Supplementary Material

1 ROTATION MATRIX AND ITS DERIVATIVE

The rotation matrix $\mathbf{R}(\boldsymbol{\theta}) \in \mathbb{R}^3$ ($\boldsymbol{\theta} = \theta\mathbf{n}$) that rotates an arbitrary vector in three-dimensional space by an angle θ ($\theta \geq 0$) around the rotation axis $\mathbf{n} = (n_1, n_2, n_3)^\top \in \mathbb{R}^3$ ($\|\mathbf{n}\| = 1$) can be expressed as follows (Géradin and Cardona, 2007):

$$\begin{aligned}\mathbf{R}(\boldsymbol{\theta}) &= \cos\theta\mathbf{I}_3 + (1-\cos\theta)\mathbf{n}\mathbf{n}^\top + \sin\theta[\mathbf{n}]_\times \\ &= \begin{bmatrix} n_1^2(1-\cos\theta)+\cos\theta & n_1n_2(1-\cos\theta)-n_3\sin\theta & n_3n_1(1-\cos\theta)+n_2\sin\theta \\ n_1n_2(1-\cos\theta)+n_3\sin\theta & n_2^2(1-\cos\theta)+\cos\theta & n_2n_3(1-\cos\theta)-n_1\sin\theta \\ n_3n_1(1-\cos\theta)-n_2\sin\theta & n_2n_3(1-\cos\theta)+n_1\sin\theta & n_3^2(1-\cos\theta)+\cos\theta \end{bmatrix}\end{aligned} \tag{S1}$$

If θ and $\mathbf{n}$ are considered as functions of the vector $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3)^\top$, the following equations hold:

$$\theta = \|\boldsymbol{\theta}\| = \sqrt{\theta_1^2+\theta_2^2+\theta_3^2}$$

$$\mathbf{n} = \frac{\boldsymbol{\theta}}{\theta} = \frac{\boldsymbol{\theta}}{\sqrt{\theta_1^2+\theta_2^2+\theta_3^2}}$$

Accordingly, defining $\mathbf{e}_1 = (1,0,0)^\top$, $\mathbf{e}_2 = (0,1,0)^\top$, and $\mathbf{e}_3 = (0,0,1)^\top$, when $\theta > 0$, the first-order derivatives of θ and $\mathbf{n}$ with respect to θ_l ($l = 1,2,3$) are calculated as

$$\frac{\partial\theta}{\partial\theta_l} = \frac{\theta_l}{\sqrt{\theta_1^2+\theta_2^2+\theta_3^2}} = n_l \tag{S2}$$

$$\frac{\partial\mathbf{n}}{\partial\theta_l} = \frac{1}{\theta^2}\left(\theta\frac{\partial\boldsymbol{\theta}}{\partial\theta_l} - \boldsymbol{\theta}\frac{\partial\theta}{\partial\theta_l}\right) = \frac{1}{\theta}(\mathbf{e}_l - n_l\mathbf{n}) \tag{S3}$$

Let δ_{kl} denote Kronecker delta. The derivatives of n_k ($k = 1,2,3$) with respect to θ_l ($l = 1,2,3$) are calculated as

$$\frac{\partial n_k}{\partial\theta_l} = \frac{1}{\theta}(\delta_{kl} - n_kn_l) \tag{S4}$$

In addition, according to Eqs. (S2) and (S3), the following equations hold:

$$\frac{\partial}{\partial\theta_l}\left(\mathbf{n}\mathbf{n}^\top\right) = \frac{1}{\theta}\left(\mathbf{e}_l\mathbf{n}^\top + \mathbf{n}\mathbf{e}_l^\top - 2n_l\mathbf{n}\mathbf{n}^\top\right) \tag{S5}$$

$$\frac{\partial}{\partial\theta_l}[\mathbf{n}]_\times = \frac{1}{\theta}([\mathbf{e}_l]_\times - n_l[\mathbf{n}]_\times) \tag{S6}$$

$$\frac{\partial\cos\theta}{\partial\theta_l} = -n_l\sin\theta \tag{S7}$$

$$\frac{\partial\sin\theta}{\partial\theta_l} = n_l\cos\theta \tag{S8}$$

Therefore, the first- and second-order derivatives of $\mathbf{R}(\boldsymbol{\theta})$ are calculated as follows:

