F-matrix

The tangential field direction at the intercept is naturally given by $\boldsymbol{s}_1 = \hat{\boldsymbol{k}} \times \hat{\boldsymbol{N}}$ while its normal can be derived via $\boldsymbol{s}_2 = \hat{\boldsymbol{N}} \times \boldsymbol{s}_1$. Together with the electric and magnetic fields, one can construct a matrix for the convenient calculation of reflection and transmission coefficients, which I will name here in compliance with Chipman, Lam and Young ([?]) as F-matrix. With $\boldsymbol{E}_{\mathrm{tf}}$, $\boldsymbol{E}_{\mathrm{ts}}$, $\boldsymbol{E}_{\mathrm{rf}}$, and $\boldsymbol{E}_{\mathrm{rs}}$ being the transmitted-fast, transmitted-slow, reflected-fast, and reflected-slow electric field, that matrix can be expressed as:

$$\boldsymbol{F} = \begin{pmatrix} \boldsymbol{s}_1 \cdot \hat{\boldsymbol{E}}_{\mathrm{tf}} & \boldsymbol{s}_1 \cdot \hat{\boldsymbol{E}}_{\mathrm{ts}} & \boldsymbol{s}_1 \cdot \hat{\boldsymbol{E}}_{\mathrm{rf}} & \boldsymbol{s}_1 \cdot \hat{\boldsymbol{E}}_{\mathrm{rs}} \\ \boldsymbol{s}_2 \cdot \hat{\boldsymbol{E}}_{\mathrm{tf}} & \boldsymbol{s}_2 \cdot \hat{\boldsymbol{E}}_{\mathrm{ts}} & \boldsymbol{s}_2 \cdot \hat{\boldsymbol{E}}_{\mathrm{rf}} & \boldsymbol{s}_2 \cdot \hat{\boldsymbol{E}}_{\mathrm{rs}} \\ \boldsymbol{s}_1 \cdot \boldsymbol{H}_{\mathrm{tf}} & \boldsymbol{s}_1 \cdot \boldsymbol{H}_{\mathrm{ts}} & \boldsymbol{s}_1 \cdot \boldsymbol{H}_{\mathrm{rf}} & \boldsymbol{s}_1 \cdot \boldsymbol{H}_{\mathrm{rs}} \\ \boldsymbol{s}_2 \cdot \boldsymbol{H}_{\mathrm{tf}} & \boldsymbol{s}_2 \cdot \boldsymbol{H}_{\mathrm{ts}} & \boldsymbol{s}_2 \cdot \boldsymbol{H}_{\mathrm{rf}} & \boldsymbol{s}_2 \cdot \boldsymbol{H}_{\mathrm{rs}} \end{pmatrix}, \tag{1}$$

where The magnetic fields are derived via

$$\boldsymbol{H} = n\boldsymbol{K} \cdot \boldsymbol{E}. \tag{2}$$

Note, that we don't have to use the normalized vector representation in case of the magnetic fields. The resulting transmission and reflection coefficients $t_{\mathrm{f/s}}$ and $r_{\mathrm{f/s}}$ can be derived by applying field-conservation when passing the intercept, whereas each coefficient is valid for both tangential and normal component. Thus,

$$\begin{aligned} &\boldsymbol{t} = \boldsymbol{F}^{-1} \cdot \boldsymbol{C}, \qquad \text{where} \\ &\boldsymbol{t} = \begin{pmatrix} t_{\mathrm{tf}} \\ t_{\mathrm{ts}} \\ t_{\mathrm{rf}} \\ t_{\mathrm{rs}} \end{pmatrix} \qquad \text{and} \qquad \boldsymbol{C} = \begin{pmatrix} \boldsymbol{s}_1 \cdot \hat{\boldsymbol{E}}_{\mathrm{inc}} \\ \boldsymbol{s}_2 \cdot \hat{\boldsymbol{E}}_{\mathrm{inc}} \\ \boldsymbol{s}_1 \cdot \boldsymbol{H}_{\mathrm{inc}} \\ \boldsymbol{s}_2 \cdot \boldsymbol{H}_{\mathrm{inc}} \end{pmatrix}. \end{aligned} \tag{3}$$

It is important to note that the $\boldsymbol{C}$-matrix of an incoming field has to be calculated for the two eigenpolarizations of the incoming medium (in an isotropic environment, these would be the S and P polarization). Hence, there are two transmission matrices $\boldsymbol{t}_1$ and $\boldsymbol{t}_2$ for either eigenpolarization of the incoming field.