Supplementary Material

Early warning signals of financial crises using persistent homology and critical slowing down: Evidence from difference correlation tests

# Supplementary Data

List of financial crises in the US market from 12/06/1990 until 03/27/2018

1. The global mini crash caused by 1997 Asian economic crisis (10/27/1997)

2. Dotcom crash (03/10/2000)

3. September 11 attack (09/11/2001)

4. 2002 stock market downturn (10/09/2002)

5. Chinese stock bubble of 2007 (02/27/2007)

6. US bear market of 2007 (10/11/2007)

7. Lehman Brothers bankruptcy (09/15/2008)

8. 2009 Dubai debt standstill (11/27/2009)

9. 2010 flash crash (03/06/2010)

10. 2015-2016 Chinese stock market crash (06/12/2015)

11. 2015-2016 US stock market sell-off (08/18/2015)

# Supplementary Tables

Table 1 Results obtained by using Kendall tau correlations of mean for power spectrum at low frequencies of $L^{1}$-norm time series.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Event number** | **Duration** | $τ$ **exceed the threshold (Yes/No)** | **The nearest financial crisis (first day of the crisis)** | **Early warning signal (EWS), false alarm (FA), false positive (FP), or false negative (FN)** |
| 1 | 12/22/1987 – 04/03/1991 | No | None | TN |
| 2 | 04/04/1991 – 11/06/1991 | Yes | None | FA |
| 3 | 11/07/1991 – 03/27/1997 | No | None | TN |
| 4 | 03/31/1997 – 04/28/1997 | Yes | None | FA |
| 5 | 04/29/1997 – 10/27/1997 | No | The global mini crash caused by the 1997 Asian economic crisis (10/27/1997) | FA |
| 10/28/1997 – 12/17/1998 | No | None | TN |
| 6 | 12/18/1998 – 04/30/1999 | Yes | None | FA |
| 7 | 05/03/1999 – 03/08/2000 | No | None | TN |
| 8 | 03/09/2000 – 03/10/2000 | Yes | Dotcom crash (03/10/2000) | EWS (1 day before the crisis) |
| 03/11/2000 – 09/11/2001 | Yes | September 11 attack (09/11/2001) | EWS (549 days before the crisis) |
| 09/12/2001 – 09/21/2001 | Yes | None | FA |
| 9 | 09/24/2001 – 10/09/2002 | No | 2002 stock market downturn (10/09/2002) | FN |
| 10/10/2002 – 02/27/2007 | No | Chinese stock bubble of 2007 (02/27/2007) | FN |
| 02/28/2007 – 10/11/2007 | No | US bear market of 2007 (10/11/2007) | FN |
| 10/12/2007 – 07/30/2008 | No | None | TN |
| 10 | 07/31/2008 – 09/15/2008 | Yes | Lehman Brothers bankruptcy (09/15/2008) | EWS (46 days before the crisis) |
| 09/16/2008 – 09/17/2009 | Yes | None | FA |
| 11 | 09/18/2009 – 11/27/2009 | No | 2009 Dubai debt standstill (11/27/2009) | FN |
| 11/28/2009 – 03/06/2010 | No | 2010 flash crash (03/06/2010) | FN |
| 03/07/2010 – 05/29/2012 | No | None | TN |
| 12 | 05/30/2012 – 07/26/2012 | Yes | None | FA |
| 13 | 07/27/2012 – 12/02/2014 | No | None | TN |
| 14 | 12/03/2014 – 06/10/2015 | Yes | None | FA |
| 15 | 06/11/2015 – 12/29/2017 | No | 2015-2016 Chinese stock market crash (06/12/2015) | FN |
| 06/11/2015 – 12/29/2017 | No | 2015-2016 US stock market sell-off (08/18/2015) | FN |
| 06/11/2015 – 12/29/2017 | No | None | TN |