$$\begin{aligned}\frac{\partial \mathbf{R}(\boldsymbol{\theta})}{\partial \theta_l} =& -n_l \sin\theta \mathbf{I}_3 + n_l \left(\sin\theta - 2\frac{1-\cos\theta}{\theta}\right)\mathbf{n}\mathbf{n}^\top \\ &+ n_l \left(\cos\theta - \frac{\sin\theta}{\theta}\right)[\mathbf{n}]_\times + \frac{1-\cos\theta}{\theta}\left(\mathbf{e}_l\mathbf{n}^\top + \mathbf{n}\mathbf{e}_l^\top\right) + \frac{\sin\theta}{\theta}[\mathbf{e}_l]_\times\end{aligned} \tag{S9}$$

$$\begin{aligned}\frac{\partial^2 \mathbf{R}(\boldsymbol{\theta})}{\partial \theta_k \partial \theta_l} =& -\left\{n_k n_l \cos\theta + (\delta_{kl} - n_k n_l)\frac{\sin\theta}{\theta}\right\}\mathbf{I}_3 \\ &+ \left\{n_k n_l \cos\theta + (\delta_{kl} - 5 n_k n_l)\frac{\sin\theta}{\theta} - 2(\delta_{kl} - 4 n_k n_l)\frac{1-\cos\theta}{\theta^2}\right\}\mathbf{n}\mathbf{n}^\top \\ &+ \left\{-n_k n_l \sin\theta + (\delta_{kl} - 3 n_k n_l)\left(\frac{\cos\theta}{\theta} - \frac{\sin\theta}{\theta^2}\right)\right\}[\mathbf{n}]_\times \\ &+ \left(\frac{\sin\theta}{\theta} - 2\frac{1-\cos\theta}{\theta^2}\right)\left\{(n_k\mathbf{e}_l + n_l\mathbf{e}_k)\mathbf{n}^\top + \mathbf{n}(n_k\mathbf{e}_l + n_l\mathbf{e}_k)^\top\right\} \\ &+ \frac{1-\cos\theta}{\theta^2}\left(\mathbf{e}_k\mathbf{e}_l^\top + \mathbf{e}_l\mathbf{e}_k^\top\right) \\ &+ \left(\frac{\cos\theta}{\theta} - \frac{\sin\theta}{\theta^2}\right)(n_k[\mathbf{e}_l]_\times + n_l[\mathbf{e}_k]_\times)\end{aligned} \tag{S10}$$

Because these equations are not valid when $\theta = 0$, the following relations should be utilized:

$$\begin{aligned}&\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta} = 0 \\ &\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1 \\ &\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta^2} = \frac{1}{2} \\ &\lim_{\theta\to 0}\left(\frac{\cos\theta}{\theta} - \frac{\sin\theta}{\theta^2}\right) = 0\end{aligned}$$

Hence, when $\theta = 0$ the first- and second-order derivatives of $\mathbf{R}(\boldsymbol{\theta})$ are calculated as follows:

$$\frac{\partial \mathbf{R}(\mathbf{0})}{\partial \theta_l} = [\mathbf{e}_l]_\times \tag{S11}$$

$$\frac{\partial^2 \mathbf{R}(\boldsymbol{\theta})}{\partial \theta_k \partial \theta_l} = -\delta_{kl}\mathbf{I}_3 + \frac{1}{2}\left(\mathbf{e}_k\mathbf{e}_l^\top + \mathbf{e}_l\mathbf{e}_k^\top\right) \tag{S12}$$

2 DERIVATIVE OF INCOMPATIBILITY VECTOR

The first- and second-order derivatives of the components of the incompatibility vector $\mathbf{C}(\mathbf{W})$ are derived which are in the compatibility matrix $\boldsymbol{\Gamma}(\mathbf{W}) = \nabla_{\mathbf{W}}\mathbf{C}(\mathbf{W})$ and in the second term of the Hessian of the augmented Lagrangian $\mathbf{H}_\mathrm{C}(\mathbf{W},\boldsymbol{\lambda}) = \nabla_{\mathbf{W}}\left(\boldsymbol{\Gamma}(\mathbf{W})^\top\boldsymbol{\lambda}\right)$, respectively. Let k ($k = 1,\dots,n_\mathrm{N}$) denote the index of node connecting to the j-th end of member i ($i = 1,\dots,n_\mathrm{M}$; $j = 1,2$). The non-zero first-order derivatives of the incompatibility vector of translation $\Delta\mathbf{U}_{ij}$ defined in Eq. (10) with respect to the components of the generalized displacement vector $\mathbf{W}$ are calculated as follows for $l = 1,2,3$:

$$\frac{\partial \Delta \mathbf{U}_{ij}}{\partial V_i^{(l)}} = -\mathbf{e}_l \tag{S13}$$

$$\frac{\partial \Delta \mathbf{U}_{ij}}{\partial \Psi_i^{(l)}} = -\frac{\partial \mathbf{R}(\boldsymbol{\Psi}_i)}{\partial \Psi_i^{(l)}} \mathbf{r}_{ij} \tag{S14}$$

$$\frac{\partial \Delta \mathbf{U}_{ij}}{\partial U_k^{(l)}} = \mathbf{e}_l \tag{S15}$$

Note that the derivatives with respect to $\Theta_k^{(l)}$, φ_h ($k = 1,\ldots,n_\mathrm{N}$, $h = 1,\ldots,n_\mathrm{H}$) are equal to zero. In addition, the derivatives with respect to $V_{i'}^{(l)}$, $\Psi_{i'}^{(l)}$, $U_{k'}^{(l)}$ ($i' \neq i$, $k' \neq k$) are also equal to zero. Therefore, the second-order derivative of $\Delta \mathbf{U}_{ij}$ with respect to each component of $\mathbf{W}$ is $\mathbf{0}$ except for the following term:

$$\frac{\partial^2 \Delta \mathbf{U}_{ij}}{\partial \Psi_i^{(l)} \partial \Psi_i^{(l')}} = -\frac{\partial^2 \mathbf{R}(\boldsymbol{\Psi}_i)}{\partial \Psi_i^{(l)} \partial \Psi_i^{(l')}} \mathbf{r}_{ij} \tag{S16}$$

If the member end is rigidly connected to the node, the non-zero first-order derivatives of the incompatibility vector of rotation $\Delta \boldsymbol{\Theta}_{ij}$ defined in the first equation of Eq. (11) with respect to the components of $\mathbf{W}$ are calculated as follows:

$$\frac{\partial \Delta \boldsymbol{\Theta}_{ij}}{\partial \Psi_i^{(l)}} = -\mathbf{e}_l \tag{S17}$$

$$\frac{\partial \Delta \boldsymbol{\Theta}_{ij}}{\partial \Theta_k^{(l)}} = \mathbf{e}_l \tag{S18}$$

Therefore, if the j-th end of member i is rigidly connected to the node, the second-order derivative of $\Delta \boldsymbol{\Theta}_{ij}$ with respect to any component of $\mathbf{W}$ is zero. According to Eqs. (4), (7), and (9), if the j-th end of member i is connected to node k via hinge h, the non-zero first-order derivatives of the incompatibility vector of rotation $\Delta \boldsymbol{\Theta}_{ij} = \boldsymbol{\Phi}_{ij}(\boldsymbol{\Psi}_i, \boldsymbol{\Theta}_k, \varphi_h)$ with respect to the components of $\mathbf{W}$ are calculated as follows:

$$\frac{\partial \Phi_{ij}^{(1)}}{\partial \Psi_i^{(l)}} = \left(\frac{\partial \mathbf{R}(\boldsymbol{\Psi}_i)}{\partial \Psi_i^{(l)}} \mathbf{t}_h^{\langle 1 \rangle} \right) \cdot \left(\mathbf{R}(\boldsymbol{\Theta}_k) \mathbf{t}_h^{\langle 2 \rangle} \right) \tag{S19}$$

$$\frac{\partial \Phi_{ij}^{(2)}}{\partial \Psi_i^{(l)}} = \left(\frac{\partial \mathbf{R}(\boldsymbol{\Psi}_i)}{\partial \Psi_i^{(l)}} \mathbf{t}_h^{\langle 1 \rangle} \right) \cdot \left(\mathbf{R}(\boldsymbol{\Theta}_k) \mathbf{t}_h^{\langle 3 \rangle} \right) \tag{S20}$$

