# Appendix A

When multiple components coexist in the same space under certain temperature and pressure, when a component reaches equilibrium in the gas-liquid phase, its fugacity in the gas-liquid phase is equal.

 （A1）

Where, *fi*,g is the fugacity of component *i* in the gas phase, Pa; *f*i,L is the fugacity of component *i* in the liquid phase, Pa.

According to Gibbs free energy and fugacity, the fugacity of gas is:

 （A2）

Where, *f* is fugacity, Pa; *G* is Gibbs free energy, J; *V*M is the molar volume of gas, 22.4 m3·mol-1.

Write the above formula as the form of compression factor, and the fugacity coefficient (*f*/*p*) is:

 （A3）

Where *Z* is the compression factor.

The equation of state of gas is a function that describes the relationship between temperature, pressure and volume of gas in a steady state. Peng Robinson (PR) equation of state is selected for calculation in this paper.

 （A4）

 （A5）

 （A6）

 （A7）

 （A8）

Where, *a*、*b* are van der Waals constants, *T*r is the ratio of actual temperature to critical temperature, dimensionless.

The main components of natural gas are methane, ethane, propane, carbon dioxide, etc. the base oil of drilling fluid is mostly white oil or diesel oil. The average carbon number C15 is taken in this calculation. The critical temperature, critical pressure, eccentricity factor and other data of each component are shown in Table A1.

**Tab.A1 The parameters of state equation of major components of natural gas**

|  |  |  |  |
| --- | --- | --- | --- |
| Component name | Critical temperature *T*c /K | Critical pressure *p*c /MPa | Eccentricity factor *ω* /kJ·kg-1·K-1 |
| Methane | 190.564 | 4.5992 | 0.01142 |
| Ethane | 305.322 | 4.8722 | 0.0995 |
| Propane | 369.89 | 4.2512 | 0.1521 |
| Carbon dioxide | 304.1282 | 7.3773 | 0.22394 |
| Water | 647.096 | 22.064 | 0.3443 |
| Base oil(C15) | 706.75 | 1.48 | 0.6918 |

Rewrite the PR equation of state into the form of compressibility factor, see formula (43).

 （A9）

 （A10）

 （A11）

The calculation formula of fugacity coefficient of single component in gas phase or liquid phase can be obtained by integrating formula (A3).

 （A12）

When there are many components in the gas-liquid mixed phase, the fugacity of each component is:

 （A13）

 （A14）

 （A15）

 （A16）

Where, *δij* is the binary interaction coefficient between two components.

The solubility of different components can be calculated from equations (A1) and (A3). The binary interaction coefficients of common natural gas components and drilling fluid based oil are shown in Table A2.

**Tab.A2 Coefficient of binary interaction between different components**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Component name | Methane | Ethane | Propane | Carbon dioxide | Base oil(C15) |
| Methane | 0 | 0.003 | 0.014 | 0.107 | 0.06 |
| Ethane | 0.003 | 0 | 0.001 | 0.132 | 0.04 |
| Propane | 0.014 | 0.001 | 0 | 0.124 | 0.025 |
| Carbon dioxide |  | 0.132 | 0.124 | 0 | 0.0145 |
| Base oil(C15) | 0.06 | 0.04 | 0.025 | 0.0145 | 0 |

# Appendix B

### Formation fluid invasion velocity model

The diffusive gas influx during drilling is mainly driven by the concentration difference. Wang et al. 2017 established a calculation model of diffusive gas influx rate considering the mass transfer process in the process of gas entering the wellbore. The diffusive gas influx rate in the wellbore during drilling fluid circulation is:

 （B1）

Where, *q*g, d is the diffusion gas invasion rate, kg·s-1; wA, c is the saturated concentration of gas, kg·m-3; *D*AB is the diffusion coefficient, m2 s-1; *k'*m is the total mass transfer coefficient, m·s-1; *H*f is the height of reservoir section, m.

Under shut in conditions, the total amount of invaded gas in the wellbore is:

 （B2）

Where, *m*g, d are the total mass of invading gas, kg.

The derivative of equation (B2) shows that the diffusion gas invasion rate under shut in conditions is:

 （B3）

When differential pressure gas invasion occurs, formation fluid penetrates into the formation under the action of differential pressure, and the gas invasion rate is calculated by Sun et al. 2017 model line.

 （B4）

Where, *q*g, p are the diffusion gas invasion rate, kg·s-1; *k* is permeability, m2; *p*e is the formation pressure, Pa; *C*t is the total compression coefficient, Pa-1.

During the calculation of multiphase flow in the wellbore, the appropriate calculation model of gas invasion velocity is selected according to the conditions and working conditions of gas invasion.

### Calculation model of wellbore fluid loss velocity

After gas influx shut in, the wellbore pressure increases gradually with the gas slippage. When the bottom hole pressure is equal to the formation pressure, the formation fluid will no longer invade the wellbore. With the further increase of wellbore pressure, when the bottom hole pressure is greater than the formation pressure, the formation fluid will leak into the formation under the action of differential pressure. Duan et al. 2021 considering the viscosity of drilling fluid and other factors, the filtration rate of drilling fluid in the wellbore is:

 （B5）

 （B6）

 （B7）

 （B8）

 （B9）

Where, *q*L, l is the leakage rate of drilling fluid, m3·s-1; *C* is the total filtration coefficient of drilling fluid, m·min-0.5; *C*1 is the viscosity filtration coefficient of drilling fluid, m·min-0.5; *φ* is the formation porosity; *C*2 is the formation filtration coefficient, m·min-0.5; *C*f is the compressibility coefficient of drilling fluid (or formation fluid), MPa-1. *C*3 is the differential pressure filtration coefficient, m·min-0.5; *C'*3 is the filtration coefficient obtained from the experiment, m·min-0.5.

### Calculation model of gas migration velocity in wellbore

The calculation model of gas migration velocity in the wellbore has been relatively mature. When using the drift flow model to calculate the gas migration velocity in the wellbore, this paper selects the gas drift velocity model established by Ishii et al. 1977 to calculate the gas migration velocity under each flow pattern.

Under the condition of bubbly flow, the drift velocity of gas in the wellbore is:

 （B10）

 （B11）

The drift velocity of gas in the wellbore under slug flow is:

 （B12）

 （B13）

Under the condition of agitated flow, the drift velocity of gas in the wellbore is:

 （B14）

 （B15）

Under the condition of annular fog flow, the gas in the wellbore is a continuous phase, the gas distribution coefficient is close to 1, and the drift velocity and distribution coefficient are respectively：

 （B16）

 （B17）

The flow pattern division and friction coefficient calculation method in the wellbore are calculated by the flow pattern division criteria and friction coefficient calculation model in the literature. If hydrate is formed in deep-water wells, the hydrate formation rate is calculated by the model adopted by Sun et al. 2017.