Table 2 Results obtained by using Pearson correlations of mean for power spectrum at low frequencies of $L^{1}$-norm time series.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Event number** | **Duration** | $r$ **exceed the threshold (Yes/No)** | **The nearest financial crisis (first day of the crisis)** | **Early warning signal (EWS), false alarm (FA), false positive (FP), or false negative (FN)** |
| 1 | 12/22/1987 – 03/27/1991 | No | None | TN |
| 2 | 03/28/1991 – 11/04/1991 | Yes | None | FA |
| 3 | 11/05/1991 – 10/27/1997 | No | The global mini crash caused by the 1997 Asian economic crisis (10/27/1997) | FN |
| 10/28/1997 – 05/11/1999 | No | None | TN |
| 4 | 05/12/1999 – 07/16/1999 | Yes | None | FA |
| 5 | 07/19/1999 – 02/22/2000 | No | None | TN |
| 6 | 02/23/2000 – 03/10/2000 | Yes | Dotcom crash (03/10/2000) | EWS (16 days before the crisis) |
| 03/11/2000 – 04/10/2000 | Yes | None | FA |
| 7 | 04/11/2000 – 09/25/2000 | No | None | TN |
| 8 | 09/26/2000 – 02/16/2001 | Yes | None | FA |
| 9 | 02/20/2001 – 09/11/2001 | No | September 11 attack (09/11/2001) | FN |
| 09/12/2001 – 10/09/2002 | No | 2002 stock market downturn (10/09/2002) | FN |
| 10/10/2002 – 02/27/2007 | No | Chinese stock bubble of 2007 (02/27/2007) | FN |
| 02/28/2007 – 10/11/2007 | No | US bear market of 2007 (10/11/2007) | FN |
| 10/12/2007 – 09/15/2008 | No | Lehman Brothers bankruptcy (09/15/2008) | FN |
| 09/16/2008 – 09/17/2008 | No | None | TN |
| 10 | 09/18/2008 – 10/30/2008 | Yes | None | FA |
| 11 | 10/31/2008 – 11/19/2008 | No | None | TN |
| 12 | 11/20/2008 – 01/09/2009 | Yes | None | FA |
| 13 | 01/12/2009 – 06/09/2009 | No | None | TN |
| 14 | 06/10/2009 – 09/21/2009 | Yes | None | FA |
| 15 | 09/22/2009 – 11/27/2009 | No | 2009 Dubai debt standstill (11/27/2009) | FN |
| 11/28/2009 – 03/06/2010 | No | 2010 flash crash (03/06/2010) | FN |
| 03/07/2010 – 06/15/2012 | No | None | TN |
| 16 | 06/18/2012 – 07/30/2012 | Yes | None | FA |
| 17 | 07/31/2012 – 11/07/2014 | No | None | TN |
| 18 | 11/10/2014 – 06/12/2015 | Yes | 2015-2016 Chinese stock market crash (06/12/2015) | EWS (214 days before the crisis) |
| 06/13/2015 – 07/15/2015 | Yes | None | FA |
| 19 | 07/16/2015 – 08/18/2015 | No | 2015-2016 US stock market sell-off (08/18/2015) | FN |
| 08/19/2015 – 12/29/2017 | No | None | TN |

Table 3 Results obtained by using Spearman rho correlations of mean for power spectrum at low frequencies of $L^{1}$-norm time series.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Event number** | **Duration** | $p$ **exceed the threshold (Yes/No)** | **The nearest financial crisis (first day of the crisis)** | **Early warning signal (EWS), false alarm (FA), false positive (FP), or false negative (FN)** |
| 1 | 12/22/1987 – 04/01/1991 | No | None | TN |
| 2 | 04/02/1991 – 11/05/1991 | Yes | None | FA |
| 3 | 11/06/1991 – 03/14/1997 | No | None | TN |
| 4 | 03/17/1997 – 04/29/1997 | Yes | None | FA |
| 5 | 04/30/1997 – 10/27/1997 | No | The global mini crash caused by the 1997 Asian economic crisis (10/27/1997) | FN |
| 10/28/1997 – 12/17/1998 | No | None | TN |
| 6 | 12/18/1998 – 05/12/1999 | Yes | None | FA |
| 7 | 05/13/1999 – 03/08/2000 | No | None | TN |
| 8 | 03/09/2000 – 03/10/2000 | Yes | Dotcom crash (03/10/2000) | EWS (1 day before the crisis) |
| 03/11/2000 – 09/11/2001 | Yes | September 11 attack (09/11/2001) | EWS (549 days before the crisis) |
| 09/12/2001 – 10/03/2001 | Yes | None | FA |
| 9 | 10/04/2001 – 10/09/2002 | No | 2002 stock market downturn (10/09/2002) | FN |
| 10/10/2002 – 02/27/2007 | No | Chinese stock bubble of 2007 (02/27/2007) | FN |
| 02/28/2007 – 10/11/2007 | No | US bear market of 2007 (10/11/2007) | FN |
| 10/12/2007 – 07/16/2008 | No | None | TN |
| 10 | 07/17/2008 – 09/15/2008 | Yes | Lehman Brothers bankruptcy (09/15/2008) | EWS (60 days before the crisis) |
| 09/16/2008 – 09/15/2009 | Yes | None | FA |
| 11 | 09/16/2009 – 11/27/2009 | No | 2009 Dubai debt standstill (11/27/2009) | FN |
| 11/28/2009 – 03/06/2010 | No | 2010 flash crash (03/06/2010) | FN |
| 03/07/2010 – 06/05/2012 | No | None | TN |
| 12 | 06/06/2012 – 07/24/2012 | Yes | None | FA |
| 13 | 07/25/2012 – 11/17/2014 | No | None | TN |
| 14 | 11/18/2014 – 06/12/2015 | Yes | 2015-2016 Chinese stock market crash (06/12/2015) | EWS (206 days before the crisis) |
| 06/13/2015 – 06/25/2015 | Yes | None | FA |
| 15 | 06/26/2015 – 08/18/2015 | No | 2015-2016 US stock market sell-off (08/18/2015) | FN |
| 08/19/2015 – 12/29/2017 | No | None | TN |