$$\frac{\partial \Phi_{ij}^{(3)}}{\partial \Psi_i^{(l)}} = \left(\frac{\partial \mathbf{R}(\boldsymbol{\Psi}_i)}{\partial \Psi_i^{(l)}} \mathbf{t}_h^{\langle 2 \rangle} \right) \cdot \left\{ \sin \varphi_h \left(\mathbf{R}(\boldsymbol{\Theta}_k) \mathbf{t}_h^{\langle 2 \rangle} \right) + \cos \varphi_h \left(\mathbf{R}(\boldsymbol{\Theta}_k) \mathbf{t}_h^{\langle 3 \rangle} \right) \right\} \tag{S21}$$

$$\frac{\partial \Phi_{ij}^{(1)}}{\partial \Theta_k^{(l)}} = \left(\mathbf{R}(\boldsymbol{\Psi}_i) \mathbf{t}_h^{\langle 1 \rangle} \right) \cdot \left(\frac{\partial \mathbf{R}(\boldsymbol{\Theta}_k)}{\partial \Theta_k^{(l)}} \mathbf{t}_h^{\langle 2 \rangle} \right) \tag{S22}$$

$$\frac{\partial \Phi_{ij}^{(2)}}{\partial \Theta_k^{(l)}} = \left(\mathbf{R}(\boldsymbol{\Psi}_i) \mathbf{t}_h^{\langle 1 \rangle} \right) \cdot \left(\frac{\partial \mathbf{R}(\boldsymbol{\Theta}_k)}{\partial \Theta_k^{(l)}} \mathbf{t}_h^{\langle 3 \rangle} \right) \tag{S23}$$

$$\frac{\partial \Phi_{ij}^{(3)}}{\partial \Theta_k^{(l)}} = \left(\mathbf{R}(\boldsymbol{\Psi}_i)\mathbf{t}_h^{\langle 2\rangle}\right) \cdot \left\{ \sin\varphi_h \left(\frac{\partial \mathbf{R}(\boldsymbol{\Theta}_k)}{\partial \Theta_k^{(l)}} \mathbf{t}_h^{\langle 2\rangle} \right) + \cos\varphi_h \left(\frac{\partial \mathbf{R}(\boldsymbol{\Theta}_k)}{\partial \Theta_k^{(l)}} \mathbf{t}_h^{\langle 3\rangle} \right) \right\} \tag{S24}$$

$$\frac{\partial \Phi_{ij}^{(3)}}{\partial \varphi_{h_{ij}}} = \left(\mathbf{R}(\boldsymbol{\Psi}_i)\mathbf{t}_h^{\langle 2\rangle}\right) \cdot \left\{ \cos\varphi_h \left(\mathbf{R}(\boldsymbol{\Theta}_k) \mathbf{t}_h^{\langle 2\rangle} \right) - \sin\varphi_h \left(\mathbf{R}(\boldsymbol{\Theta}_k) \mathbf{t}_h^{\langle 3\rangle} \right) \right\} \tag{S25}$$

Therefore, the second-order derivatives of $\boldsymbol{\Phi}_{ij}$ are $\mathbf{0}$ except for the following terms:

$$\frac{\partial^2 \Phi_{ij}^{(1)}}{\partial \Psi_i^{(l)} \partial \Psi_i^{(l')}} = \left(\frac{\partial^2 \mathbf{R}(\boldsymbol{\Psi}_i)}{\partial \Psi_i^{(l)} \partial \Psi_i^{(l')}} \mathbf{t}_h^{\langle 1\rangle} \right) \cdot \left(\mathbf{R}(\boldsymbol{\Theta}_k) \mathbf{t}_h^{\langle 2\rangle} \right) \tag{S26}$$

$$\frac{\partial^2 \Phi_{ij}^{(2)}}{\partial \Psi_i^{(l)} \partial \Psi_i^{(l')}} = \left(\frac{\partial^2 \mathbf{R}(\boldsymbol{\Psi}_i)}{\partial \Psi_i^{(l)} \partial \Psi_i^{(l')}} \mathbf{t}_h^{\langle 1\rangle} \right) \cdot \left(\mathbf{R}(\boldsymbol{\Theta}_k) \mathbf{t}_h^{\langle 3\rangle} \right) \tag{S27}$$

