




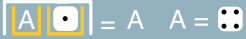











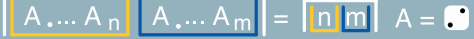





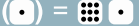















Supplementary Figures

	Symbolic	James	Symbolic	Black-and-white Full Shorthand	
(A)	B^A	(A)	B^A	[A]	
	$\log(A)$	[A]	$\log(A)$	[A]	
	A	A	$B^{\log(A)}$	[[A]]	A
	A	A	$\log(B^A)$	[[A]]	A
	-A	<A>	$-B^{\log(A)}$	[[A]]	A
	-A	<A>	$-\log(B^A)$	[[A]]	A
(B)	$-B^A$	<(A)>	$-B^A$	[A]	
	$-\log(A)$	<[A]>	$-\log(A)$	[A]	
	--A	<<A>>	$-B^{\log(-A)}$	[[[A]]]	[[A]]
	$\log(-A)$	[<A>]	$\log(-A)$	[[[A]]]	[A]

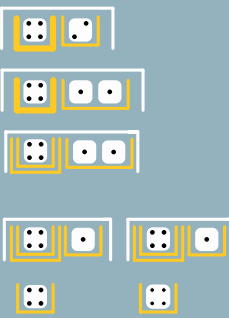
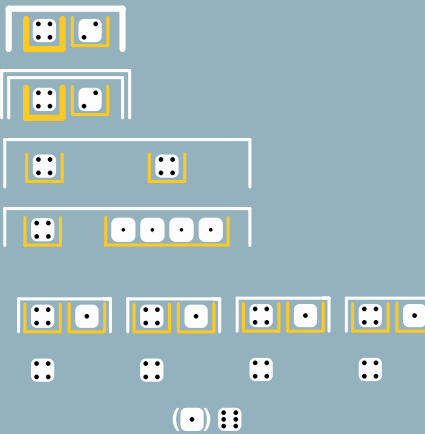
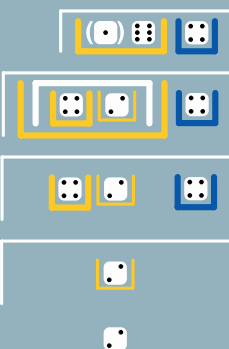
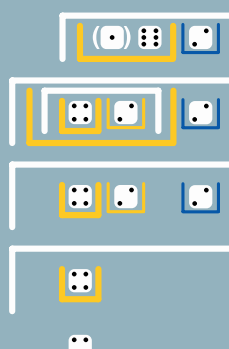
Supplementary Figure 1. A copy of Figure 6 in bracket notation. The bracket notation has the advantage that, as nesting increases, the notation expands horizontally but not vertically. Yet it has the disadvantage that it is less iconic, less clear.

Basis for next step	Next step	Symbolic interpretation
$(A_1 + \dots + A_n) / (A_1 + \dots + A_m)$	$\boxed{A \dots A_n} \boxed{A \dots A_m}$	$(A_1 + \dots + A_n) / (A_1 + \dots + A_m)$
$A = \boxed{A} \boxed{\bullet}$ (several times)	$\boxed{\boxed{A} \boxed{\bullet} \dots \boxed{A} \boxed{\bullet}}_n \boxed{\boxed{A} \boxed{\bullet} \dots \boxed{A} \boxed{\bullet}}_m$	$((A \times 1)_1 + \dots + (A \times 1))_n / ((A \times 1)_1 + \dots + (A \times 1))_m$
$\boxed{A} \boxed{B} \boxed{A} \boxed{C} \dots = \boxed{A} \boxed{B C \dots}$ with $B = C = \dots = \bullet$	$\boxed{\boxed{A} \boxed{\bullet} \dots \boxed{\bullet}}_n \boxed{\boxed{A} \boxed{\bullet} \dots \boxed{\bullet}}_m$	$(A \times (1_1 + \dots + 1_n)) / (A \times (1_1 + \dots + 1_m))$
rearranging dots into (unspecified) numbers n and m	$\boxed{\boxed{A}} \boxed{n} \boxed{\boxed{A}} \boxed{m}$	$(A \times n) / (A \times m)$
$\boxed{B} = B$ with $B = \boxed{A} \boxed{n}$	$\boxed{A} \boxed{n} \boxed{A} \boxed{m}$	$(A \times n) / (A \times m)$
$\boxed{B} = \boxed{B}$ with $B = \boxed{A} \boxed{m}$	$\boxed{A} \boxed{n} \boxed{\boxed{A}} \boxed{m}$	$(A \times n) / (A \times m)$
$\boxed{B C} = \boxed{B} \boxed{C}$ with $B = \boxed{A}$ $C = \boxed{m}$	$\boxed{A} \boxed{n} \boxed{\boxed{A}} \boxed{\boxed{m}}$	$(A \times n) / A / m$
$\boxed{B} = B$ with $B = \boxed{A}$ $\boxed{C} = C$ with $C = \boxed{m}$	$\boxed{A} \boxed{n} \boxed{A} \boxed{m}$	$(A \times n) / A / m$
$B B = \bullet$ with $B = \boxed{A}$	$\boxed{n} \boxed{m}$	n / m

