

## Supplementary Material

## 1 ADDITIONAL PPD RESULTS

Posterior Predictive Distribution (PPD) figures for all nuclei analyzed in this work, and similar to Fig. 8 in the original paper, are shown in Figs. S1 and S2.

In Figs. S3, S4 and S5 we show the energy residuals for the different nuclei in our work at each order with error bars. We note here that to get the error bars we added the experimental error, the theory truncation error and the error that comes from sampling the LECs in quadrature. To get the truncation error we calculate the theory error at each point in the sample space and then marginalize over the theory error parameters, W,  $\bar{c}_{odd}$  and  $\bar{c}_{even}$ . We also note that since the errors are highly correlated across levels, one cannot determine whether or not the errors are too large—it's not clear how many actual degrees of freedom we have.



**Figure S1.** The posterior predictive distribution of energy-level residuals for several different nuclei. The dark and light red bands show the truncation error plus the experimental error at 68% and 95% credible levels respectively. The lighter blue lines connect the energy residuals calculated from the distribution of the LECs. The solid black line represents the median of the distribution and the dashed lines indicate the 16th and 84th percentiles. The correlation shown on the plot is the highest correlation between any LEC and any error-model parameter.  $I_{br}$  was determined from the distribution of W. The dashed purples line show the lower and upper limits of  $I_{br}$  while the solid purple line shows the median. The insert on the plot shows the residuals on the first 5 levels with an altered y-axis scale.



**Figure S2.** The posterior predictive distribution of energy-level residuals for several different nuclei. The dark and light red bands show the truncation error plus the experimental error at 68% and 95% credible levels respectively. The lighter blue lines connect the energy residuals calculated from the distribution of the LECs. The solid black line represents the median of the distribution and the dashed lines indicate the 16th and 84th percentiles. The correlation shown on the plot is the highest correlation between any LEC and any error-model parameter.  $I_{br}$  was determined from the distribution of W. The dashed purples line show the lower and upper limits of  $I_{br}$  while the solid purple line shows the median. The insert on the plot shows the residuals on the first 5 levels with an altered y-axis scale.



Figure S3. Energy residuals for the different nuclei in our work. The error bars are determined by combining the experimental error, the theory truncation error and the error that comes from sampling the LECs.



**Figure S4.** Energy residuals for the different nuclei in our work. The error bars are determined by combining the experimental error, the theory truncation error and the error that comes from sampling the LECs.



**Figure S5.** Energy residuals for the different nuclei in our work. The error bars are determined by combining the experimental error, the theory truncation error and the error that comes from sampling the LECs.