$$\frac{\partial^2 \Phi_{ij}^{(3)}}{\partial \Psi_i^{(l)} \partial \Psi_i^{(l')}} = \left(\frac{\partial^2 \mathbf{R}(\boldsymbol{\Psi}_i)}{\partial \Psi_i^{(l)} \partial \Psi_i^{(l')}} \mathbf{t}_h^{\langle 2\rangle} \right) \cdot \left\{ \sin\varphi_h \left(\mathbf{R}(\boldsymbol{\Theta}_k) \mathbf{t}_h^{\langle 2\rangle} \right) + \cos\varphi_h \left(\mathbf{R}(\boldsymbol{\Theta}_k) \mathbf{t}_h^{\langle 3\rangle} \right) \right\} \tag{S28}$$

$$\frac{\partial^2 \Phi_{ij}^{(1)}}{\partial \Theta_k^{(l)} \partial \Theta_k^{(l')}} = \left(\mathbf{R}(\boldsymbol{\Psi}_i)\mathbf{t}_h^{\langle 1\rangle}\right) \cdot \left(\frac{\partial^2 \mathbf{R}(\boldsymbol{\Theta}_k)}{\partial \Theta_k^{(l)} \partial \Theta_k^{(l')}} \mathbf{t}_h^{\langle 2\rangle} \right) \tag{S29}$$

$$\frac{\partial^2 \Phi_{ij}^{(2)}}{\partial \Theta_k^{(l)} \partial \Theta_k^{(l')}} = \left(\mathbf{R}(\boldsymbol{\Psi}_i)\mathbf{t}_h^{\langle 1\rangle}\right) \cdot \left(\frac{\partial^2 \mathbf{R}(\boldsymbol{\Theta}_k)}{\partial \Theta_k^{(l)} \partial \Theta_k^{(l')}} \mathbf{t}_h^{\langle 3\rangle} \right) \tag{S30}$$

$$\frac{\partial^2 \Phi_{ij}^{(3)}}{\partial \Theta_k^{(l)} \partial \Theta_k^{(l')}} = \left(\mathbf{R}(\boldsymbol{\Psi}_i)\mathbf{t}_h^{\langle 2\rangle}\right) \cdot \left\{ \sin\varphi_h \left(\frac{\partial^2 \mathbf{R}(\boldsymbol{\Theta}_k)}{\partial \Theta_k^{(l)} \partial \Theta_k^{(l')}} \mathbf{t}_h^{\langle 2\rangle} \right) + \cos\varphi_h \left(\frac{\partial^2 \mathbf{R}(\boldsymbol{\Theta}_k)}{\partial \Theta_k^{(l)} \partial \Theta_k^{(l')}} \mathbf{t}_h^{\langle 3\rangle} \right) \right\} \tag{S31}$$

$$\frac{\partial^2 \Phi_{ij}^{(3)}}{\partial \varphi_h^2} = -\left(\mathbf{R}(\boldsymbol{\Psi}_i)\mathbf{t}_h^{\langle 2\rangle}\right) \cdot \left\{ \sin\varphi_h \left(\mathbf{R}(\boldsymbol{\Theta}_k) \mathbf{t}_h^{\langle 2\rangle} \right) + \cos\varphi_h \left(\mathbf{R}(\boldsymbol{\Theta}_k) \mathbf{t}_h^{\langle 3\rangle} \right) \right\} = -\Phi_{ij}^{(3)} \tag{S32}$$

$$\frac{\partial^2 \Phi_{ij}^{(1)}}{\partial \Psi_i^{(l)} \partial \Theta_k^{(l')}} = \left(\frac{\partial \mathbf{R}(\boldsymbol{\Psi}_i)}{\partial \Psi_i^{(l)}} \mathbf{t}_h^{\langle 1\rangle} \right) \cdot \left(\frac{\partial \mathbf{R}(\boldsymbol{\Theta}_k)}{\partial \Theta_k^{(l')}} \mathbf{t}_h^{\langle 2\rangle} \right) \tag{S33}$$