Supplementary Figure 2. Proof of the last additional theorem of Figure 7. The justification of each next step in the derivation is shown (left), the step itself (middle), and a symbolic interpretation in terms of multiplication and division (right). A mathematically equivalent symbolic interpretation in terms of powers and logarithms would be more straightforward but less instructive, and due to symbolic math's rather baroque notation of powers and logarithms (see Figure 7), also considerably harder to read. All logarithms and exponents are assumed to have the same arbitrary base B. Keeping in mind that $\log(1)=0$, the other additional theorems of Figure 7 are considered self-evident and left without proof.

Basis for next step	Next step	Symbolic interpretation
(A) 4 x 2 rearrange  A =  B = C =   rearrange	    	4 x 2 4 x (1 + 1) (4 x 1) + (4 x 1) 4 + 4 8
(B) 6 / 2 rearrange  	   	6 / 2 (2+2+2) / 2 3 / 1 3
(C) 2 / 6 rearrange  	   	2 / 6 2 / (2 + 2 + 2) 1 / 3 $\frac{1}{3}$
(D) -173 / 41  rearrange  rearrange  C =  A =   A =  A = 	       	- (100+70+3)/(40+1) - (90+10+70+3)/(40+1) - (41+41+41+41+9) / 41 - (41+41+41+41)/41 - (9/41) - (1+1+1+1)/1 - 9/41 - 4 $\frac{9}{41}$

Supplementary Figure 3. A copy of Figure 8 in which, for added clarity, white and black upright containers have been replaced with yellow and blue ones.

Basis for next step	Next step	Symbolic interpretation
(A) $\log(4) \times 2$ rearrange change notation $[A B C] = [A B] [A C]$ $A = \begin{array}{ c } \hline \bullet \bullet \\ \hline \end{array}$ $B = C = \begin{array}{ c } \hline \bullet \\ \hline \end{array}$ $[A \bullet] = A$ $A = \begin{array}{ c } \hline \bullet \bullet \\ \hline \end{array}$ (twice)		$\log(4) \times 2$ $\log(4) \times (1 + 1)$ $\log(4) \times (1 + 1)$ $\log(4) \times 1 + \log(4) \times 1$ $\log(4) + \log(4)$
(B) 4^2 change notation using the final result of the previous example's derivation rearrange $[A B C \dots] = [A B] [A C] \dots$ $A = \begin{array}{ c } \hline \bullet \bullet \\ \hline \end{array}$ $B = C = D = E = \begin{array}{ c } \hline \bullet \\ \hline \end{array}$ $[A \bullet] = A$ $A = \begin{array}{ c } \hline \bullet \bullet \\ \hline \end{array}$ (four times) rearrange		$B^{(\log(4) \times 2)}$ $B^{(\log(4) \times 2)}$ 4×4 $4 \times (1 + 1 + 1 + 1)$ $(4 \times 1) + (4 \times 1) + (4 \times 1) + (4 \times 1)$ $4 + 4 + 4 + 4$ 16
(C) $\log_4(16)$ using the 4^2 -example's derivation in reverse order $[A] = A$ $A = \begin{array}{ c } \hline \bullet \bullet \bullet \bullet \\ \hline \end{array}$ $A A = \blacksquare$ $A = \begin{array}{ c } \hline \bullet \bullet \\ \hline \end{array}$ $[A] = A$ $A = \begin{array}{ c } \hline \bullet \\ \hline \end{array}$		$\log(16) / \log(4)$ $\log(4) / \log(4)$ $\log(4) \times 2 / \log(4)$ $B^{(\log(2))}$ 2
(D) $\sqrt{16}$ using the 4^2 -example's derivation in reverse order $[A] = A$ $A = \begin{array}{ c } \hline \bullet \bullet \bullet \bullet \\ \hline \end{array}$ $A A = \blacksquare$ $A = \begin{array}{ c } \hline \bullet \bullet \\ \hline \end{array}$ $[A] = A$ $A = \begin{array}{ c } \hline \bullet \\ \hline \end{array}$		$16^{1/2}$ $(4^2)^{1/2}$ $4^{(2/2)}$ $B^{(\log(4))}$ 4

Supplementary Figure 4. A copy of Figure 9 in which, for added clarity, white and black upright containers have been replaced with yellow and blue ones.