VTI anisotropic media inversion method based on the exact reflection coefficient equation

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**Appendix**

Appendix A

The Cauchy distribution and modified Cauchy distribution are show in equation (A-1). From the expression of modified Cauchy distribution, when the model parameters r increases, .

. (A-1)

**Figure A-1** displays the comparison of the sensitivity to the model parameter between the Gauss distribution, Cauchy distribution, modified Cauchy distribution and Exponential distribution. As shown in the figure, the modified Cauchy distribution has "long tails" on both sides compared to the other three common distributions. This helps to retain the weak reflection information in the data, thus protecting the thin reflection layer information and improving the resolution of the inversion.



**Figure A-1** Comparison between the constraint criterions of the Gauss distribution, Cauchy distribution, modified Cauchy distribution and Exponential distribution.

We adopted the modified Cauchy distribution as the prior distribution, which is given by:

, (A-2)

where *Φi*=(*Di*)T*ψ−1Di*, of which *ψ* is a 5×5 covariance matrix that contains the statistical correlations among the five parameters and can be used for the elimination of the statistical correlations, *μ* is the means of model parameters, and *N* is the size of model parameters. Because the inversion method is ultimately converted into solving the perturbations of the five elastic parameters, the covariance matrix *ψ* is the covariance matrix of the perturbations in the process of practical calculation. Matrix *D* is a 5×5*N* matrix defined as:

. (A-3)

According to Bayes' theorem, the posterior distribution for model parameters is meet with: *P*(*m*|*d*) ∝*P*(*m*|*d*) *P*(*m*). Substituting the likelihood distribution in equation (12) and the prior distribution in equation (A-2) into Bayes' theorem and ignoring normalization constants, we can get:

, (A-4)

where, .

One solution of the inverse problem is the maximizing the posterior distribution, which is equivalent to minimizing the following objective function:

. (A-5)

It is assumed that the noise terms for the observed data are uncorrelated and have a Gaussian distribution. Therefore, the covariance matrix in the objective function can be simplified as a diagonal matrix. The noise variance of seismic data is σpp So equation (A-5) can be simplified as:

, (A-6)

where *β*=*σpp*controls the weight of the priori information.

The objective function is a complex nonlinear expression about the elastic parameters. Considering the factors such as the volume of calculation, the convergence speed, and convergence accuracy, we use the idea of generalized linear inversion to solve the above problems in order to obtain the updated iterative formula for five elastic parameters. Compared with the method of using the second-order Taylor expansion to unfold the objective function directly, this method has quicker convergence. In ‘Constructing objective function based on Bayesian theory’ part paragraph 1, the forward modeling can be written in classical form of ***d***=***Gm***+***n***, using a Taylor Series expansion on the forward operator ***G*** at the initial model parameters *m0*, we obtain the equation (19).

Appendix B

We statistically analyzed the logging data of the work area to see the feasibility of inversion using Cauchy distribution. The results are shown in **Figure B-1**. In the figure, the curve of the traditional Gaussian distribution is also plotted. It can be seen that the Cauchy distribution can better portray the distribution of the parameters. This shows the rationality to assume the model parameters are Cauchy distribution.

|  |  |  |  |
| --- | --- | --- | --- |
| **(A)** |  | **(B)** |  |
| **(C)** |  |  |  |
| **(D)** |  | **(E)** |  |

**Figure B-1** Probability density distribution statistical results calculated by logging data, the red line represents Gaussian distribution, the black line represents Cauchy distribution. Each parameter: **(A)** P-wave velocity; **(B)** S-wave velocity; **(C)** density; **(D)** *ε*; **(E)** *δ*.