$$\frac{\partial^2 \Phi_{ij}^{(2)}}{\partial \Psi_i^{(l)} \partial \Theta_k^{(l')}} = \left(\frac{\partial \mathbf{R}(\boldsymbol{\Psi}_i)}{\partial \Psi_i^{(l)}} \mathbf{t}_h^{\langle 1\rangle} \right) \cdot \left(\frac{\partial \mathbf{R}(\boldsymbol{\Theta}_k)}{\partial \Theta_k^{(l')}} \mathbf{t}_h^{\langle 3\rangle} \right) \tag{S34}$$

$$\frac{\partial^2 \Phi_{ij}^{(3)}}{\partial \Psi_i^{(l)} \partial \Theta_k^{(l')}} = \left(\frac{\partial \mathbf{R}(\boldsymbol{\Psi}_i)}{\partial \Psi_i^{(l)}} \mathbf{t}_h^{\langle 2\rangle} \right) \cdot \left\{ \sin\varphi_h \left(\frac{\partial \mathbf{R}(\boldsymbol{\Theta}_k)}{\partial \Theta_k^{(l')}} \mathbf{t}_h^{\langle 2\rangle} \right) + \cos\varphi_h \left(\frac{\partial \mathbf{R}(\boldsymbol{\Theta}_k)}{\partial \Theta_k^{(l')}} \mathbf{t}_h^{\langle 3\rangle} \right) \right\} \tag{S35}$$

$$\frac{\partial^2 \Phi_{ij}^{(3)}}{\partial \Psi_i^{(l)} \partial \varphi_h} = \left(\frac{\partial \mathbf{R}(\boldsymbol{\Psi}_i)}{\partial \Psi_i^{(l)}} \mathbf{t}_h^{\langle 2\rangle} \right) \cdot \left\{ \cos\varphi_h \left(\mathbf{R}(\boldsymbol{\Theta}_k) \mathbf{t}_h^{\langle 2\rangle} \right) - \sin\varphi_h \left(\mathbf{R}(\boldsymbol{\Theta}_k) \mathbf{t}_h^{\langle 3\rangle} \right) \right\} \tag{S36}$$

$$\frac{\partial^2 \Phi_{ij}^{(3)}}{\partial \Theta_k^{(l)} \partial \varphi_h} = \left(\mathbf{R}(\boldsymbol{\Psi}_i)\mathbf{t}_h^{\langle 2\rangle}\right) \cdot \left\{ \cos\varphi_h \left(\frac{\partial \mathbf{R}(\boldsymbol{\Theta}_k)}{\partial \Theta_k^{(l')}} \mathbf{t}_h^{\langle 2\rangle} \right) - \sin\varphi_h \left(\frac{\partial \mathbf{R}(\boldsymbol{\Theta}_k)}{\partial \Theta_k^{(l')}} \mathbf{t}_h^{\langle 3\rangle} \right) \right\} \tag{S37}$$

3 AUGMENTED LAGRANGIAN METHOD

Let $\mathbf{W}^k$, $\boldsymbol{\lambda}^k$, and s_k denote the values of the generalized displacement $\mathbf{W}$, the Lagrange multiplier $\boldsymbol{\lambda}$, and the penalty parameter s in the k-th iteration of the process of the augmented Lagrangian method, respectively. The function $G(\mathbf{W}^k)$ is defined as

$$G(\mathbf{W}^k) = \frac{1}{2}\mathbf{C}(\mathbf{W}^k)^\top \mathbf{C}(\mathbf{W}^k)$$

In addition, the binary function $O(\mathbf{W}^k, \boldsymbol{\lambda}^k)$ that indicates convergence of the optimization problem (21) with sufficient accuracy is defined as

$$O(\mathbf{W}^k, \boldsymbol{\lambda}^k) = \begin{cases} 1 & \text{(Optimization process is converged)} \\ 0 & \text{(Otherwise)} \end{cases}$$

Algorithm 1 presents the process of obtaining the solution $\mathbf{W}^*$ and the corresponding Lagrange multiplier $\boldsymbol{\lambda}^*$ of problem (15), using the augmented Lagrangian method (Birgin and Martínez, 2012). The load factor Λ is given, and the process terminates when the largest absolute value in the components of the incompatibility vector $\mathbf{C}(\mathbf{W})$, represented as follows, is less than or equal to $\epsilon_{\mathrm{tol}} > 0$:

$$\|\mathbf{C}(\mathbf{W})\|_\infty = \max_i |C_i(\mathbf{W}^k)|$$

where $C_i(\mathbf{W})$ $(i = 1, \ldots, n_{\mathrm{C}})$ is the i-th component of $\mathbf{C}(\mathbf{W})$. In the numerical examples of this study, the parameters in Algorithm 1 for updating the penalty parameter are set as $\bar{s} = 1 \times 10^4$, $s_{\min} = 1 \times 10^{-16}$, $s_{\max} = 1 \times 10^6$, $\gamma = 1.2$, and $\alpha = 0.5$.

