

## Supplementary Material

## 1 APPENDIX: GENERALIZED SPECTRAL DENSITIES OF DIFFERENT SPHERICAL CAVITIES

To show that the lineshapes of the generalized spectral densities are sensitive to the material and size of a spherical cavity, we demonstrate the generalized spectral densities of molecules inside a silver spherical cavity and a aluminium spherical cavity, where the radius of the inner sphere of the cavity ( $R_2$ ) can be either 60 nm or 300 nm, and the thickness of the shell is set to be 100 nm. For simplicity, both of the molecules are located at the center of the cavity, with the magnitude of their transition dipole moments given by 1 Debye. The resulting generalized spectral densities are shown in Fig. S1.



**Figure S1.** Frequency dependence of generalized spectral densities inside (A) a silver spherical cavity and (B) an aluminum spherical cavity.  $R_2$  is the radius of the inner sphere of the cavity. The molecules are located at the center of the cavity, and the magnitude of their transition dipole moments is set to be 1 Debye.