particle $\mathcal{A}$, Yuan et al. (2015) proved

$$\begin{aligned} C^{\mathbb{A}}_{\mathrm{RE}}(\rho_{\mathcal{A}}) &= H(A|\mathcal{B}) = H(A) - S_{\mathrm{VN}}(\rho_{\mathcal{A}}) \\ C^{\mathbb{B}}_{\mathrm{RE}}(\rho_{\mathcal{A}}) &= H(B|\mathcal{B}) = H(B) - S_{\mathrm{VN}}(\rho_{\mathcal{A}}). \end{aligned} \tag{S2}$$

Thus, Eq. S1 can be expressed in terms of the reletive entropy of coherence

$$C^{\mathbb{A}}_{\mathrm{RE}}(\rho) + C^{\mathbb{B}}_{\mathrm{RE}}(\rho) \geq -\log_2 c - S_{\mathrm{VN}}(\rho). \tag{S3}$$

The entropic uncertainty relations proposed by S*á*nches-Ruiz (1998) and Korzekwa et al. (2014) are

$$H(A) + H(B) \geq -\frac{1+\sqrt{2c'^2-1}}{2}\log_2\left(\frac{1+\sqrt{2c'^2-1}}{2}\right) - \frac{1-\sqrt{2c'^2-1}}{2}\log_2\left(\frac{1-\sqrt{2c'^2-1}}{2}\right), \tag{S4}$$

$$H(A) + H(B) \geq -\log_2 c + S_{\mathrm{VN}}(\rho)[2 + \log_2 c], \tag{S5}$$

with $c'^2 = c = \max_{i,j} |\langle a_i|b_j\rangle|^2$. Using $h(x) = -x\log_2 x - (1-x)\log_2(1-x)$ and relations in Eq. S2 , Eq. S4 and Eq. S5 can be rewritten in terms of relative entropy of coherence by

$$C^{\mathbb{A}}_{\mathrm{RE}}(\rho) + C^{\mathbb{B}}_{\mathrm{RE}}(\rho) \geq h\left(\frac{1+\sqrt{2c-1}}{2}\right) - 2S_{\mathrm{VN}}(\rho) \tag{S6}$$

$$C^{\mathbb{A}}_{\mathrm{RE}}(\rho) + C^{\mathbb{B}}_{\mathrm{RE}}(\rho) \geq -[1 - S_{\mathrm{VN}}(\rho)]\log_2 c \tag{S7}$$

2 MORE EXPERIMENTAL RESULTS

2.1 Fidelities of prepared states

For each prepared state $\rho(\tau)$, we reconstructed the density matrix $\rho^{\exp}(\tau)$ with $N_s = 2000$ samples using QST, and the fidelity of the prepared states is calculated by $\mathcal{F} = \mathrm{Tr}\sqrt{\sqrt{\rho(\tau)}\rho^{\exp}(\tau)\sqrt{\rho(\tau)}}$. The results are shown in TABLE S1.

τ	$\mathcal{F}$
0	0.9999±0.0005
0.1	0.9998±0.0005
0.2	0.9998±0.0003
0.3	0.9992±0.0003
0.4	0.9967±0.0006
0.5	0.9997±0.0003
0.6	0.9995±0.0002
0.7	0.9993±0.0002
0.8	0.9993±0.0002
0.9	0.9996±0.0002
1	0.9941±0.0006

Table S1. Fidelities of the experimentally prepared states.

2.2 Comparison of purity with CS algorithm and QST

In this section, we give the comparison of purity obtained from CS algorithm and QST with the same N_s samples. For each prepared state $\rho^{\exp}(\tau)$, we calculate the purity using the same number of samples N_s in two algorithms. The accuracy of estimated purity is reflected by the distances (error) between experimental value and its ideal value. The average errors over 11 prepared states are shown in Figure S1. We observe that the advantage of classical shadow algorithm still hold when $N_s \leq 800$ even for the single-qubit states.

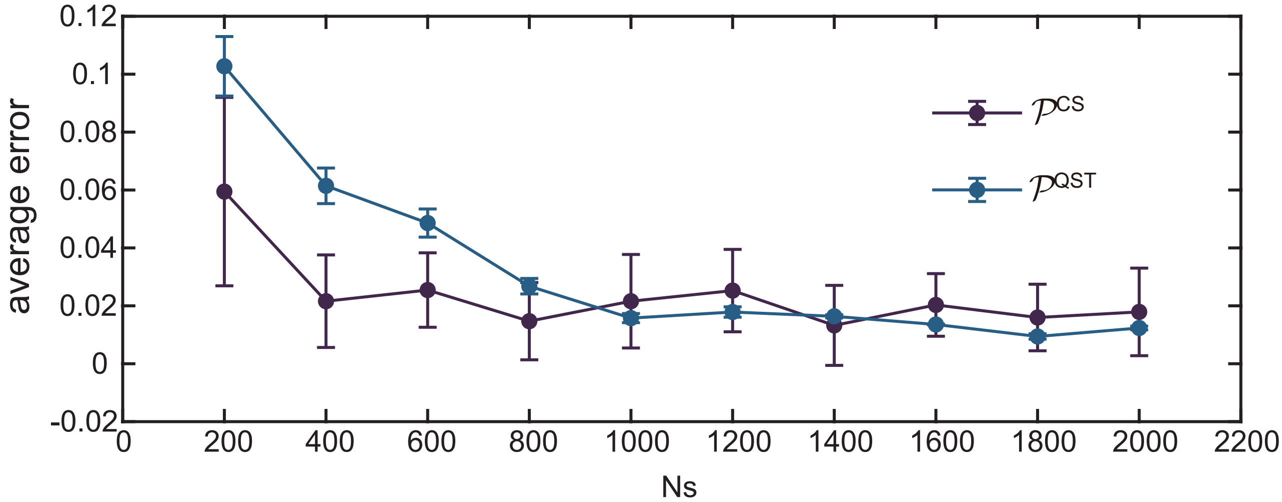

Figure S1. The results of the accuracy of purity obtained using the same number of samples N_s with CS algorithm and QST, respectively

REFERENCES

Berta M, Christandl M, Colbeck R, Renes JM, Renner R. The uncertainty principle in the presence of quantum memory. *Nature Physics* **6** (2010) 659–662. doi:10.1038/nphys1734.

Yuan X, Zhou H, Cao Z, Ma X. Intrinsic randomness as a measure of quantum coherence. *Phys. Rev. A* **92** (2015) 022124. doi:10.1103/PhysRevA.92.022124.

Sánches-Ruiz J. Optimal entropic uncertainty relation in two-dimensional hilbert space. *Physics Letters A* **244** (1998) 189–195. doi:https://doi.org/10.1016/S0375-9601(98)00292-8.

Korzekwa K, Lostaglio M, Jennings D, Rudolph T. Quantum and classical entropic uncertainty relations. *Phys. Rev. A* **89** (2014) 042122. doi:10.1103/PhysRevA.89.042122.