REFERENCES

Birgin, E. G. and Martínez, J. M. (2012). Augmented Lagrangian method with nonmonotone penalty parameters for constrained optimization. *Computational Optimization and Applications* 51, 941–965. doi:10.1007/s10589-011-9396-0

Géradin, M. and Cardona, A. (2007). *Flexible Multibody Dynamics: A Finite Element Approach* (Wiley)

Algorithm 1 Augmented Lagrangian method

Input: $\mathbf{W}^0 \in \Omega, \boldsymbol{\lambda}^1, \epsilon_{\text{tol}} > 0, \bar{s} > 0, 0 < s_{\min} < s_{\max}, \gamma > 1, 0 \leq \alpha \leq 1$

Output: $\mathbf{W}^* = \mathbf{W}^k$, $\boldsymbol{\lambda}^* = \boldsymbol{\lambda}^k$

$k \leftarrow 1$, $\beta_k \leftarrow 0$,
$s_k \leftarrow \min\left\{\max\left\{s_{\min}, \bar{s}\frac{\max\{1,\Pi(\mathbf{W}^0)\}}{\max\{1,G(\mathbf{W}^0)\}}\right\}, s_{\max}\right\}$

while $O(\mathbf{W}^k, \boldsymbol{\lambda}^k) = 0$ and $\|\mathbf{C}(\mathbf{W}^k)\|_\infty > \epsilon_{\text{tol}}$ **do**

  Solve Problem(21) with $\boldsymbol{\lambda} = \boldsymbol{\lambda}^k$, and let the solution be $\mathbf{W}^k$.

  $\boldsymbol{\lambda}^{k+1} \leftarrow \boldsymbol{\lambda}^k + s_k \mathbf{C}(\mathbf{W}^k)$

  if $k = 1$ **then**

    $\beta_{k+1} \leftarrow \beta_k$,
    $s_{k+1} \leftarrow \min\left\{\max\left\{s_{\min}, \bar{s}\frac{\max\{1,\Pi(\mathbf{W}^k)\}}{\max\{1,G(\mathbf{W}^k)\}}\right\}, s_{\max}\right\}$

  else if $\|\mathbf{C}(\mathbf{W}^k)\|_\infty \leq \epsilon_{\text{tol}}$ **then**

    if $k \geq 3$ and $\|\mathbf{C}(\mathbf{W}^{k-1})\|_\infty \leq \epsilon_{\text{tol}}$ and $O(\mathbf{W}^{k-1}, \boldsymbol{\lambda}^{k-1}) = O(\mathbf{W}^k, \boldsymbol{\lambda}^k) = 0$ **then**

      $\beta_{k+1} \leftarrow \beta_k + 1$,
      $s_a \leftarrow \min\{\gamma^{\beta_k} s_{\min}, 1\}$, $s_b \leftarrow \max\{\gamma^{-\beta_k} s_{\max}, 1\}$,
      $s_{k+1} \leftarrow \min\left\{\max\left\{s_a, \bar{s}\frac{\max\{1,\Pi(\mathbf{W}^k)\}}{\max\{1,G(\mathbf{W}^k)\}}\right\}, s_b, s_k\right\}$

    else

      $\beta_{k+1} \leftarrow \beta_k$, $s_{k+1} \leftarrow c_k$

    end if

  else

    $\beta_{k+1} \leftarrow \beta_k$

    if $\|\mathbf{C}(\mathbf{W}^k)\| \leq \alpha \|\mathbf{C}(\mathbf{W}^{k-1})\|$ **then**

      $s_{k+1} \leftarrow s_k$

    else

      $s_{k+1} = \max\{\gamma s_k, \gamma^{\beta_k} s_{\min}\}$

    end if

  end if

  $k \leftarrow k + 1$

end while